INVESTIGATING SCHOOL MATHEMATICS

TEACHERS' EDITION

LET'S DO

LET'S TALK

LET'S USE

	Kindergarten	Book One	Book Two
		Units a, b, c, and d	**Units e, f, g, and h**
A Sets, Logical Reasoning, and Patterns	Concept of sets: P13–14 Comparisons of sizes: P1–8, P11–12, P15 Similarities and differences: P17–32 Patterns: P9–10, P16 Maze: P32	Comparing sets: a1–9 Equivalent sets: a5–8 Cardinal number of a set: a13–14 Empty set: a19 Union of sets and addition: b1–13, b25–32, b41 Sets and subtraction: b15–23, b33–37, b45 Sets of ten: c1–4 Skip counting: d54, d62 Logic: a59–60, d42 Patterns: c14	Cardinal number of a set: e1–7 Maze: e40, g22 Sets and addition: e43 Sets and subtraction: e47 Counting sequences: e15, 62; f36; g6, 11, 13; h38 Informal logic: e52, 57; f55; g12, 34, 37; h20, 50 Patterns: f10 Attribute pieces: g23 Puzzle problems: g34, g62, h8 Multiplication and sets: g50–52, g55, g57
B Numeration and Place Value	More than and less than concept: P33–37, P43–47 Matching, one-to-one correspondence: P37–42 Numerals and numbers 0 to 10: R1–7, R11–12, R17–32 Recognition of number of a set: R8–10, R13–16, R37–38	Introducing ten: c1–4 Grouping by tens: c5–13 Counting to 99: c15–26, d44 One more than: a9, a27, 29, 31, 33, 35 Greater than and less than: a41–49, d55–61 Numeral writing: a15–23, a27–36	Numeral writing: e3 2- and 3-digit numerals: e7–11, f13–18, g1–13 Counting to 99: e13–17, h62 Greater than and less than: e17–23, g12 Roman numerals: e32–39 Expanded notation: f49, g16–17 Regrouping: h33, h41–44, h52–56 Fraction numerals: h21–29
C Addition and Subtraction of Whole Numbers	Addition combinations 1 to 10 (intuitive): R32–36, R39–44 Subtraction related to sums of 10 or less (intuitive): R45–46 Order of numbers 1 to 10: R48	Addition and subtraction facts, sums through 9: b1–51, c35–45 Zero principle for addition: b9 Inverse relation (+ and −): c57–60, d49–52 Missing addends: c53–56, d45–53 Sums of ten: d1–11 Addition and subtraction facts, sums greater than 10: d13–23, d35–43 Commutative (order) principle: d41, d51	Addition and subtraction facts, sums of 10 or less: e41–51, e59 Missing addends: e53–61 Inverse relation (+ and −): e57–58, h13–14 Basic principles: f1–9 Addition and subtraction facts, sums to 18: f11–35, g35–47, h1–19 Addition and subtraction without regrouping: f47–61, g15–21 Addition and subtraction with regrouping: h31–61
D Multiplication and Division of Whole Numbers			Multiplication concepts: g49–61
E Fractional Numbers		Introduction to fractions, $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$: d25–33	Halves, thirds, and fourths: h21–29
F Problem Solving and Applications		Solving money picture stories: c47–51 Discussions: a2, 14, 26, 42; b2, 16, 26, 40; c16, 36, 46, 54; d2, 14, 26, 46, 56 Picture story for zero: a20	Money problems: e25–31, g14 Addition and subtraction problems: e60; f22, 32, 34, 54, 60; g20, 46, 60; h16, 18, 45, 58, 60 Multiplication problems: g59–60 Addition and subtraction problems with regrouping: h45, h58, h60 Discussions: e2, 26, 42; f48
G Estimation			Introductory concepts leading to regrouping: h31, h39
H Number Theory		Skip counting sequences: d54 Skip counting even numbers: d62	Odd and even numbers: e12 Skip counting: e62, f36, h38 Nomograph addition: f24 Square numbers: f62
I Measurement		Concept of length: b53–57 Centimetres: b57–61 Length of paths: b62, d34 Readiness for area concepts: c62 Readiness for volume concepts: d24	Time: e33–39 Centimetres: f37–40 Perimeter: f41–42 Area: f43–44 Litres: f46 Volume: g48
J Geometry, Number Line, and Graphs	Recognition of simple shapes: P17, P27, P28, P30, P48	Number line: a47; b11, 21, 29, 35, 43, 47; d5, 19–22, 48, 54 Recognition of basic shapes: a12, a50–54 Open and closed curves: a55 Segments and paths: a57 Same size figures: a59–61 Symmetry: d25	Number line: e6, 46, 49; f12, 16, 21, 29; g54, 58; h35 Triangles, squares, rectangles, circles: g23–26 Segments: g27–28 Congruence: g28–30 Similarity: g31–32 Graphing: h48
K Special Topics		Telling time: c27–33 Money: c45–51 Let's have fun: a12, 24, 40, 50, 62; b14, 24, 38, 52, 62; c14, 26, 34, 44, 52, 62; d12, 24, 34, 44, 54, 62	Time: e33–39 Roman numerals: e32–39 Money: e25–31, g14 Magic squares: h8 Let's have fun: e12, 24, 32, 40, 52, 62; f10, 24, 36, 46, 62; g14, 22, 34, 48, 62; h8, 20, 30, 38, 48, 62

[Continued on inside back cover

Teachers' Edition to accompany

Investigating School Mathematics

ROBERT E. EICHOLZ

PHARES G. O'DAFFER

CHARLES R. FLEENOR

Collaborator, Teachers' Edition
THERESA BURKE

Collaborators, Reference Material and Metrication
JOHN BATES
DONALD IRWIN
J. NORMAN C. SHARP
JAMES SHERRILL

ADDISON-WESLEY (CANADA) LTD.
DON MILLS, ONTARIO • READING, MASSACHUSETTS • MENLO PARK, CALIFORNIA • LONDON

This book is in the
ADDISON-WESLEY MATHEMATICS SERIES FOR GRADES K–8

Investigating School Mathematics for Grades K–8

School Mathematics I and II for Grades 7 and 8

Success with Mathematics 1 and 2 for Grades 7 and 8

Supplementary Materials available for use with
ADDISON-WESLEY MATHEMATICS SERIES FOR GRADES K–8

Big Book Primer • Duplicator Masters for Grades 1–8
Supplementary Workbooks for Grades 3–8
Achievement Tests for Grades 1–8 • Diagnostic Tests for Grades 3–8
Getting Ready Books • Mathematics Manipulative Aids
Enrichment Booklets for Grades 1–2 Practice Booklets for Grades 1–2

ISBN 0-201-01130-1

ABCDEFGH 7987654

Foreword

The *Investigating School Mathematics* series co-ordinates the precise concepts of modern mathematics with an approach that stimulates the child to actively participate in his own learning experiences. The series provides for the necessary mastery of basic number skills, and presents the material in a way that emphasizes the exciting, creative nature of mathematics. As the child becomes involved in exciting explorations and investigations, the structure and beauty of mathematics unfolds. The children are encouraged to investigate and discover ideas for themselves, to look for interesting patterns and relationships, and to develop their own generalizations. New and fascinating topics are explored not solely for their mathematical value, but also because they stimulate interest and motivate children to put forth their best efforts.

In our view, the development of a sound mathematical structure need not be hindered by an exciting, activity-oriented approach. Rather, the activity approach can and should reinforce the child's experiences as he investigates mathematical topics in an orderly, structured fashion. The same, sound mathematical structure that was called "modern" in the 1960's is present in *Investigating School Mathematics*. The important difference in this new series lies in its approach. The child learns through continual active participation in activities and investigations that lead to the unfolding and discovery of each new idea.

As each new concept unfolds, the child is given an opportunity to investigate the ideas by using a wide variety of manipulative materials and activities. Then, through guided discussion, he is led to a deeper understanding of the ideas and their relation to the overall structure of mathematics. Following the investigation and discussion, he is provided with sufficient problem-solving practice to develop speed and accuracy.

The *Investigating School Mathematics* series is unprecedented in its careful provision for individual differences. Throughout each text, the child is challenged to do what he *can* do, not what someone else *thinks* he can do. Each child has the opportunity to experience individual success in an environment that stresses co-operation and communication rather than competition. This careful provision for individual differences makes the *Investigating School Mathematics* series unusually adaptable to such diverse teaching situations as ungraded schools, individual or small-group instruction, or whole-class instruction.

The essence of the *Investigating School Mathematics* series is reflected in the beliefs to which we are committed: that there are fundamental mathematical concepts which can be isolated and set forth with sharpness and clarity; that these concepts, when truly understood, provide powerful tools for extending knowledge; that children of every level should be encouraged to actively participate, to think, to question, and to seek understanding; that, although a certain body of knowledge must be passed on to each generation from preceding generations, the individual creativity of each new generation must not be stifled by pedagogy which forces upon its pupils patterns of thought which have served us well in the past but which may be inadequate for the future.

Mathematics can be successfully taught in this spirit. At every stage in the learning of mathematics, the discovery of new relationships can be a delight. It is in this spirit that *Investigating School Mathematics* has been written.

The authors wish to express their appreciation to Ball State University and to the Educational Research Council of Greater Cleveland, where many of the ideas were generated and tested for the *Elementary School Mathematics* series, which served as forerunner of *Investigating School Mathematics;* to Edith Biggs and the Nuffield Project in England, for their leadership in bringing the activity-oriented laboratory approach into prominence; to Mrs. Nancy Hildebrand, whose contributions to the teachers' manuals for *Elementary School Mathematics* are still reflected in this manual; to Theresa Burke, who assisted in the preparation of this manual by bringing, from a wealth of classroom experience, many of the activities and teaching suggestions found in each lesson; and finally, to the many teachers and children who have proved that studying mathematics can be an exciting and stimulating experience in the elementary school.

*Contents of Learning Units **P** and **R***

Mathematics of the Primer Program

A significant number of mathematical concepts are introduced in the Primer program for *Investigating School Mathematics*. Because of the age of the children, however, the depth of coverage of these topics is often quite limited. The following list indicates the major concepts introduced in the Primer program.

Classification and Sorting
One-to-One Matching
Sets and Subsets
Union and Intersection of Sets
Order in Counting
Introductory Concepts of Addition and Subtraction
Introductory Geometric Concepts
Basic Concepts of Measurement

This list of mathematical concepts may seem formidable for the Primer program, but it should be understood that many of the concepts are introduced only on an intuitive level. For example, the concept of the union of two sets is vital to the idea of addition of numbers, so the child is asked to identify the number of two specific sets and then the number in all. Thus, using the idea of the union of two disjoint sets is preparation for a later understanding of the concept of addition. This is done easily without ever mentioning the word union, or addition, or any of the symbolism involved.

The first module in learning Unit P (the Yellow Module) is concerned primarily with comparisons, such as larger-smaller, tallest-shortest, longest-shortest, alike-different. The last lesson in this module is concerned with introductory concepts of the idea of a set group or collection of objects. This lesson is given in preparation for the set concepts which will be developed in the next module. The second module in Learning Unit P (the Orange Module) has to do with classification and sorting. The children are given an opportunity to sort and classify according to similarities and differences, such as "use," "external attributes," "color," "size," "shape." The third module in Learning Unit P (the Red Module) deals with primitive number concepts and introduces one-to-one matching and the concept of more-and-less on a pre-number basis. Emphasis is also given to the concept of *one more* in preparation for introduction of the numbers 0 through 10 in Learning Unit R.

The Yellow Module of Learning Unit R introduces the numbers 0 through 5. By using the pre-number work from the third module in the first learning unit, the numbers 0 through 5 are introduced as concepts associated with equivalent sets. That is, by matching two sets of three, the child begins to see that there is something common between these sets and it is that idea which we learn to refer to as "threeness" or "the number 3." The numbers 1 through 5 are introduced prior to the introduction of the number 0. This is done simply because the basic concept of the number 0 is more abstract to the children than that of the numbers 1 through 5. The Orange Module of Learning Unit R introduces the numbers 6 through 10. Techniques similar to those used for introducing the numbers 0 through 5 are used for the numbers 6 through 10. Again, the child uses his pre-number experiences and works with the idea of equivalent sets to recognize the sameness between sets that have the same number. The Red Module of Learning Unit R deals with intuitive concepts of addition and subtraction. The child is given a number of opportunities to identify the number of two different sets and then the number in all. Much of the work in this third module has to do with the basic concept of addition, although it could be interpreted in terms of subtraction depending upon which numbers one thinks about first. The concept of subtraction is introduced in the last lesson of the module through the use of first identifying the number in all and then identifying the number of each of two designated subsets.

Teachers' Bibliography

To gain a broader comprehension of the overall mathematical development in the *Investigating School Mathematics* series, the following books should be extremely helpful.

Biggs, E., and J. MacLean, *Freedom to Learn: An Active Learning Approach to Mathematics* (Don Mills, Ont.: Addison-Wesley, 1969).

Boyer, C. B., *A History of Mathematics* (New York: Wiley, 1968).

Copeland, R., *How Children Learn Mathematics: Teaching Implications of Piaget's Research* (New York: Macmillan, 1970).

Dienes, Z. P., *Building Up Mathematics* (London: Hutchinson Educational Ltd.).

Elliott, H. A., J. MacLean, and J. Jordan, *Geometry in the Classroom: New Concepts and Methods* (Toronto, Ont.: Holt, Rinehart and Winston of Canada, 1968).

Forbes, J., and R. Eicholz, *Mathematics for Elementary Teachers* (Menlo Park, Calif.: Addison-Wesley, 1971).

Huff, M. E., D. A. Irwin, *Activities in Geometry* (Don Mills, Ont.: Addison-Wesley 1973).

Lavatelli, C. S., *Piaget's Theory Applied to an Early Childhood Curriculum* (New York: Learning Research Associates, 1970).

Members of the Association of Teachers of Mathematics, *Notes on Mathematics in Primary Schools* (New York: Cambridge University Press, 1967).

National Council of Teachers of Mathematics, *Insights into Modern Mathematics* (23rd Yearbook, 1957); *The Growth of Mathematical Ideas, Grades K-12* (24th Yearbook, 1959); *Enrichment Mathematics for the Grades* (27th Yearbook, 1963); *Topics in Mathematics for Elementary School Teachers* (29th Yearbook, 1964); *More Topics in Mathematics for*

Elementary School Teachers (30th Yearbook, 1969) (Washington D.C.: National Council of Teachers of Mathematics).

Newman, J. R., *The World of Mathematics* (New York: Simon and Schuster, 1956).

Nuffield Mathematics Project, *I Do, and I Understand* (New York: Wiley, 1967).

School Mathematics Study Group, *Studies in Mathematics, Volume IX, A Brief Course in Mathematics for Elementary School Teachers,* Revised Edition (Stanford University, 1963).

The Schools Council, *Mathematics in Primary Schools,* Curriculum Bulletin No. 1 (Available from Selective Educational Equipment, Newton, Mass., 1964).

Williams, E. M., and H. Shuard, *Elementary Mathematics Today, Grades 1–8* (Menlo Park, Calif.: Addison-Wesley, 1972).

Design Features of the Primer

The Primer program is organized into two 48-page learning units. Each learning unit is divided into three 16-page modules. The pages of the first learning unit are identified by the letter P (pre-number activities) and those for the second unit are identified by the letter R (readiness activities). The first module of each learning unit is color coded yellow in the upper outside corner of each page, while the second and third modules are color coded orange and red, respectively.

Each lesson of the Primer is presented on the front and back of a sheet, rather than on facing pages. Thus, the sheets may either be torn out and used one at a time or used in the book itself.

The first lesson of each module is made up of an investigation page and a discussion page. The investigation page is titled "Let's do" and the discussion page "Let's talk." The investigation is designed to provide the child with an opportunity to explore the germ of the concept developed throughout the module. This part of the lesson should be child-centred insofar as possible. That is, the children should be given considerable freedom to explore and investigate the concept involved. Following the investigation phase of the lesson, the reverse side of the sheet gives the children an opportunity to discuss the ideas and allows the teacher to further develop the ideas and prepare the children for the various utilization phases of the module that follow. The subsequent lessons present children with the opportunity to practice, develop, and extend the concepts and skills of the module.

Each lesson has detailed teacher commentary to provide objectives, pre-book activities, and follow-up suggestions. The last lesson in each module is made up of two parts. The front side of the sheet, titled "Show you know," can be used as a module review and, possibly, as an evaluation instrument to determine the child's understanding of the concepts and ideas presented in the module. The back side of the sheet, titled "Let's have fun," is intended as an interesting mathematical side trip and should be treated with a light touch, in the spirit of having fun with mathematics.

The investigation phase of the first lesson is distinguished by a blue band on the left side of the page, and the discussion phase of the lesson is distinguished by a green band on the right side of the page. Each key lesson in the Primer features demonstration art at the top of the page to assist you in introducing the activity required on that page. This art also serves to remind the children of the particular task to be done on the page.

General Approach and Teaching Suggestions

The pre-text section of this manual, "Activities to Develop Mathematical Awareness," is suggested for use with individuals and small groups. Children should be encouraged to use their natural instincts to explore and make decisions for themselves. Select or devise those activities which will be most appealing to the particular children with whom you are working. Their interest is of primary importance if they are to enjoy these premathematical experiences. Some of these premathematical experiences are incorporated into the primer text itself, namely in the first unit. This material might be used intermittently with the pre-text activities.

All the pages in the Primer student text are reproduced in this Teachers' Edition in full color, with annotated answers. Notes for each module of both units are provided to orient you to the contents and objectives of the module. Detailed page lesson notes and suggested activities appear next to each pair of pages (or lesson) in each module. The pre-book activity is not only a preparation for the lesson but is often investigative in nature. Thus, it corresponds to the investigation phase of the child-centred teaching strategy used throughout the *Investigating School Mathematics* program. The pages themselves provide for the discussion and utilization phases of this strategy, and the follow-ups provide suggestions for extensions of the concepts treated in each lesson. Adjust the suggested activities to the needs of the children and to your own most successful teaching techniques. There will be times when you will want to adapt one of your own favorite activities for the purpose of pre-book activity or follow-up. The notes are supplied as suggestions and in no way are intended to stifle your own effective teaching methods and creative efforts. Many other excellent ideas for activity-centred learning for early childhood may be found in the book *Workjobs,* by Mary D. Lorton (Addison-Wesley, 1972).

Lesson Schedule

When the Primer is to be used in a kindergarten program, the games and activities suggested in the pre-text section of this Teachers' Edition are intended to be used during the first three or four months of the school year. During this time, the suggested activities will help the

children become adjusted to the school environment and to the degree of formality conducive to participation in group activities. If the Primer is used as a part of the Grade 1 program, this readiness period should be shortened.

When the children start to work in the Primer, plan the activities so that the children cover about one pair of pages every other day. At the Grade 1 level, you should plan to cover at *least* one pair of pages per day when you have started the text material. If you then move into Book 1, some of the material there may be omitted, depending upon the maturity of your pupils and the general aims of the curriculum in your local educational system.

Vocabulary

Since there are virtually no words in the Primer, the reading vocabulary problem is completely eliminated. All necessary instructions must be given orally by the teacher.

The mathematical terms most frequently used within each module are given in the introductory notes preceding the module. It will be necessary for you to study this list in order to use these terms naturally as you teach the lessons.

The vocabulary is not intended as a list for the children to memorize. If you use the words in their proper contexts, the children should gain an intuitive understanding of their meanings. They should then use the words naturally as their understanding develops and as they begin to verbalize their ideas.

Evaluation of Progress

Planning meaningful activities through the skilful use of materials is perhaps your most important role in guiding children through the pre-text awareness phase of the *Investigating School Mathematics* Primer program. Enabling each child to discover and to explore the relationships in his world, and helping him to gain skill in self-direction and to seek and select information are the long-range goals of the Primer program.

Because the overall objectives differ, evaluation in the modern kindergarten program presents a problem different from that of other grade levels. Your best basis for assessing a child's understanding will be daily observation. This may be supplemented by appropriate testing programs, but your personal observation will remain the most important part of any testing program.

About Resources for Active Learning

With the investigative approach it is important to know what materials are available for activity-oriented classrooms. The lists in the module introductions and in the teaching suggestions for many individual lessons offer you some suggestions. If one or two of these resources are available, it is hoped that you will be able to use them or adapt the ideas.

In the module introductions there are three kinds of resources listed. The "General Activities" would be useful as ongoing activities throughout the module, as a review of concepts, and as practice for basic skills. The "Manipulative Devices" and "Commercial Games" can be used to support the lessons in the module and throughout that unit. The resources listed for a specific lesson are more clearly related to the performance objective for that lesson.

At the time of this writing, those high quality resources which directly complement the active-learning approach were included. Try to become acquainted with those materials that have become available subsequently. Your principal, mathematics supervisor, or resource teacher may be able to provide information about these more recently marketed materials. Such materials can often be obtained from your local supplier, but a listing of some specific sources, to help you get started, is provided following the "Bibliography of Resources."

Tips on technique: start gradually, choose discreetly, be flexible, experiment, relax, and have fun and learn with the children.

Bibliography of Resources for Active Learning

Bates, J., D. Irwin, and G. Hamilton, *Developmental Math Cards* (Don Mills, Ont.: Addison-Wesley, 1970).

Buckeye, D., *NR Math Activities,* Vol. 1 (Troy, Mich.: Midwest Publications, 1972).

Dienes, Z. P., and E. W. Golding, *Exploration of Space and Practical Measurement* (New York: Herder and and Herder, 1966).

Dienes, Z. P., and E. W. Golding, *Sets, Numbers and Powers* (New York: Herder and Herder, 1966).

Dumas, E., *Math Activities for Child Involvement* (Boston: Allyn and Bacon, 1971).

Elementary Science Study: *Attribute Games and Problems; Match and Measure; Primary Balancing; Tangrams* (St. Louis: Webster Division, McGraw-Hill, 1968).

Huff, M. E., D. A. Irwin, *Activities in Geometry* (Don Mills, Ont.: Addison-Wesley, 1973).

Lorton, M., *Workjobs: Activity-Centered Learning for Early Childhood Education* (Menlo Park, Calif.: Addison-Wesley, 1972).

Mathex: Matching and Graphing No. 1; Numeration No. 2; Geometry No. 4; Measurement and Estimation No. 5 (Toronto, Ont.: Encyclopaedia Britannica Publications Ltd., 1970. Available from Encyclopaedia Britannica Educational Corp., Chicago).

Neufeld, E., and J. Lucas, *Developing Number Experiences,* Kit A (New York: Holt, Rinehart and Winston, 1966).

———, *Think and Color* (San Leandro, Calif.: Educational Science Consultants, 1971).
Nuffield Mathematics Project: *Beginnings* 1; *Mathematics Begins* 1; *Pictorial Representation* 1 (New York: Wiley, 1967–1969).
Read, R., *Tangrams: 330 Puzzles* (New York: Dover Publications, 1965).
Rockcastle, V., F. Salamon, and V. Schmidt, *Elementary School Science,* Primer (Menlo Park, Calif.: Addison-Wesley, 1972).
Sealey, L., *Early Numbers Multi-Group Lab,* Activity Cards and Teacher's Guide (Englewood Cliffs, N. J.: Responsive Environments Corp., 1969, 1970).
Wirtz, R., M. Botel, M. Beberman, and W. W. Sawyer, *Math Workshop: Games and Enrichment Activities* (Chicago: Encyclopaedia Britannica Educational Corp., 1964).

Suppliers of Resources for Active Learning

Addison-Wesley (Canada) Ltd., Don Mills, Ont.
CCM School Materials, Inc., Chicago, Ill.
Childcraft Educational Corp., New York, N. Y.
Creative Publications, Palo Alto, Calif.
Cuisenaire Company of America, Inc., New Rochelle, N. Y.
Educational Playsystems, Inc., New York, N. Y.
J. L. Hammett Co., Braintree, Mass.
Jack Hood School Supplies Co. Ltd., Stratford, Ont.
Ideal School Supply Co., Oak Lawn, Ill.
Lakeshore, San Leandro, Calif.
Learning Research Associates, New York, N. Y.
Mafex Associates, Inc., Johnstown, Penn.
Math Media, Inc., Danbury, Conn.
Metric-Aids Ltd., Toronto, Ont.
Milton Bradley, Springfield, Mass.
Moyer-Vico Ltd., Weston, Ont.
Responsive Environments Corp., Englewood Cliffs, N. J.
Scott, Foresman and Company, (Gage Educational Publishing Ltd., Agincourt, Ont.)
Selective Educational Equipment, Newton, Mass.
Teaching Resources, Boston, Mass.
Webster Division, McGraw-Hill Book Co., McGraw-Hill Ryerson Ltd., Scarborough, Ont.

Children's Bibliography

Alain, *One, Two, Three, Going to Sea* (New York: William R. Scott, 1965). Counting.
Ardizzone, E., *Johnny the Clockmaker* (New York: Walck, 1960). Time.
Barr, C., *Jeff and the Fourteen Eyes* (New York: Walck, 1958). Counting and numerical order; Multiples and pairs.
———, *Seven Chicks Missing* (New York: Walck, 1962). Cardinal numbers; Counting and numerical order.
Baum, A., *One Bright Monday Morning* (New York: Random House, 1962). Number; Order; Days.
Berkley, E., *Big and Little, Up and Down* (New York: William R. Scott, 1960). Patterns and shapes; Size.
———, *Early Concepts of Size and Direction* (New York: William R. Scott, 1960). Measurement; Size.
Bishop, C., *Twenty-two Bears* (New York: Viking Press, 1964). Cardinal numbers; Counting and numerical order.
Borten, H., *Do You See What I See?* (New York: Abelard-Schuman, 1959). Patterns and shapes.
Branley, F. M., *Big Tracks, Little Tracks* (New York: Crowell, 1960). Patterns and shapes; Size.
Brenner, B., *Mr. Tall and Mr. Small* (New York: William R. Scott, 1966). Comparison and size.
Buckley, H. E., *The Little Boy and the Birthdays* (New York: Lothrop, Lee & Shepard, 1965). Calendar.
Budney, B., *A Kiss Is Round* (New York: Lothrop, Lee & Shepard, 1954). Patterns and shapes.
Caudill, R., *A Pocketful of Cricket* (New York: Holt, Rinehart and Winston, 1964). Number line.
Chwast, S., and M. Moskof, *Still Another Number Book* (New York: McGraw-Hill, 1971). Counting.
Colman, H., *Watch That Watch* (New York: William Morrow, 1962). Time.
Elkin, B., *Six Foolish Fishermen,* Revised Edition (Chicago: Children's Press, 1965). Cardinal numbers; Counting and numerical order.
Everson, D., *Mrs. Popover Goes to the Zoo* (New York: William Morrow, 1963). Cardinal Numbers; Counting and numerical order.
Federico, H., *Numbers* (New York: Golden Press, 1963). Cardinal numbers; Counting and numerical order.
Fehr, H. F., *Five Is 5,* Little Owl Book (New York: Holt, Rinehart and Winston, 1963). Cardinal numbers.
Francoise (Seignobore), *Jeanne-Marie Counts Her Sheep* (New York: Scribner's, 1951). Counting and numerical order.
Freeman, M., *Finding Out About Shapes* (New York: McGraw-Hill, 1969). Shapes.
Friskey, M., *Chicken Little, Count to Ten* (Chicago: Children's Press, 1964). Counting and numerical order.
Froman, R., *Bigger and Smaller,* Young Math Book (New York: Crowell, 1971). Relativity of size.
Gans, M., *Three Presents for Jamie* (Chicago: Children's Press, 1969). Concept of 3.
Geisel, T. S., *One Fish, Two Fish, Red Fish, Blue Fish* (New York: Random House, 1960). Counting and numerical order; One-to-one correspondence.
Georgiou, C., *The Clock* (Irvington-on-Hudson, N. Y.: Harvey House, 1967). Time.
Gifford, A., *Ten Little Foxhounds* (Chicago: Children's Press, 1969). Counting.
Hawkinson, L., *Picture Book Farm* (Chicago: Children's Press, 1971). Observing.
Hoban, R., *Bedtime for Frances* (New York: Harper and Row, 1960). Time.

Howell, R., *Everything Changes* (New York: Atheneum, 1968). Time.
Hutchins, P., *Clocks and More Clocks* (New York: Macmillan, 1970). Time.
Ipcar, D., *Ten Big Farms* (New York: Knopf, 1958). Ordinal numbers.
Jean, P., *Pattie Round and Wally Square* (New York: Obolensky, 1965). Patterns and shapes.
Kohn, B., *Everything Has a Shape* (Englewood Cliffs, N. J.: Prentice-Hall, 1964). Patterns and shapes.
Krasilovsky, P., *The Very Little Boy* (New York: Doubleday, 1962). Comparison.
Kuskin, K., *Square as a House* (New York: Harper and Row, 1960). Rhymes of comparison.
McGinley, P., *Wonderful Time* (Philadelphia: Lippincott, 1960). Time.
McLeod, E., *One Snail and Me* (Boston: Little, Brown, 1961). Cardinal numbers; One-to-one correspondence.
O'Brien, T., *Odds and Evens*, Young Math Book (New York: Crowell, 1971). Number.
Oxenbury, H., *Numbers of Things* (New York: Franklin Watts, 1968). Numbers and numerals.
Podendorf, I., *Shapes: Sides, Curves and Corners* (Chicago: Children's Press, 1970). Shapes.
———, *Things Are Alike and Different* (Chicago: Children's Press, 1970). Classification.
Rand, A., *Little 1* (New York: Harcourt, Brace, Jovanovich, 1962). Cardinal numbers.
Ravielli, A., *The World Is Round* (New York: Viking Press, 1963). Patterns and shapes.
Schwarz, L. *In 1 Little House* (Chicago: Follett, 1966). Counting.
Sesame Street Books, *The Sesame Street Book of Numbers* (Boston: Little, Brown, 1970). Cardinal numbers.
———, *The Sesame Street Book of Shapes* (Boston: Little, Brown, 1970). Shapes.
Slobodkin, L., *Millions and Millions* (New York: Vanguard Press, 1965). Number.
Stengle, H., *Busy Builders* (Chicago: Children's Press, 1969). Color and number.
Sullivan, J., *Round Is a Pancake*, Little Owl Book (New York: Holt, Rinehart and Winston, 1963). Shapes.
Tudor, T., *Around the Year* (New York: Walck, 1957). Calendar.
Vogel, I. M., *1 Is No Fun, But 20 Is Plenty* (New York: Atheneum, 1965). Counting.
Waller, L., *A Book to Begin On: Time* (New York: Holt, Rinehart and Winston, 1959). Time.
Watson, N., *Annie's Spending Spree* (New York: Viking Press, 1957), Money.
Watts, M., *Weeks and Weeks* (New York: Abelard-Schuman, 1962). Calendar.
Wildsmith, B., *Brian Wildsmith's 1, 2, 3's* (New York: Franklin Watts, 1965). Shapes.
———, *Brian Wildsmith's Puzzles* (New York: Franklin Watts, 1971). Teasers and braintwisters.
Wing, H. R. *Ten Pennies for Candy*, Little Owl Book (New York: Holt, Rinehart and Winston, 1963). Coins.
———, *What Is Big?* Little Owl Book (New York: Holt, Rinehart and Winston, 1963). Size.
Ziner, F., and P. Galdone, *Counting Carnival* (New York: Coward-McCann, 1962). Counting rhymes.

Activities to Develop Mathematical Awareness

To provide important background for the introduction of beginning mathematical concepts, meaningful activities in the six areas of awareness are outlined in this pre-text section of the Teachers' Edition. Materials, games, and procedures are suggested to informally develop vocabulary and to help children gain an intuitive understanding of basic mathematical concepts, such as sets and geometric figures. Take advantage of the opportunities for exploration of quantitative concepts that are inherent in the many social experiences that occur daily in the kindergarten program. Matching activities, such as straws to milk or pairs of boots to children; collecting coins for field trips or parties; manipulating play objects such as beads, blocks, pegboard, or dominoes; and comparing strategies and testing hunches in playing group games are examples of extremely valuable sources of mathematical learning. Planning significant activities through skilful use of materials is perhaps your most important role in guiding children through this pre-text phase of the Primer program.

Area I: *Recognition of Sets*

VOCABULARY

set

Develop the meaning of the word *set* informally by discussing sets of objects brought by the children for sharing time, as well as those sets evident in the classroom. The children can name or identify the numbers of these sets. Examples of sets that might be brought by the children include rock or shell collections, books, marbles, jacks, jump ropes, or plastic animals, ships, planes, cars, and the like. Sets to recognize in the classroom might be scissors, paint and brushes, chalk or crayons, puzzles, beads, books, sweaters, mittens, boots, and so on.

Many opportunities occur to group some or all the members of a set and also intuitively develop the idea of a subset. Samples of groupings follow.

(1) *In games:* Sets of boys, girls, red socks, blue jeans, and black tennis shoes.

(2) *On the playground:* Sets of swings, monkey bars, slides, trees, balls, and children in these activities.

(3) *In the playhouse:* Sets of dishes, glasses, dolls, blocks, tools, books, chairs, and rooms.

(4) *In the classroom:* Sets of red crayons, sharpened pencils, clean erasers, red chairs, clocks, green-skinned teachers, flowering plants, red boots, yellow raincoats, wheeled toys, and giants.

POSSIBLE ACTIVITIES

The games that follow should help the children perceive sets and intuitively understand parts of sets (subsets).

1. Set Spy

This game is like the game "I spy." The teacher must describe one aspect of a set. For example, "I spy a set whose members are red," or "I spy a set whose members have wheels," and so on. The children are to respond, "Is it the set of red chairs?" or "Is it the set of tricycles?" Do not criticize if the children don't use such vocabulary as *set, member of the set,* and the like; but continue to use it naturally in the activities in this section.

2. Pirates' Plunder

A small satchel or sack and objects that the children can identify easily are needed for this game. One member of the class is to put familiar classroom objects in the bag and give hints about what is in the container until someone guesses what the set is. For example, the bag might contain crayons, or the teacher's bell, or erasers from the board. In addition to teaching the set language, this kind of activity helps children sharpen their observation skills.

3. Family Scrapbook

To help children think about sets in their own personal relationships, provide them with a quantity of old magazines, catalogues, paper, paste, and scissors. Have each child make a family scrapbook by cutting out sets of pictures to represent the members of his family (including himself), his pets, home, the family car, his dream room, the clothes he would like, possessions he owns or would like to own, and so on.

4. Story Sets

Read aloud an interesting story and have the children identify sets in the story. Some suggestions for reading are listed below.

Barr, Catherine, *Jeff and Fourteen Eyes.*
Gag, Wanda, *Millions of Cats.*
Ipcar, Dahlov, *Brown Cow Farm.*
Kay, Helen, *One-Mitten Lewis.*
Otto, Margaret, *Three Little Dachshunds.*
Thompson, George S., *Sparrow Socks.*

You may prefer to use traditional titles, such as:

Cinderella
Goldilocks and the Three Bears

The Little Engine That Could
Peter Rabbit
Snow White and the Seven Dwarfs
The Three Little Pigs

5. Pocket Detective

As a continuing activity, secretly put a familiar set in the pocket of your coat, smock, or apron. Ask the children to identify the set from your clues during their sharing-time activities. If you change the set regularly, the game will hold the children's interest for several days.

Area II: *Recognition of Geometric Shapes*

VOCABULARY

circular shape	**square shape**
corner	**rectangular shape**
edge	**triangular shape**
side	

Do not attempt to have the children distinguish between circular shape and circle, square shape and square, and so on, at this level. Introduce the shapes with flannel, construction paper, or cardboard regions of various sizes and colors. Ask the children to find the shapes in objects both in the classroom and at home. Sample lists of objects that are frequently available follow.

(1) *Circular shapes:* Coins, checkers, circular box and bottle tops, jar rings, bracelets, buttons, and the clock face.

(2) *Square shapes:* Floor tile, square box tops, face of a block, square chair seats, bottom of the milk carton, sections of the chalkboard, and square window panes.

(3) *Triangular shapes:* Triangular prism, triangular rulers, bridge structures, chair supports, the rhythm instrument, and a pipe-cleaner triangle.

(4) *Rectangular shapes:* Window and door frames, dominoes, desks, books, stamps, and most boxes.

In handling and discussing the geometric shapes, children seldom have trouble distinguishing between round objects and those having corners. However, let them feel and manipulate both kinds of objects until they understand that cornered objects always have more than two sides and that there are as many sides as corners on such objects.

POSSIBLE ACTIVITIES

A variety of activities will help children discriminate among the four geometric shapes introduced. Some possible approaches follow.

1. Show Me

First, guide the children in making an envelope of shapes to use in this and other geometric games. Give each child colored construction paper on which several of each geometric shape have been duplicated in rows. (Make each side or diameter a minimum of 3 cm.) Hold up a large geometric shape from a set you made for demonstration, ask someone to name it, and then ask the children to cut the strip of shapes off their papers when they find the one that corresponds to the shape just named. After the four shapes have been named, give the children time to cut out all the individual shapes and store them in an envelope.

At another time, you can ask the children to take out their envelopes of geometric shapes. Say, "Show me a square," "Show me a shape with three sides," or "Show me a shape with four sides alike . . ." Use descriptive and formal terms in asking the children to find and identify shapes.

2. Hidden Shapes

As another identification activity, place a variety of heavy cardboard cutouts in a large covered oatmeal box called "The Mystery Box." Instruct the children to reach in, pick up one shape, feel, manipulate, and name it without looking. You may wish to blindfold the children with a wide stretchy headband or something similar for this activity. Continue the game until the box is empty, and then have the children name the shapes as they replace them.

As a variation for practice in verbal identification, have a child select a shape and hold it behind his back. He should then give hints until someone guesses what shape he is hiding.

3. Shape Detective

Give the children a chance to identify the geometric curves in various reproduced pictures. Sample directions might be, "Draw over all the triangles with a green crayon, all the squares with a black crayon, all the circles with a blue crayon," and so on. Or, for a variation of this, distribute a page of geometric regions that will result in a recognizable figure or animal when colored, and direct the children to color the rectangles orange, the triangles yellow, and so on.

4. Draw the Shape

Encourage children to experiment with drawing the four given geometric curves on a large sheet of paper in any design pleasing to them. Perhaps they will need to practice tracing over the shapes, particularly circles, before drawing them on their own. Show the children a method of making circles, similar to making manuscript *o*'s, by starting at the top and drawing counterclockwise (for right-handed children).

5. Designs

To help the children get a "feel" for geometric curves, let them manipulate buttons, ceramic tile, precut pieces

of wood, cereal, macaroni, seeds, beans, or rice and arrange these into a pleasing design on a sheet of paper before gluing them down. Gluing them onto heavy, corrugated cardboard covered with burlap will make a permanent hanging. If three-dimensional materials are not available, give the children shapes of colored construction paper and ask them to arrange a pleasing design on a background sheet before pasting them down.

Area III: *Developing Visual Memory*

Developing visual memory is helpful for the future involvement in more abstract activities. The following games are samples of those that can help children improve their total perception.

1. I'm Going on a Lion Hunt

This variation of "Follow the Leader" gives the children practice in perception and makes them aware of changes of motion. Appoint a leader as the hunter who announces "I'm going on a lion hunt and whoever wants to can follow me." The children are to follow him around the room, mimicking his actions as he stomps through the grass, crawls over a fence, wades the stream, climbs a tree, sights the lion, and so on. At any moment he chooses, he may say, "There's a lion. Bang!" and simultaneously stomp his foot loudly. The other children should then race back to their seats. The first one in his own seat, other than the leader, is the next hunter. This kind of game should help the children learn to observe carefully.

2. Disappearing Sets

Place easy-to-see sets of objects on a display table. Ask the children to identify the sets and then hide their eyes. Remove one set and move the objects together before calling heads up. Ask someone to tell which set disappeared. You might use sets of paintbrushes, toy cars, juice cans, dolls, toy trains, blocks, play dishes, rhythm instruments, and the like. For a variation of the game on a flannelboard, use sets of objects, such as stars, chicks, rabbits, hearts, or geometric shapes. Be sure to close the gap of space when the sets are removed. Or, use only one set of shapes and remove just one object. Then ask the children to show you an example similar to the missing members among the shapes in their envelope of cutout shapes.

3. Reappearing Sets

Use sets such as those suggested for Game 2 above, but while the children hide their eyes, add one set. Ask the children which set appeared. Variations similar to those for Game 2 are possible, except that now you must ask which set or object has been added in the reappearing set.

4. Match the Shapes

Make sure each child has an envelope of geometric shapes and a sheet of dark paper. Place a simple set of three geometric shapes on the flannelboard and call attention to them. Then remove them from view and have the children try to choose the same three shapes from their envelopes and place them on their papers. When all have had a few minutes to do this, replace the shapes, one at a time, on the flannelboard. As you do this, have the children hold up a similar shape if they have chosen the proper one.

Area IV: *Following Directions*

Much of the vocabulary necessary to enable the children to compare sets can be developed along with improving the children's skill in understanding and following directions in groups and as individuals.

VOCABULARY

behind	**on**
between	**outside**
bottom	**over**
in front of	**under**
inside	**top**

Develop a small group of related words at the same time, i.e., *inside, on; under, over; behind, between, in front of;* and so on. As you give directions for games and group activities, use both objects and geometric shapes, and emphasize the natural use of the vocabulary introduced in this and the other three units.

POSSIBLE ACTIVITIES

1. Circle Games

To introduce the terms *inside, outside,* and *on,* display perception cards with a cross in various positions inside, outside, or on geometric curves. Or play circle games in which the children sing directions such as, "Put your right foot in, pull your right foot out, put your right foot in, and shake it all about . . ."

As a variation, place hula hoops or circles of rope on the playground or gym floor. Give the children directions like these: "Girls stand inside the circles"; "Boys stand outside the hoops"; "Boys wearing tennis shoes stand on the circle . . ." Asking questions to which the children must respond is another good device to develop this vocabulary.

2. Directions Game

Help the children make their own game sheets by giving directions like the following and demonstrating them on an area of the chalkboard blocked off to represent a piece of paper.

(1) Make a large circle near a corner at the top.

(2) Make a square in the other top corner.
(3) Draw a triangle at the bottom of the page, under the circle.
(4) Color a square inside the circle.

When the children have completed their own game sheets, use the sheets for a directions game by saying, "Place a toy car inside the rectangle"; "Put a checker on the circle"; "Stand one dinosaur outside the triangle and one dinosaur inside the square"; and so on. Or, play "Simon Says" to continue following directions and further develop understanding of the vocabulary.

3. What's Between

To develop the meaning of *between*, place several shapes on the flannelboard in a row. Discuss the shapes between those on the extreme ends. Then, eliminate middle shapes and ask children to place specific shapes according to your directions between the shapes already there.

4. Geometric Collage

Give the children geometric shapes cut from many materials and textures, such as velveteen, corrugated board, burlap, satin, poster board, or flannel, along with things like yarn, string, mesh vegetable bags, buttons, tiles, and the like. Let them manipulate the shapes, and when they get a design they like, give them white glue to make a collage that pleases them.

Area V: *Ordering, Classifying, and Comparing Sets*

VOCABULARY

as many as (not counting)	**longest**
behind	**shorter**
beside	**shortest**
enough	**smaller**
larger	**smallest**
largest	**taller**
longer	**tallest**

Develop the vocabulary by using familiar objects and taking advantage of sets that crop up in sharing-time activities or in the classroom. Examples of questions you can ask about sets:

(1) Did Roger bring as many toy horses as Jimmy brought cowboys?
(2) Is there enough milk for everyone?
(3) Are there as many boys as girls here today?

The height of children, trees in the school yard, and backs of chairs are samples of things to compare to bring out the meaning of *taller* and *shorter* and the superlative forms *tallest* and *shortest*. You can develop *longer* and *shorter* by comparing lengths of rope, baseball bats, sheets of paper, rulers, metre sticks, shoes, middle fingers, or mittens.

POSSIBLE ACTIVITIES

1. A Set Apiece

Personal cartons of small objects for each child are useful in a variety of set games. Ask the children to bring small cartons such as those used for cottage cheese, eggs, cigars, shoes, or the freezer. Plastic animals or small toys, such as soldiers, jacks, bracelets, blocks, bottle caps, or beads make suitable objects. You can begin by asking the children to find all the objects of a given variety. Then refine your directions to include a specific variety of a certain color. Direct the children to place the objects on a line, using *beside, in front of, behind,* and similar directions. Ask them to find the smallest and the largest objects in their box and to place them at the end of the line. You can change these directions to include many different kinds of activities.

2. Sort Out

Give children envelopes of squares, circles, triangles, and rectangles in two sizes and three or four colors. Have them sort the shapes according to color, then according to shape, then according to size. Ask them to show their largest triangle on their desks. Then ask them to place one of their smaller circles inside the triangle. Continue with directions such as "Find a small square. Place it beside a large circle. Put a blue triangle in the square. Put a yellow triangle in the circle," and so on according to some preplanned directions. Emphasize positional vocabulary, color words, and the names of regions, according to the maturity of your group. This activity can be used many times throughout the year.

3. Order Up

Give the children envelopes of assorted sizes of the four geometric shapes. Direct them as follows.

(1) Put the squares in order from smallest to largest.
(2) Put the circles in order from largest to smallest.

If you have number rods ranging from a centimetre to 10-centimetre size, ask the children to arrange them in order from largest to smallest and vice versa. Some children can do this successfully with odd lengths of wood or wire and even with twigs from the playground.

4. What's Missing

Display large paper geometric regions with smaller regions of the same or different shapes cut from the inside of the region. Supply the children with a variety of shapes and sizes from which to choose the proper one to fit the cutout regions. Have some of the proper shapes just a little too small and others just a little too large to fit that region, so that the child must discriminate carefully to choose the proper one. Vary the game by showing a centre shape and asking the children to find the region that fits around the given circle, and so on.

Area VI: *Discovering Patterns*

VOCABULARY

alike **different** **pattern**

You may wish to begin the exercises in pattern development by building three-dimensional figures from blocks, beads, cartons, or boxes found in the primary classroom. For example, build a pattern using natural colored solids or those of a single color to emphasize the shapes used. Ask the children to copy the tower or pattern using their own blocks. If the children's blocks are colored, emphasize that color is irrelevant in this particular pattern. Use the terms *like, different,* and so on in this process.

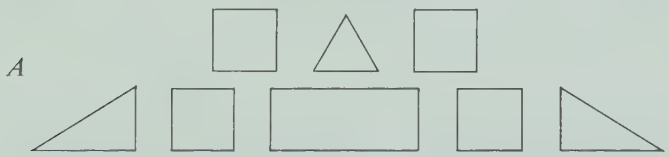
POSSIBLE ACTIVITY

1. Copy-Kat

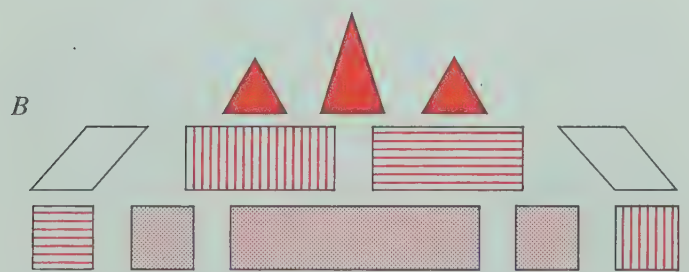
Ask the children to work in pairs. Direct one child to build a tower or any other structure he fancies and then direct his partner to make one just like it. Next, the second child should build a different pattern or structure for the first child to copy. If the children use colored blocks, be sure to have the builder tell the copier whether both color and shape must be copied.

2. Make My Pattern

Show patterns on the board (*A*) and have the children reproduce them in three dimensions using solid blocks.



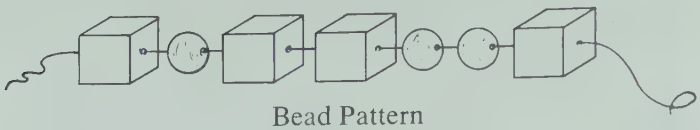
After they have learned to reproduce the pattern by shapes, color the board patterns (*B*) and have them reproduced by shape and by color.



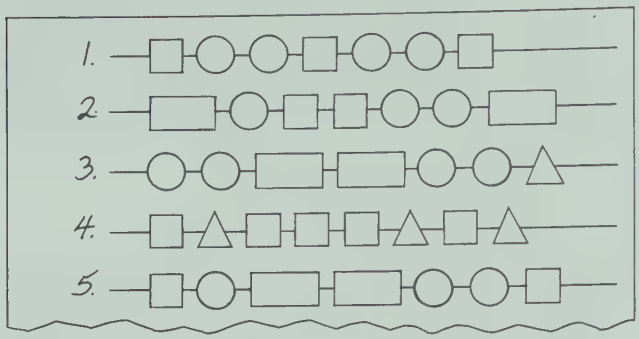
Teacher's Patterns

3. Choose My Mate

Build a simple pattern with beads shaped like the four solids, and ask the children to choose the pattern from a chart showing many patterns. This phase of discriminating is a transition between reproducing a concrete, solid figure by building a similar figure and looking at a pattern on a two-dimensional surface in order to reproduce such a figure.



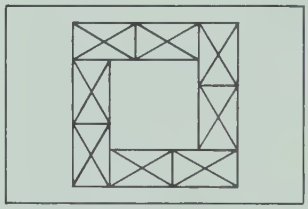
Bead Pattern



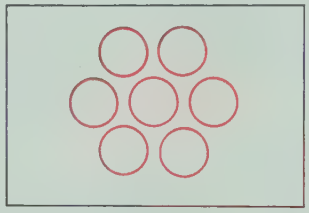
Pattern Chart

4. Geo-patterns

A. Prepare display-size perceptual cards showing patterns using one color and one size of one geometric shape. Give each child an envelope containing eight each of three sizes of each of the four geometric shapes that have been introduced, and ask them to copy a pattern from one of the cards you display. Then urge them to create their own patterns using like shapes of one color and size. Examples of display perceptual cards follow.



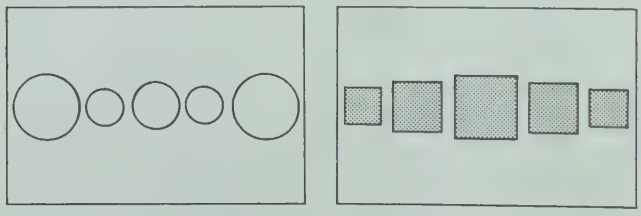
Same size, shape, and color



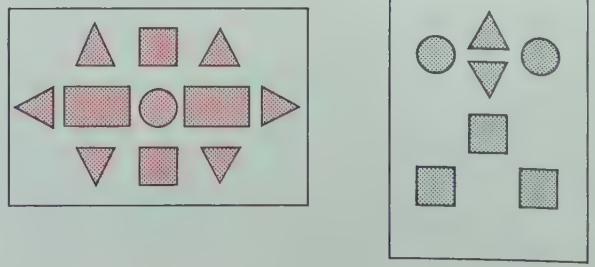
Same size, shape, and color

B. Vary the geo-pattern activity by changing a different dimension. Examples follow.

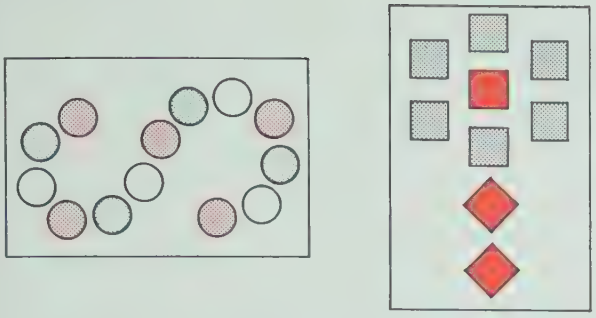
Change the size:



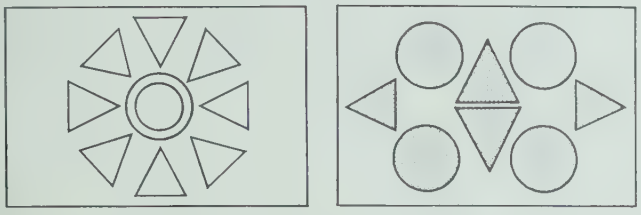
Change the shape:



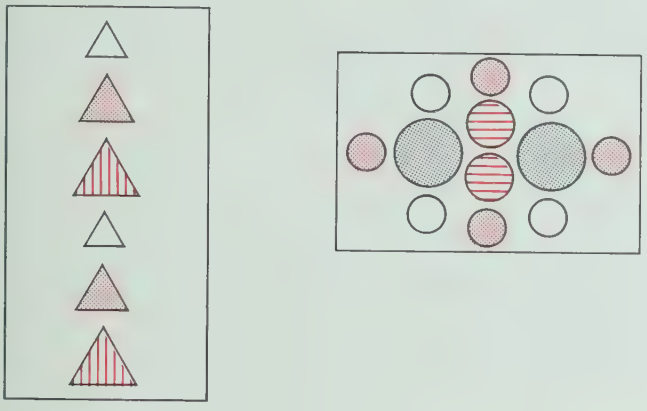
Change the color:



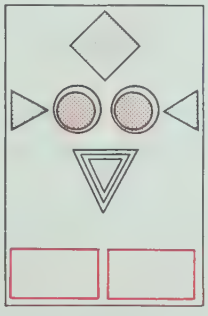
Change the size and shape:



Change the size and color:



Change the size, shape, and color:



Other variations of geo-patterns include patterns in which the pieces differ in two or more dimensions.

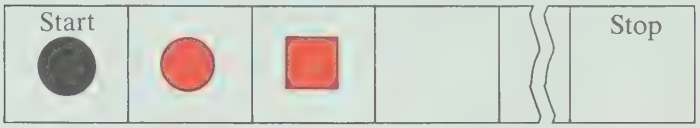
5. Cell Games

After the children are fairly accomplished at pattern reproductions such as those previously suggested in Game 4, devise a cell game in which the children take turns putting shapes in a given cell according to previously agreed upon rules.

Each pair of children needs a strip marked into cells (eight or ten cells are sufficient in the beginning) and a pair of envelopes with eight pieces each of three sizes of the four common geometric shapes (square, circle, triangle, rectangle), in at least two different colors. The winner is the child who completes the "Stop" cell; or if neither player gets that far, the winner is the child who played the last shape correctly.

A. For example, a "1-D" game means that to play correctly each player must select a shape that is different from the last in only one dimension (or characteristic).

Sample Plays in "1-D" Game



B. For a "2-D" game, the rules require the children to change two dimensions.

Sample Plays in "2-D" Game

C. In a "3-D" game, each player must choose a shape which is different in three characteristics from the last one played. If there are 15 cells, it becomes increasingly hard to play as the supply of shapes decreases.

Sample Plays in "3-D" Game

Many variations of the games and activities included in this activities section of the Teachers' Edition can be adapted and used for motivation or review throughout the childrens' work in the Primer text which follows.

Comparisons

Pages p-1 to p-16

General Objectives

To introduce children to working on the printed page
To introduce the comparison of two or more objects
To develop the idea of positional relationships
To increase awareness of similarities and differences among objects
To provide experience in working with spatial relationships

The children are asked to compare two or more objects with respect to size, height, and length, and then to use and understand positional terms as they place marks to indicate answers. Most children will be familiar with these topics because of the readiness activities, so the primary objective of this unit is to give them experience in working on a printed page.

First, drawings of geometric regions are introduced, and the children are asked to apply positional and comparative terms to these drawings. Then the next few pages should help children learn to discriminate among objects by developing their awareness of similarities. The last lesson in this unit provides background for discovering patterns and experience with spatial relationships.

Do not expect children to master the difference between comparatives and superlatives. As you give directions for the text pages, however, be sure to use the comparative form when two objects are to be compared, and the superlative form when three or more objects are to be compared. Children should recognize this distinction though they may be unable to use the words correctly themselves. Your own careful use of these words will promote correct usage by the children.

Throughout this unit and those that follow, your paramount concern should be to develop clear understanding of the ideas rather than mastery of the terminology. Make certain that the children's beginning experiences with paperwork are pleasant and successful.

Mathematics

The concept of a *relation* is one of the most important ideas in mathematics. All the terms of comparison and position introduced in this unit refer to relations. Although it is unlikely that there will be significant carryover from the relations studied here to the more abstract mathematical relations, it is important that this first exposure to a significant concept be correctly developed.

In mathematics, a relation is defined as a *set of ordered pairs*. Rather than elaborate on this definition, we simply emphasize that a relation involves *pairs* of things. Note that terms of position and comparison refer to pairs of objects. For example, a mark is *below* only with respect to another object; a mark is *inside* only with respect to something else; a kite is *higher* only with respect to another object; an object is *larger, longer,* or *taller* only with respect to something else.

The superlative use of the terms constitutes an extension of the relation idea. For example, we could compare three blocks of different colors, such as red, green, and blue. If we say that the red block is *largest,* we simply mean that the following *two* relations are true: The red block is larger than the green block, and the red block is larger than the blue block.

The statements $87 > 25$ and $5 + 3 = 8$ are examples of relations that children will meet very early in their mathematical experiences. The relation between 87 and 25 is "greater than." The relation between $5 + 3$ and 8 is "equals." Another relation between the pair (5, 3) is "the difference of," which is 2. As children progress to more advanced ideas in the secondary school, special relations called *functions* command major attention.

Teaching Yellow Module, Unit P

Materials

assorted cartons and boxes (egg cartons or oatmeal, cigar, or small shoe boxes), one per child
crayons for each child
envelopes containing geometric regions, three sizes each of four shapes, one envelope per child
felt geometric regions (squares, circles, triangles, and rectangles of several sizes)
felt set materials
flannelboard
heavy construction paper
kindergarten blocks
objects for comparisons (blocks, beads, buttons, jar lids, and so on)
paper markers for placeholders
pipe cleaners or lengths of yarn, ribbon, or string
yellow squares, orange squares, large square, large circle, orange strip, and brown strip
tagboard

Vocabulary

above	higher	lowest	smaller
below	highest	on	smallest
beside	inside	outside	square
bigger	larger	over	taller
biggest	largest	rectangle	tallest
bottom	last	row	third
circle	longer	second	top
first	longest	shorter	triangle
fourth	lower	shortest	under

The vocabulary list constitutes the words most likely to arise in a discussion of the material in the unit. Do not feel obliged to use all the words. Use only those that seem natural for the situations which arise in your class discussions and activities with respect to the Primer lessons.

While the materials listed are useful for development of the ideas of this unit, you can also make effective use of materials found in an ordinary elementary classroom. For example, you can speak of the *longest* paintbrush or the picture *over* the bookcase. Children should be given such verbal reinforcement even after being introduced to the Primer pages.

Lesson Schedule

The amount of time you spend on this module will depend upon the readiness of the children. If you developed some of the activities in the pre-text section of this manual during the first part of the year, most children will be ready to work with this material without any difficulty by late January or February.

Before being presented the first pages in the Primer, children should be able to (1) sit and listen to a story and then participate in a discussion of the story in an orderly fashion, (2) carry out simple one- and two-step directions, and (3) do simple coloring and drawing. Pace the activities to the capabilities of your group, just as you do with listening and reading readiness skills.

Evaluation of Progress

Much of the material in this module will be used again in presenting future lessons, and the children's achievement in this unit will be evident as you continue with the remainder of the Primer material. Therefore, your evaluation of the children's progress with positional and comparative relationships should be a continuing process throughout the year.

Resources for Active Learning

General Activities

Developmental Math Cards, "Paint a Splash," A^{3}2, Addison-Wesley

Elementary School Science, Primer, Teachers' Edition, T21–T22, T29–T32, Addison-Wesley

Exploration of Space and Practical Measurement, "Games . . . Geometry," pp. 35–37; "Games . . . Measurement," pp. 59–62, Herder and Herder

Nuffield Project: *Beginnings* 1, pp. 2–57; *Mathematics Begins* 1, "Relations," pp. 6–9, Wiley

Workjobs, Matching Activities, pp. 50–55, Addison-Wesley

Manipulative Devices

Attribute Blocks and Games (Learning Research Assoc.; Teaching Resources; Webster, McGraw-Hill)

Blocks (Childcraft)

Conservation of Number Sorting Cards (Teaching Resources)

Discovery Blocks (Educational Teaching Aids)

Geoboards (Addison-Wesley)

Mosaic and Primary Shapes (Learning Research Assoc.; Responsive Environments Corp.)

Sorting and Sets (Teaching Resources)

Commercial Games

Colors and Shapes (Creative Playthings; local supplier)

Remember, Remember (Responsive Environments Corp.)

(For additional suggested manipulative devices and commercial games, see the introductory notes for the Orange Module and the Red Module of Unit P.)

Activities for Continued Use

The following activity is based on a conservation task developed by Jean Piaget, who has done some interesting research regarding children's conceptual development. Many children at this age have not developed the ability to distinguish between the concept of length and that of position in space. Thus, when one of two parallel sticks of the same length is pushed forward, such children will judge that it is now longer than the other since "its end sticks out farther." The first activity described here should help you identify those children who cannot conserve length, and who would benefit from activities designed to develop this concept.

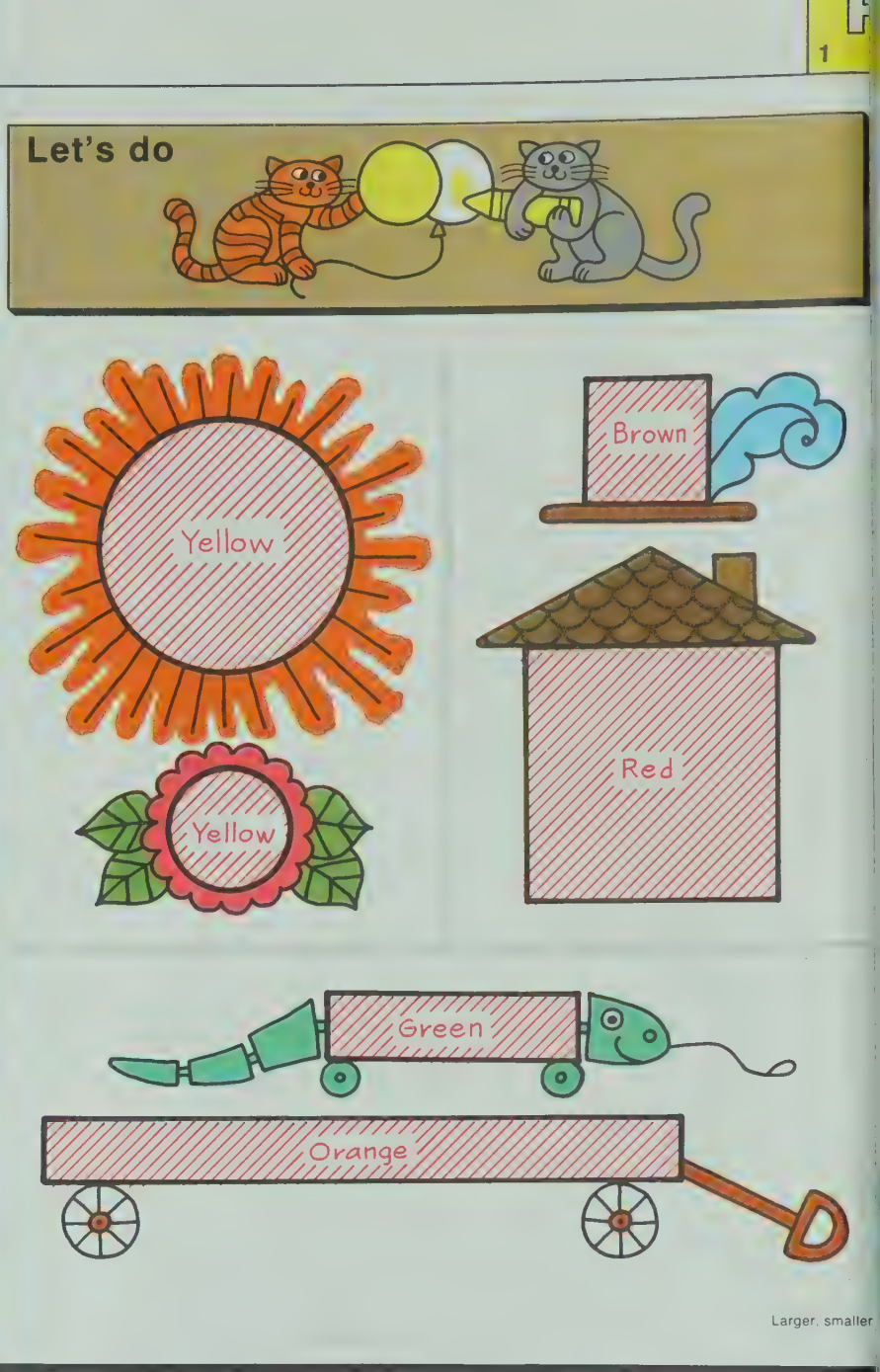
Materials: two thin, different-colored sticks of the same length

Procedure: Place the two sticks so that they are parallel. Ask a child, "Is the (blue) stick as long as the (red) stick?" When the child responds "Yes," move the (blue) stick to the right. Ask, "Now which is longer?" If the child considers only the relative positions of the right ends of the sticks, he will perceive the (blue) stick as longer. Such a child has not "conserved length."

Investigation
Page p-1

Since this is the first page of the Primer, call attention to the demonstration art at the top of the page. Art such as this will accompany many pages and provides an opportunity to discuss the activity for the page.

Explain to the children that they are to use the 6 cutout pieces you have previously prepared with this page. Ask them to try to match their pieces to a picture by placing each piece on top of a picture so that the cutouts fit on the pictures. After they have matched the cutouts to the pictures, they should color each figure like the corresponding cutout. As the children work, encourage discussion using terms such as larger, smaller, bigger, shorter, longer, round, square, and oblong. However, do not stress the vocabulary at this time. Other lessons will develop these relational terms more specifically. Treat this activity as an introductory investigation and encourage an attitude of free play and informality.



Purpose

To introduce the child to comparison relations

Preparation

Materials

cutouts (to fit the 6 shapes as shown): small yellow circle, small brown square, large red square, large yellow circle, orange strip, and green strip. (Store the squares, strips, and circles for use in the next modules.)
envelopes
crayons (the colors of the cutouts)

Distribute the 6 figures which you have previously cut out. Encourage discussion of their colors and shapes, but do not stress the names of the shapes. For example, some children may simply refer to the "round ones," the "square ones," or the "long ones." The important point is for the children to have an opportunity to become familiar with these shapes for the investigation.

Discussion
Page p-2

The illustration on page p-2 is provided as a basis for discussion of the same concepts introduced on the opposite side. This page may be used as a separate lesson or it may immediately follow the completion of page 1. Use the illustration to suggest a story. Use such phrases as "the larger deer," "the smaller deer," "the taller tree," "the shorter tree," "the longer bridge," "the shorter bridge," "the higher bird," "the lower bird", and other phrases which stress likenesses and differences in other ways the children may suggest. Also use phrases which include the positional words such as *over, under, beside, on, in,* and so on.

Follow-up

Guide the children in playing a game in which one child describes the position of an object and the other children try to guess the object he has in mind. Place special emphasis on the positional terms mentioned above, and on the ordinals showing position, such as first, second, and third. You might begin such a game yourself by describing, without pointing, the location of various objects in the classroom.

It would also be helpful to use many labels around the classroom. Tables or shelves may be labelled first, second, third, and so on. Also, positional phrases may be used as labels, such as "under the table" (for the wastebasket under a table) or "on the shelf" (in a closet or cupboard).

Teaching
Page p-3

Call attention to the two birds at the top of the page. Have the children note that there is an X beside the larger bird and a circle or ring around the smaller bird. Help them find the rabbits in the first frame. Give directions like those in the example. For the second frame, have the children put a mark *on* the larger frog and *over* the smaller frog. At the bottom of the page, ask the children to put a mark *over* the larger animal and *under* the smaller. Finally, for the last frame, ask them to put a check on top of the larger animal and ring the smaller.

Since this is the first page of the Primer on which the child writes, carefully supervise the activity of each child to make sure that he is working on the right problem and that he clearly understands what is to be done. As you give directions for this page, move around the class. Make certain that each child has understood your directions.

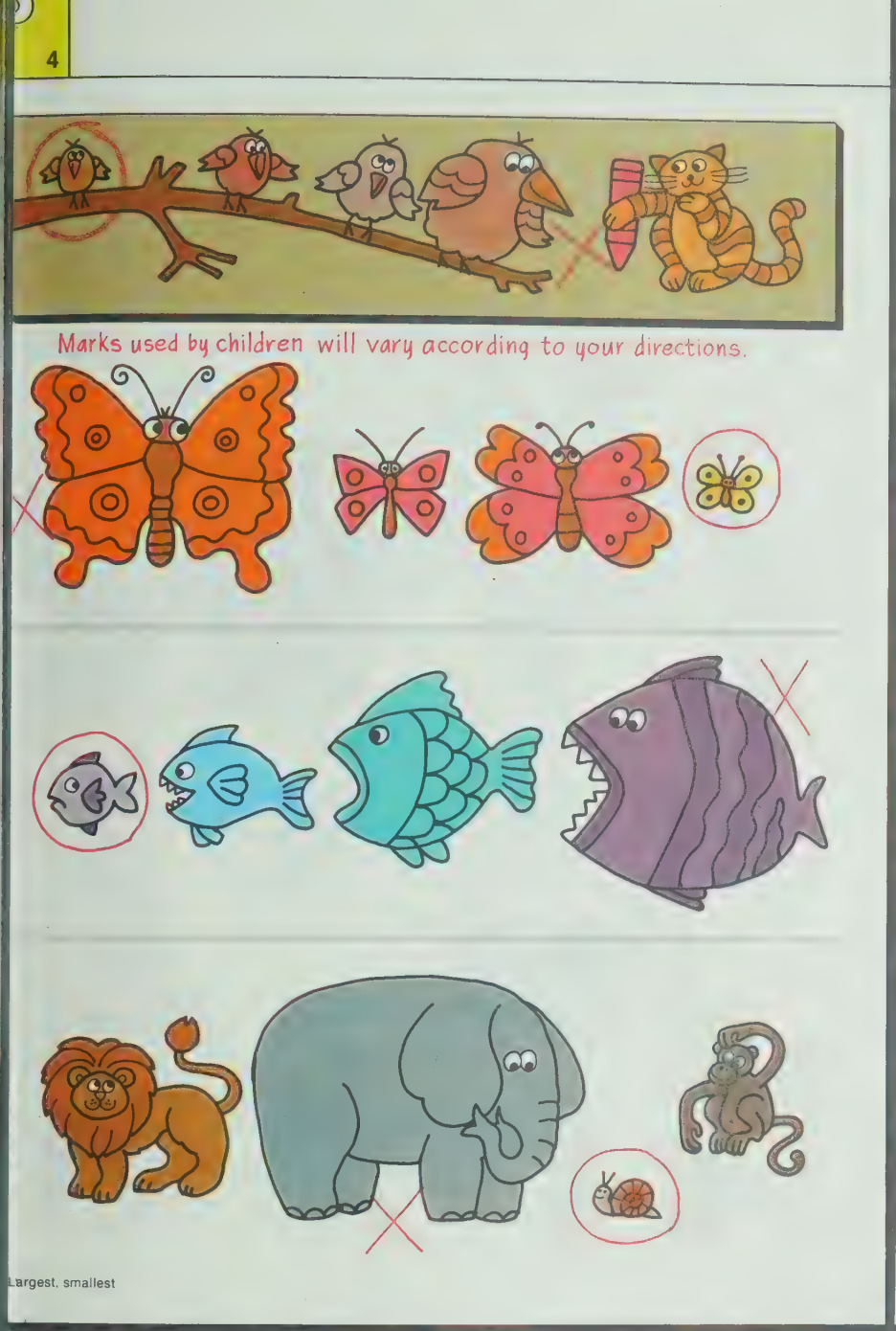
Objective

Given illustrated objects which are obviously different in size, the child will be able to identify the largest and smallest.

In this lesson children are introduced to the comparative relations *smaller, larger, smallest,* and *largest.* Besides comparing only two items, they are given a group of objects and encouraged to find the one object which is smallest or the one which is largest in each group.

Pre-Book Activity

Begin by reviewing the words *larger* and *smaller* with respect to pairs of objects (balls, chairs, books, or blocks). When the children recall these concepts of size, have them compare three or more objects, such as blocks, paint jars, books, and so on, stressing the words *largest* and *smallest.* Ask the children to describe, without pointing, the location of various objects in the classroom. They can play a game in which one child describes the position of an object and the other children try to guess what he has in mind. During this activity, place special emphasis on common positional terms such as *under, over, on, top, bottom, row, front, back,* and *last,* and on the ordinals showing position, such as *first, second,* and *third.* When you feel that the children are ready for chalkboard activities, draw on the board a pair of cats or rabbits that obviously have different sizes. Co-ordinate the use of positional terms with the activity of comparing objects by asking one child to come to the board and place a check mark (✓) under the smaller object, and another child to mark the larger object. After several examples, compare three or more objects, using the terms largest



Teaching
Page p-4

Direct the children's attention to the row of birds in the demonstration art at the top of the page. Bring out the fact that a ring has been put around the smallest bird and an X has been put beside the largest bird. Then call attention to the butterflies. Stress the words *first* and *row* with respect to the butterflies. Point out to the children that this is called "the first row." By stressing the words *first, second, third,* and so on, you will help the children become accustomed to using these words. Use the positional terms to have the children mark in a special way the largest and the smallest butterflies. Continue with the rest of the exercises, having the children mark the largest and smallest in each row. Emphasize the words "second row" and "third row," and identify the objects in the row by name. You might also introduce other words used to describe size, such as *tiny* and *huge*.

Resources for Active Learning

Developmental Math Cards, "Bigger-Smaller," A2-3, Addison-Wesley

Elementary School Science, Primer, Teachers' Edition, "Large-Small," T23–T24, Addison-Wesley

Math Activities, "Bigger-Smaller," p. 77, Allyn and Bacon

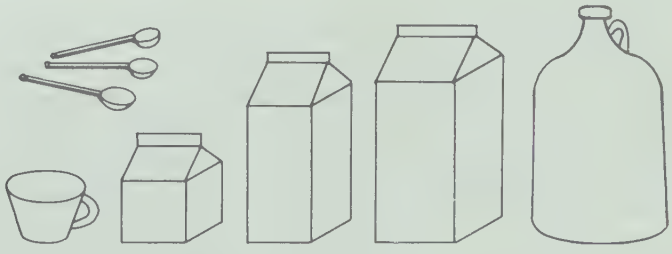
Mathex: Measurement and Estimation No. 5, p. 3, Encyclopaedia Britannica Educational Corp.

and smallest. Also, as you refer to these objects use the ordinal number names (first, second, third, and so on). Your continued use of these terms throughout the year will help the children understand, and eventually use, these words correctly.

Follow-up

Provide the children with an area for work with water and for work with sand. Also provide them with measuring spoons and a set of different-sized plastic containers. Encourage the children to use the graduated objects, measuring out quantities of water and quantities of sand into other containers. Help them to talk about their measuring instruments using words such as larger, smaller, largest, and smallest.

Clay might be used for the same type of activity. In this case, the children should simply form masses of clay that are of different sizes, and then arrange them in some order.



Teaching
Page p-5

Call attention to the two birds in the demonstration art. Point out that an X has been placed beside the taller bird and a ring is being put around the shorter bird. Then call attention to the first frame. Have the children mark, according to your directions, the taller ostrich and then the shorter ostrich. Have the children find the next two objects in the next frame and, again, mark the taller and the shorter according to your directions. Continue directing them to mark the taller and the shorter in each frame at the bottom of the page.

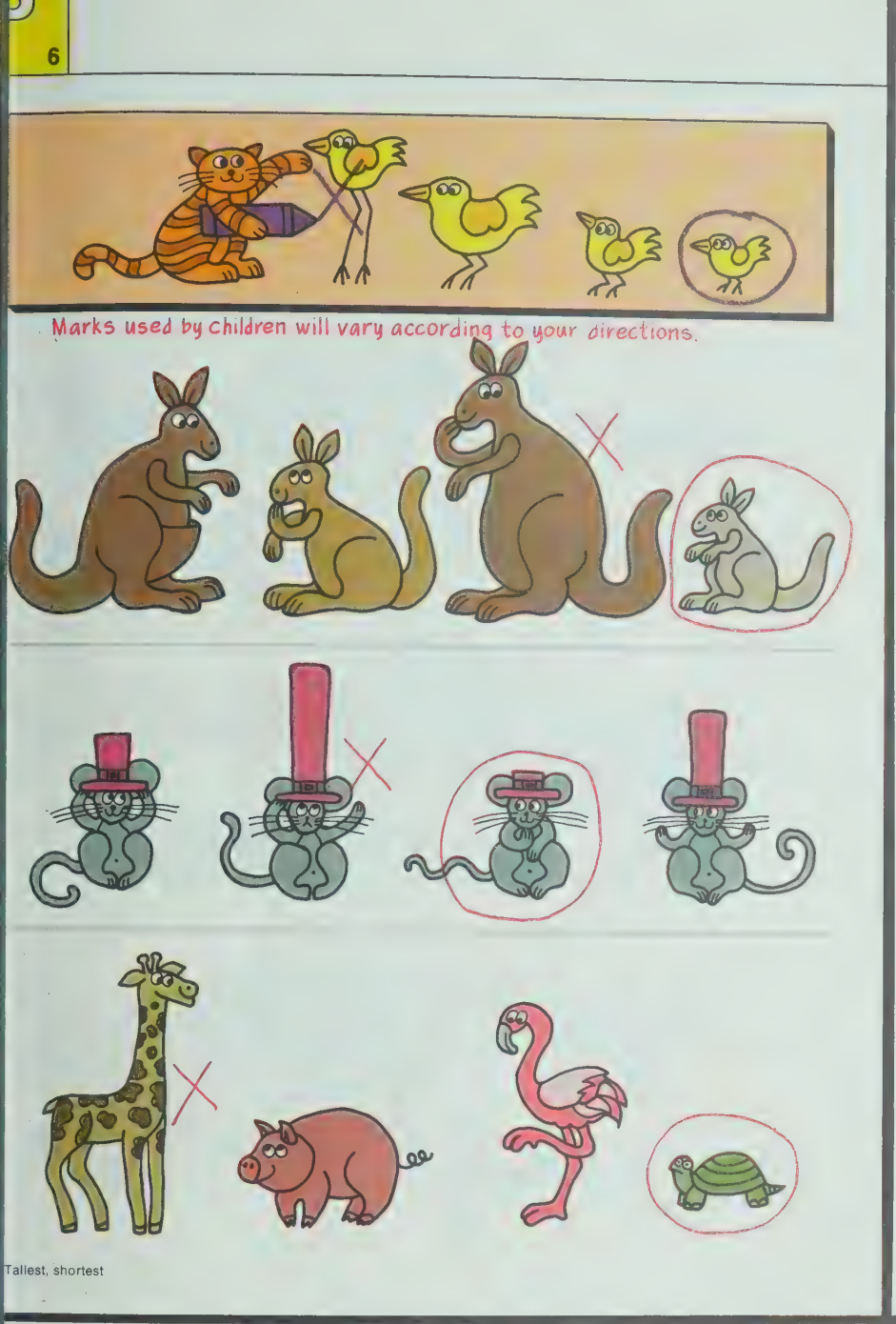
Objective

Given illustrations of taller and shorter objects, the child will be able to identify the taller (shorter) or the tallest (shortest) object.

This lesson develops the concepts of tall and short. The comparatives *taller than* and *shorter than,* as well as the superlatives *tallest* and *shortest* are included. Many pre-book activities are possible and it is recommended that you use the vocabulary with them before directing the children to work on the printed page.

Pre-Book Activity

Prepare for this pair of pages by using the terms taller or shorter and tallest or shortest in connection with familiar objects in the classroom. Using the terms taller and shorter, compare pairs of children in the class, their desks, and their chairs according to height. To develop such an idea further, hang on the wall a poster-size piece of newsprint paper or chart paper. Give each child an opportunity to stand next to the chart and have his height measured. Print next to each mark the name of the child being measured. In your discussion, you might also compare three children and stress the concept of the tallest of the three or the shortest of the three. A chart such as this might be used throughout the year and measurements might be taken every month so that the children can keep track of their height.



Teaching
Page p-6

Call attention to the demonstration art and observe with the children that it shows a group of birds which are standing in order of height. Point out that an X has been placed by the tallest bird and a ring put around the shortest bird. Then discuss the first frame.

Ask the children to place an X beside the tallest kangaroo and circle the shortest kangaroo in the picture at the top of the page. When they have completed this, call their attention to the mice and hats pictured in the next frame. Have them mark the tallest hat and the shortest hat according to your directions. Give similar directions for the last row at the bottom of the page.

Resources for Active Learning

Developmental Math Cards, "Card Height," A²4; "Taller-Shorter," A²16, Addison-Wesley

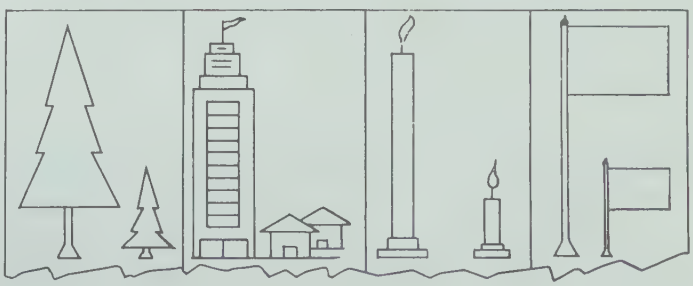
Elementary School Science, Primer, Teachers' Edition, "Short-Tall," T27–T28, Addison-Wesley

Mathex: Measurement and Estimation No. 5, "Tallest, Shortest," pp. 1–3, Encyclopaedia Britannica Educational Corp.

Follow-up

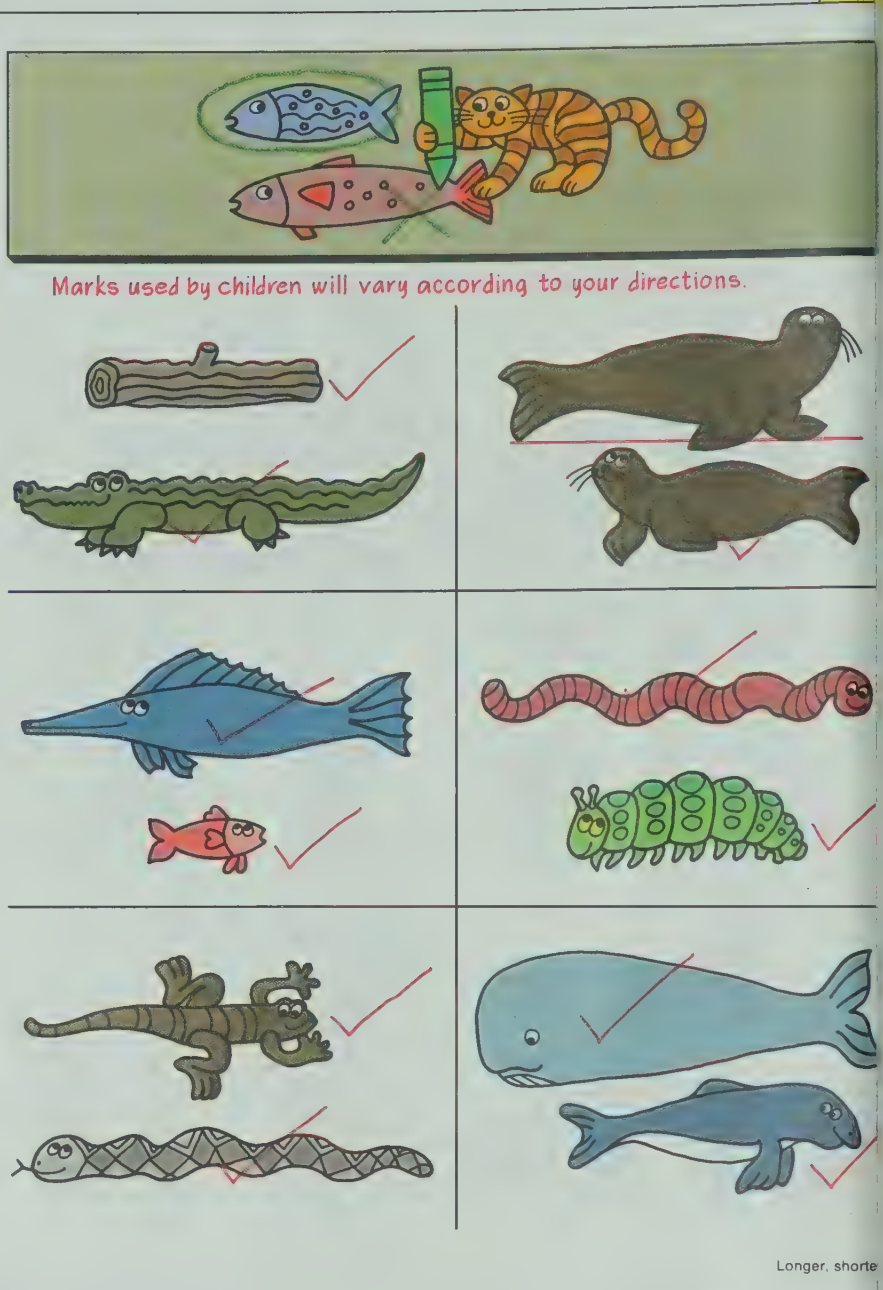
For a game to develop concepts of taller and shorter and to use ordinals in context, pass out different sizes of cardboard cylinders. (The tubes inside rolls of paper towels, waxed paper, or aluminum foil might be cut into various lengths.) Group the children around a low demonstration table and select two children to stand their cylinders on the table while the other children tell which is taller or shorter. Ask the two children to try to change the other children's opinions by rearranging the cylinders. Then ask a third child to place on the table a cylinder he thinks is taller or shorter than both of these already there, and develop the ideas of tallest and shortest. Use other suitable material to maintain interest until all of the children have had a chance to experiment and to participate.

To give children more practice in following directions and working on a printed page, duplicate a worksheet like that below. Give directions one at a time for children having difficulty and provide individual help, if necessary. Directions might be: "Circle the larger tree," or "Color it green," or "Mark a check on the shorter building." Pictorial directions may be put on the board for the more capable children. For example, show the larger figures colored blue and the smaller figures colored yellow.



Teaching
Page p-7

Use the fish at the top to introduce this page. Then call attention to the log and crocodile in the first frame. Have the children place a check mark *on* the longer object and a check mark *beside* the shorter object. Look next at the seals in the second frame. Have the children draw a line *under* the longer seal and place a mark *on* the shorter seal. In the third frame, tell the children to place a check *on* the longer fish and another check *beside* the shorter fish. Complete the page by having the children mark the pairs of objects in the remaining frames according to similar directions. Note that this page can be an exercise in following directions as well as identifying shorter and longer objects.



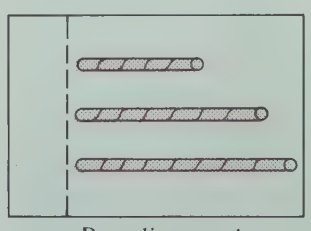
Objective

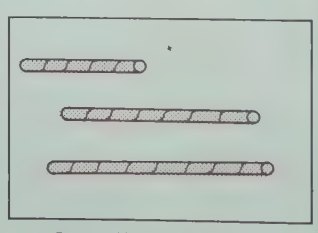
Given objects or pictures of objects which are obviously different in length, the child will be able to identify the longer (shorter) or the longest (shortest) object.

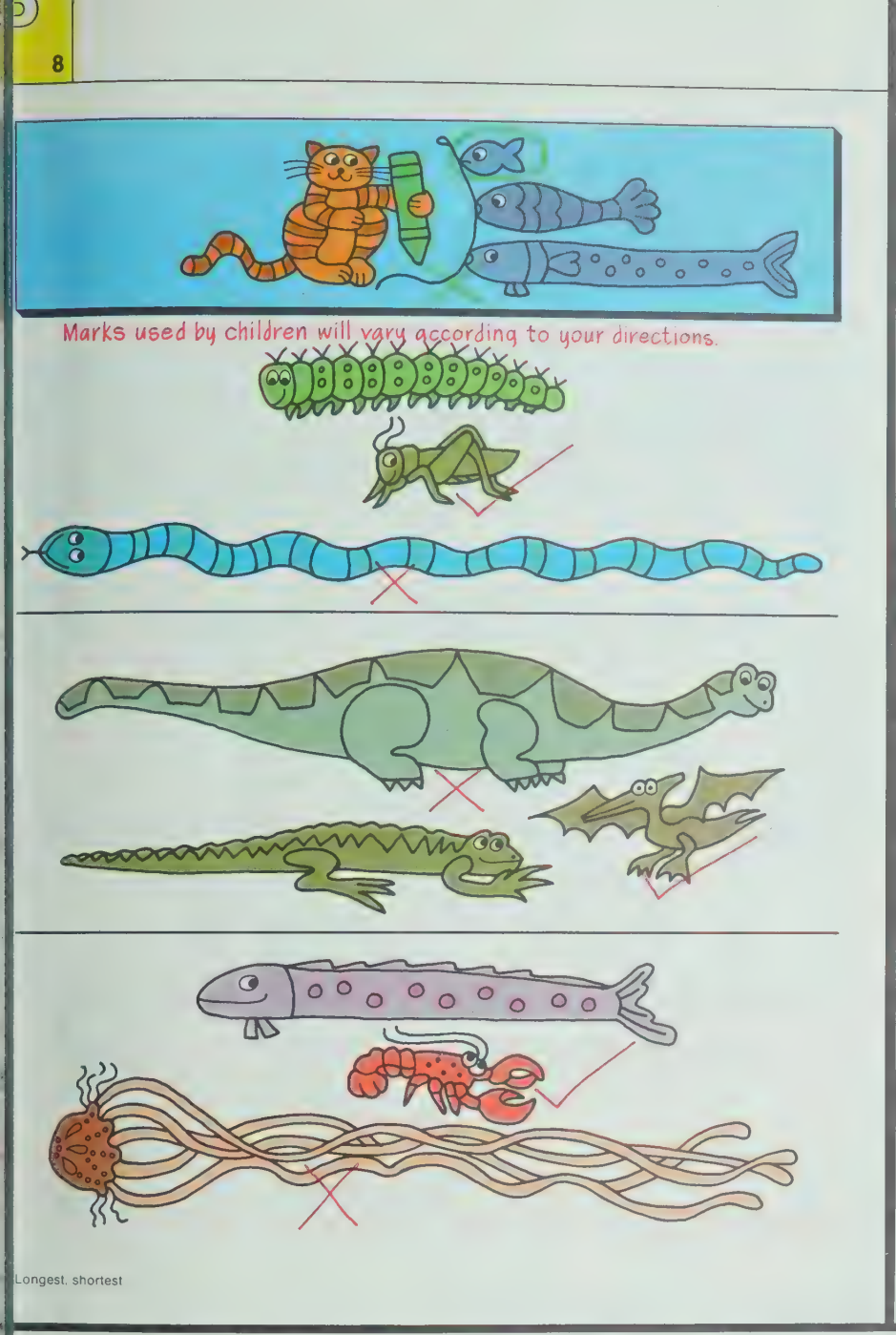
Comparisons related to the concepts of long and short are developed in this lesson. Again it is recommended that extensive use of the vocabulary precede actual work on the printed page.

Pre-Book Activity

Prepare sets of straws which contain straws cut in obviously different lengths. Give one set to each group of three or four children or, if you choose, to each individual child. Also give each group a large piece of construction paper. Explain to the children that you would like them to compare their straws and to line them up from shortest to longest. Supply them with glue and ask them to paste their straws in order of size from left to right or from top to bottom. Note that some children will not use a base line.


Base line used


Base line not used



Teaching
Page p-8

Again use the demonstration art to introduce the page. Then ask children to look at the first frame. You might want to identify the objects in order from top to bottom as caterpillar, grasshopper, and snake. Then ask the children to place a check mark *on* the shortest animal and an X *under* the longest. Give similar directions for the dinosaur, bird, and lizard. Observe with the children that even though the bird's wings are open it is still shorter than the lizard. In the final frame, point out the fish, the lobster, and the jellyfish and again ask children to identify by marks on or beside the shortest and longest sea animals.

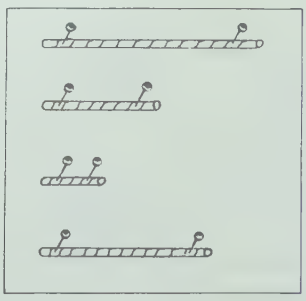
Resources for Active Learning

Elementary School Science, Primer, Teachers' Edition, "Short-Long," T25–T26, Addison-Wesley

Mathex: Measurement and Estimation No. 5, "Longest, Shortest," p. 1, Encyclopaedia Britannica Educational Corp.

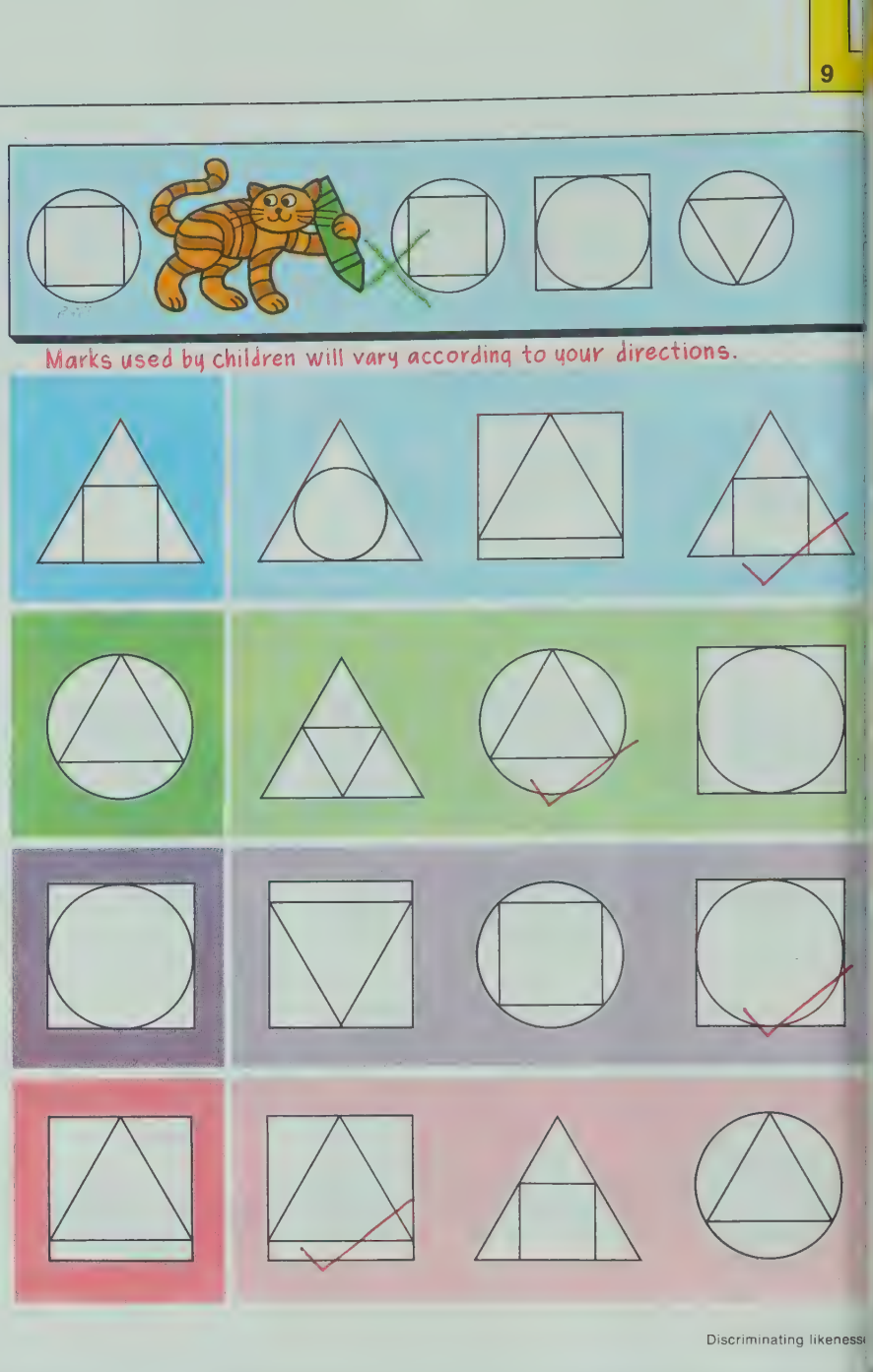
Follow-up

To a flannelboard or posterboard, attach drinking straws or pipe cleaners that are obviously different in lengths. Have children cover their eyes with a blindfold and see if they can pick out the shortest and longest straws just by feeling with their fingers. (See illustration at the right.) It would be helpful to keep the entire board covered with a cloth or piece of paper when children are not using it. This same activity may be simplified by giving a blindfolded child four straws of different length. Then ask him if he can tell which straw is shortest and which is longest. Then ask him to arrange all four in order, from shortest to longest. The child will find this easier if he has a table top to use as a base.



Teaching
Page p-9

Call the children's attention to the demonstration art and ask them to tell why the tiger has placed an X beside the square with the circle around it. Help them to see that this figure is the same as the first one in this row. Then call the children's attention to the figure at the left of the first frame. Ask the children to find another figure in this first row just like the first figure. If some children are unable to keep within a single row, give them markers of approximately 5 by 25 cm made of oaktag or construction paper. They can use such markers to keep their places, if necessary. Tell them to mark the figure according to your directions ("Make a circle around it," "Put a check mark under it," or "Color the inside of it green"). Make a quick check to see that the children have understood and followed your directions. Point out the first figure in the second row and tell the children that they are to find a figure in the row which is like it and mark it as you request. Complete the remaining rows in the same manner.



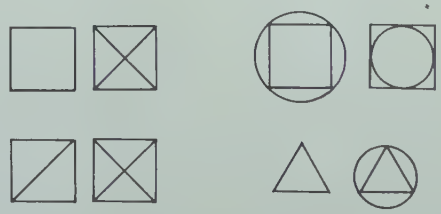
Objective

Given a set of geometric illustrations, the child will be able to identify those which are alike.

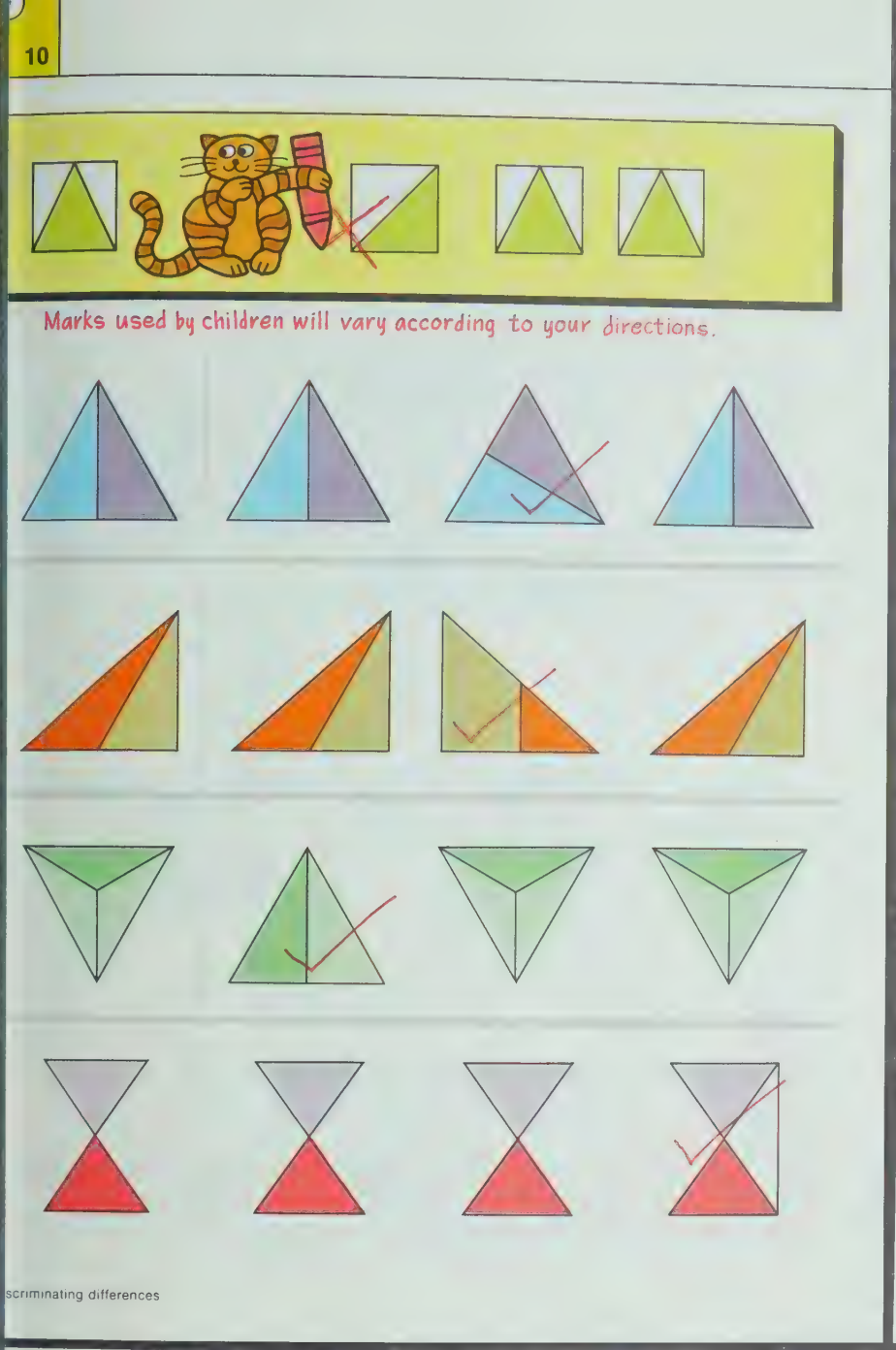
This lesson introduces the child to discriminating likenesses among geometric illustrations. Since the discrimination is a visual discrimination, the pre-book activity is designed to help the child become more aware of what he sees.

Pre-Book Activity

Prepare sets of pictures which can be used to point out visual likenesses. For example, show two pictures of women performing a household task. Point out the similarities and differences. Use other pairs of pictures in the same manner. Try to choose pictures that have many similarities. If possible, show some pairs of the same picture. Then use pairs of large geometric illustrations somewhat like those on page p-9. You might make pairs such as the following.



Encourage children to describe how the pictures are alike and how they are different.



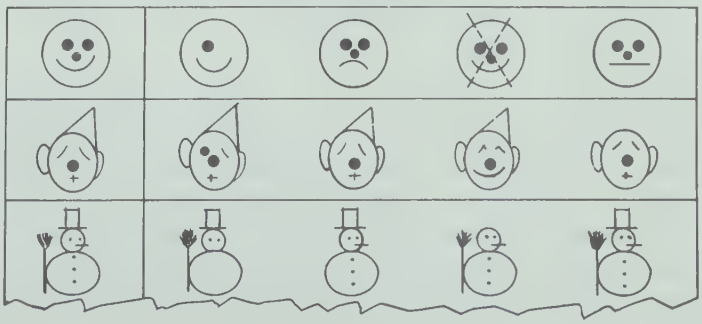
Teaching
Page p-10

Again use the demonstration art to introduce the page. Then call attention to the top row. Tell the children to find in this same row the object that is different from the first and put a check mark on it. Ask those children who are capable to finish the page independently. Continue giving row-by-row directions for children who require them.

Follow-up

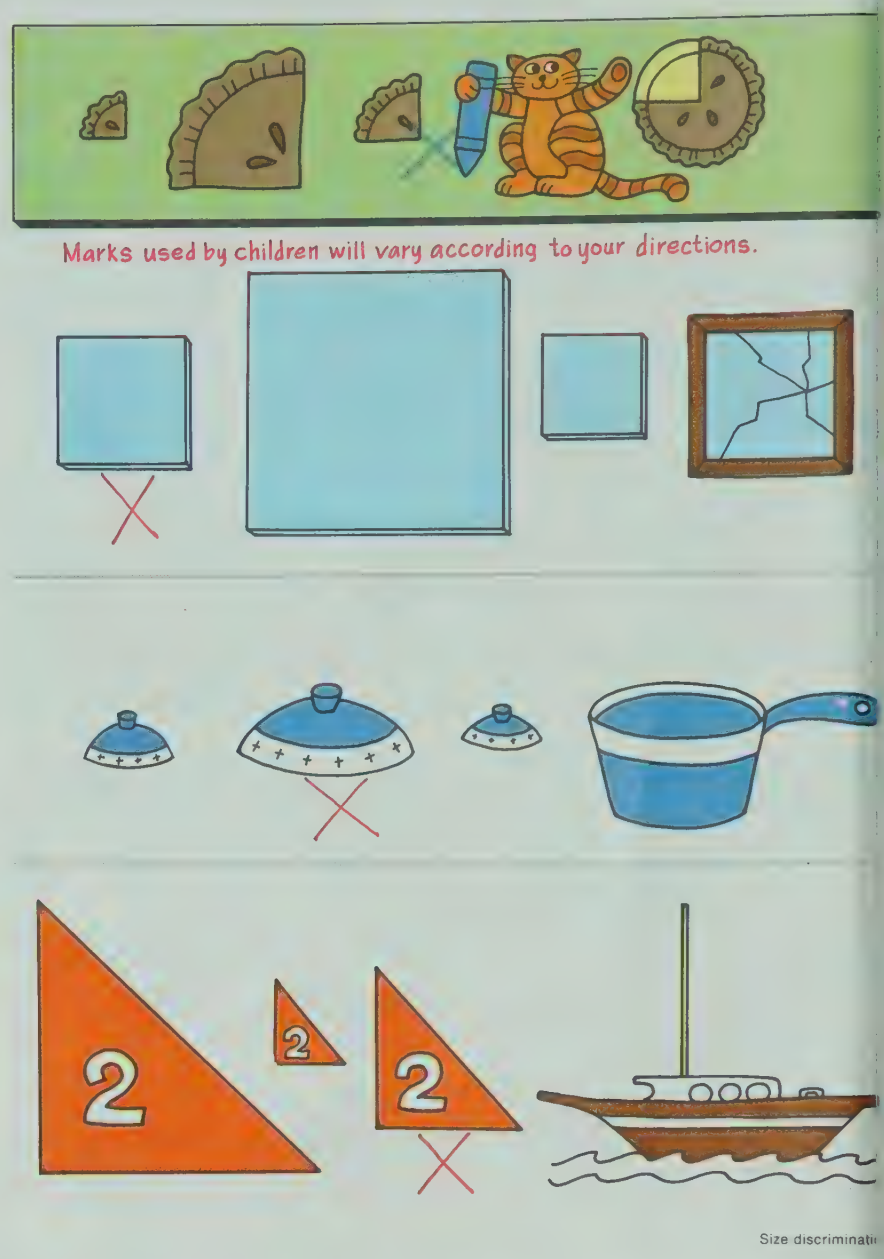
After the children have had some experience in choosing objects that have like characteristics, provide them with individual boxes of sets of objects. These can be egg cartons, cottage-cheese boxes, oatmeal boxes, cigar boxes, or envelopes that you have prepared previously. In each box, have objects such as lengths of ribbon, buttons, yarn, string, rubberbands, can lids, bracelets, pipe cleaners, squares of tagboard, and so on. Place a square region or a piece of yarn on the flannelboard or overhead projector. Ask the children to find as many as they can of the things in their boxes or envelopes which match the displayed shape. Then place several square regions so that their sides are parallel and have the children place all their shapes on their desks in the same position. Next, turn the squares to stand on the corner and again ask the children to place their shapes in a similar position. Continue the activity using other shapes and objects.

To provide more experience with visual discrimination you might prepare a worksheet similar to the following. Direct the children to mark with an X the picture which matches the first one in each row.



Teaching
Page p-11

Discuss the demonstration art with the children. Encourage them to explain why the X mark has been put beside the third piece of pie. Observe with them that only this piece of pie would fit in the space cut out of the whole pie. Then focus attention on the first frame. Discuss with the children the fact that there are three panes of glass and a broken window. Ask, "Which glass fits the broken window?" Direct them to mark their choice with an X. In the next frame, help children identify the lids and the saucepan. Ask, "Which lid should be used for this saucepan?" Finally, for the last frame, ask, "Which sail would best fit the sailboat?" As you work through this page, stress the size discrimination that is necessary in order to choose the proper item.



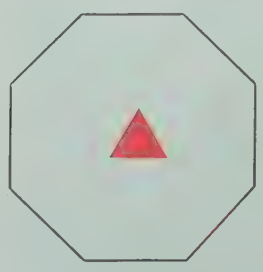
Objective

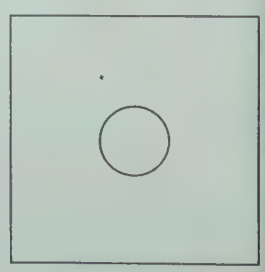
Given appropriate illustrations, the child will be able to match objects which fit together according to size.

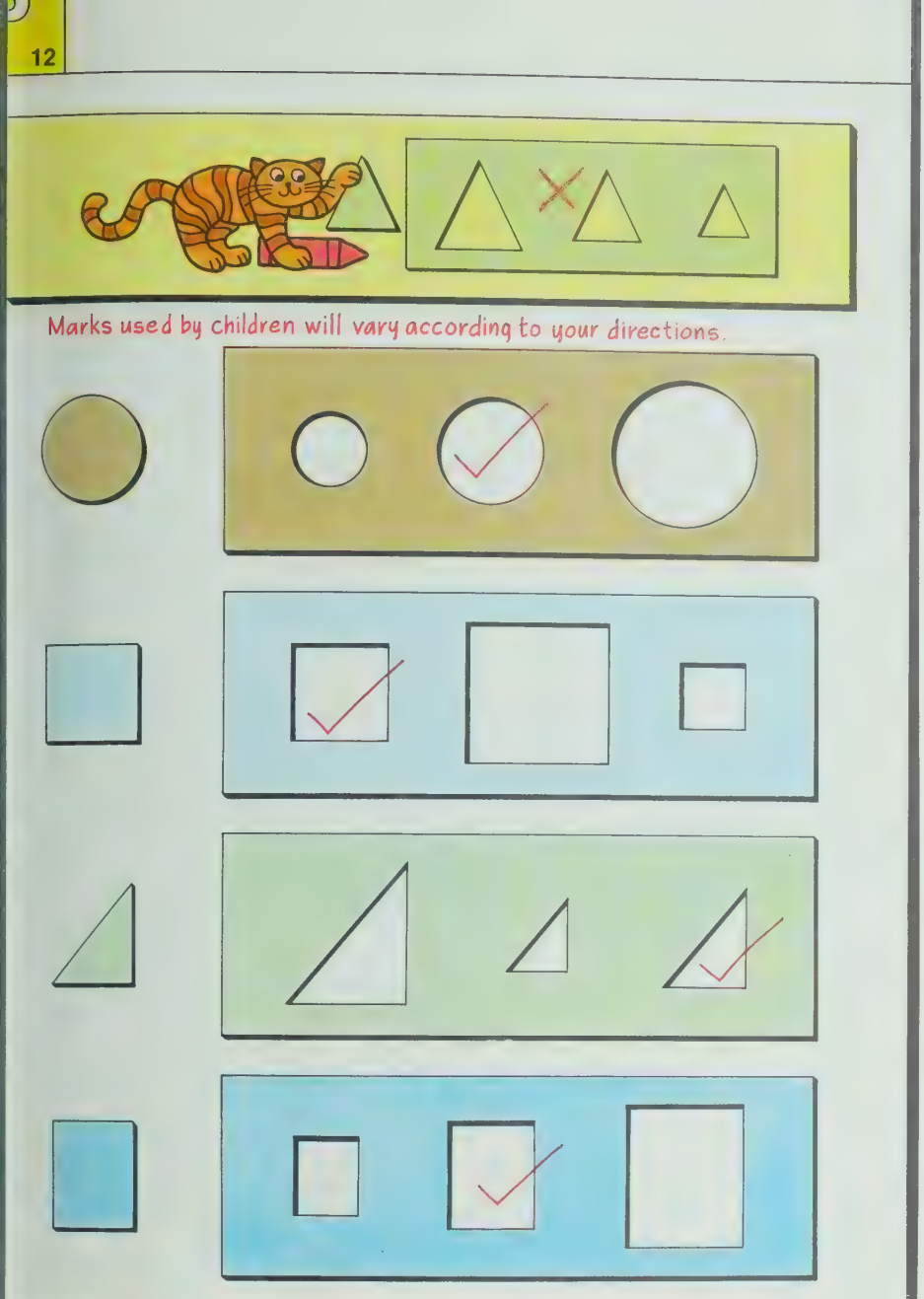
This lesson stresses the importance of size discrimination. On both pages, children are given an opportunity to choose from among three similar items the one which fits the corresponding illustration.

Pre-Book Activity

Display large geometric regions which have had other geometric shapes cut from their centres. The geometric regions should be large enough to allow the cutout portion to be at least 8 centimetres across. The flannelboard would be a good device to use with this activity. Make a variety of other sizes of the same shape. Then have the children try various shapes in the hole until the one that actually fits is found. Try to involve as many children as possible in this activity. Below are suggestions for the geometric regions.







Teaching
Page p-12

Directions similar to those for page p-11 are appropriate here. After discussing the demonstration art, ask the children to look at the first frame. Explain that the circular shape on the left has been cut from one of the three holes on the right. Direct them to put a check mark on the hole from which the circular shape must have been cut. Develop the remaining frames similarly.

Resources for Active Learning

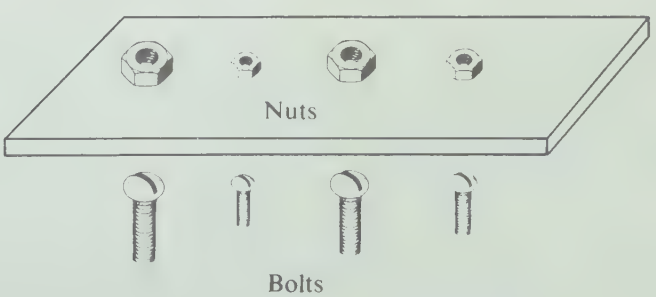
NR Math Activities, "Topology," No. 8511, Midwest Publications

Workjobs, "Jars and Lids," pp. 24–25, Addison-Wesley

Follow-up

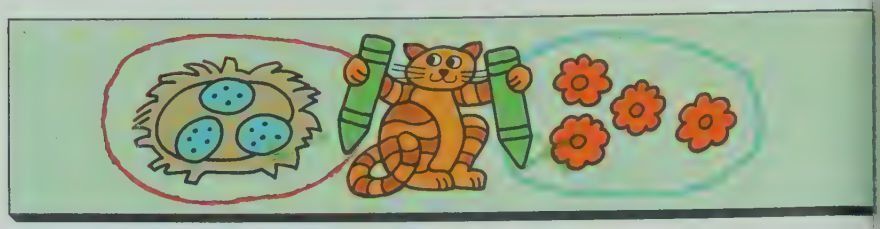
Activities which stress size discrimination would be a good follow-up for this lesson. For example, give the children a set of jars of different sizes and their lids and ask them to find the lids which fit on the proper jars. Give them a set of plastic containers and lids, again of different sizes, and ask them to fit the lids on the containers. Give them a set of measuring cups which fit one into the other and ask them to order these one into the other.

The use of puzzles such as the Tower of Hanoi would be appropriate at this time. You might also prepare a board on which you have glued nuts which vary in size and a matching set of bolts. Ask the children to see if they can find the bolts which fit into the nuts and screw them in. (See illustration at the right.)



Teaching
Page p-13

Use the red and blue rings in the demonstration art to review the concept of inside and to develop the concept of sets. Lead into the ringed sets shown on the tree. Ask the children to mark the group of butterflies inside the red ring with a check and the group of owls inside the blue ring with an X. Point out that we can think of the set of butterflies in the tree. Similarly, the blue ring reminds us that we can think of the set of owls. Continue this discussion by pointing out that the objects in a set need not be circled. We can think of the set of squirrels, the set of apples, the set of birds, and so on. If the children are capable, introduce the idea of the empty set by asking them to think about the set of horses that are in the tree or the set of flying dogs in the tree. However, the concept of empty set is rather sophisticated for most children of this age, so do not belabor this idea.



Set concept

Objective

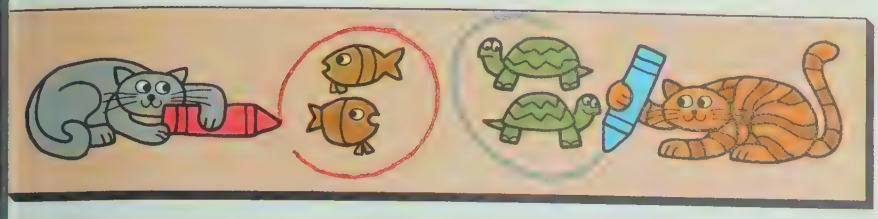
Given an actual or illustrated collection of objects, the child will be able to think of certain subsets of these objects as a single entity or set.

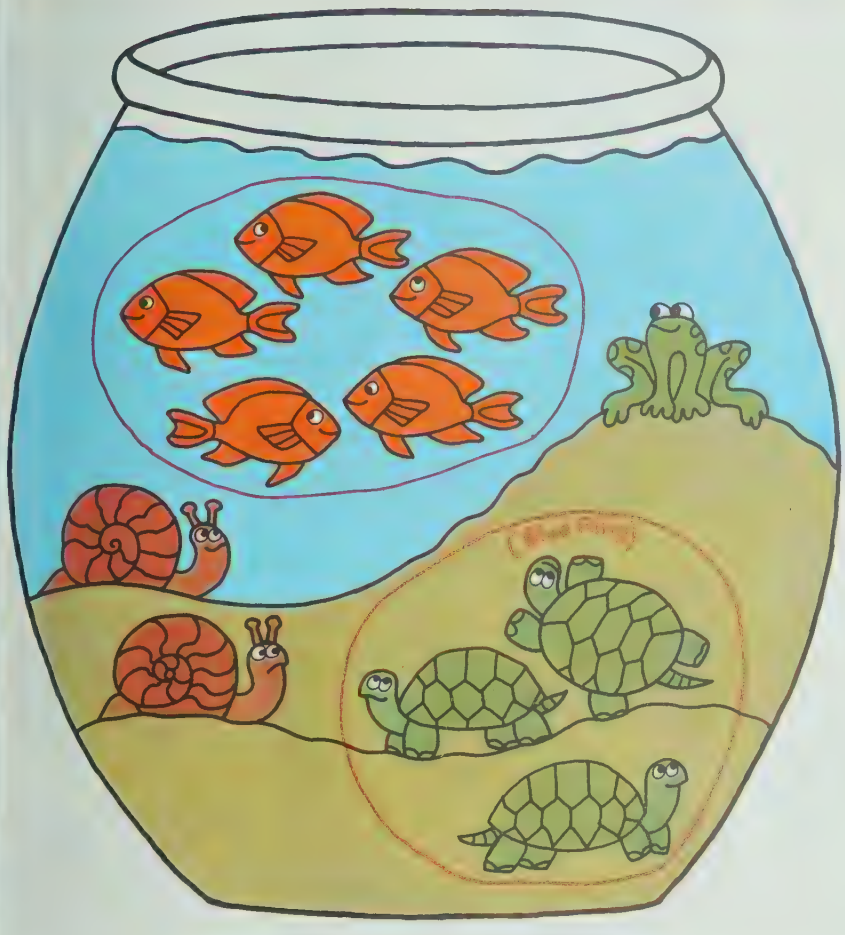
The main objective of this lesson is to encourage the children to think about sets, groups, or collections of objects as single things. For example, the child might experience thinking of the set of birds in a tree, the set of children in the classroom, the set of spoons on a table, the books on the second shelf, or the like.

Pre-Book Activity
Materials

jars, buttons, beans, screws, rubber bands, pieces of gravel, nuts, erasers, bits of paper, washers

Prepare collections of various objects for groups of three or four children to sort. Give an assortment of the items listed above to each group of children. It would be helpful to provide jars or plastic containers into which children can sort their items. Then use the collections in the jars to introduce the concept and terminology of *set*. For example, we can talk about all of the buttons and say that we have a *set* of buttons. As long as we can say about something on the table "This is a button" or "This is not a button," the *set* of buttons is a well-defined idea. We can restrict our idea, if we choose, by talking about the *set of buttons in the jar;* this is a subset of our set of buttons. Use examples such as this to develop the children's concept of a set. Remember that the objective is to help children think about a collection of objects as a single entity.





Teaching
Page p-14

The illustration on page p-14 provides further material to use in your discussion of sets. Point out to the children that the demonstration art shows them what you would like them to do: put a red ring around the fish and a blue ring around the turtles. Observe with the children that they might think of the set of fish or the set of turtles. Then encourage them to discuss sets of anything they did not ring, such as the set of snails and the set of frogs. You might ask them to ring the snails and frog, but stress the idea that objects need not be put in a ring in order for us to think of them as a set.

Follow-up

Give the children many opportunities to sort objects into various collections. Again the jars will be useful for such activities. Use materials such as blocks, pencils, paper, paint brushes, pieces of chalk, keys, sunflower seeds, flowers, stamps, buttons, erasers, safety pins, needles, snaps, rings, screws, and nails. Have prepared for the children a picture showing a home and a picture showing a school. Ask the children to sort the objects into two piles, one which they would use at home and another which they would use at school.

Another set of pictures to be used in this manner might be outdoors and indoors. Instead of giving actual objects to sort for these pictures, prepare pictures on cards. For example, you might prepare pictures of a lawn mower, dish pan, garden hose, automobile, chair, patio chair, bed, and so on. As children work, use set terminology when speaking of the collections that they form as sets and speaking of the items in the collection as members of a set; many children will gradually follow your example in using these terms.

Teaching
Page p-15

Children will need careful directions for each frame. Call their attention to the first frame. Ask them to find the smallest animal and to put a ring around it. Then ask them to mark an X on the largest animal. Move to the second frame and direct the children to put a ring around the shortest animal and to mark an X on the longest animal. Finally, call attention to the last frame and ask the children to mark an X on the wheel which fits the bicycle. You might point out that the wheel on the back of the bicycle should be the same size as the wheel on the front.

Show you know

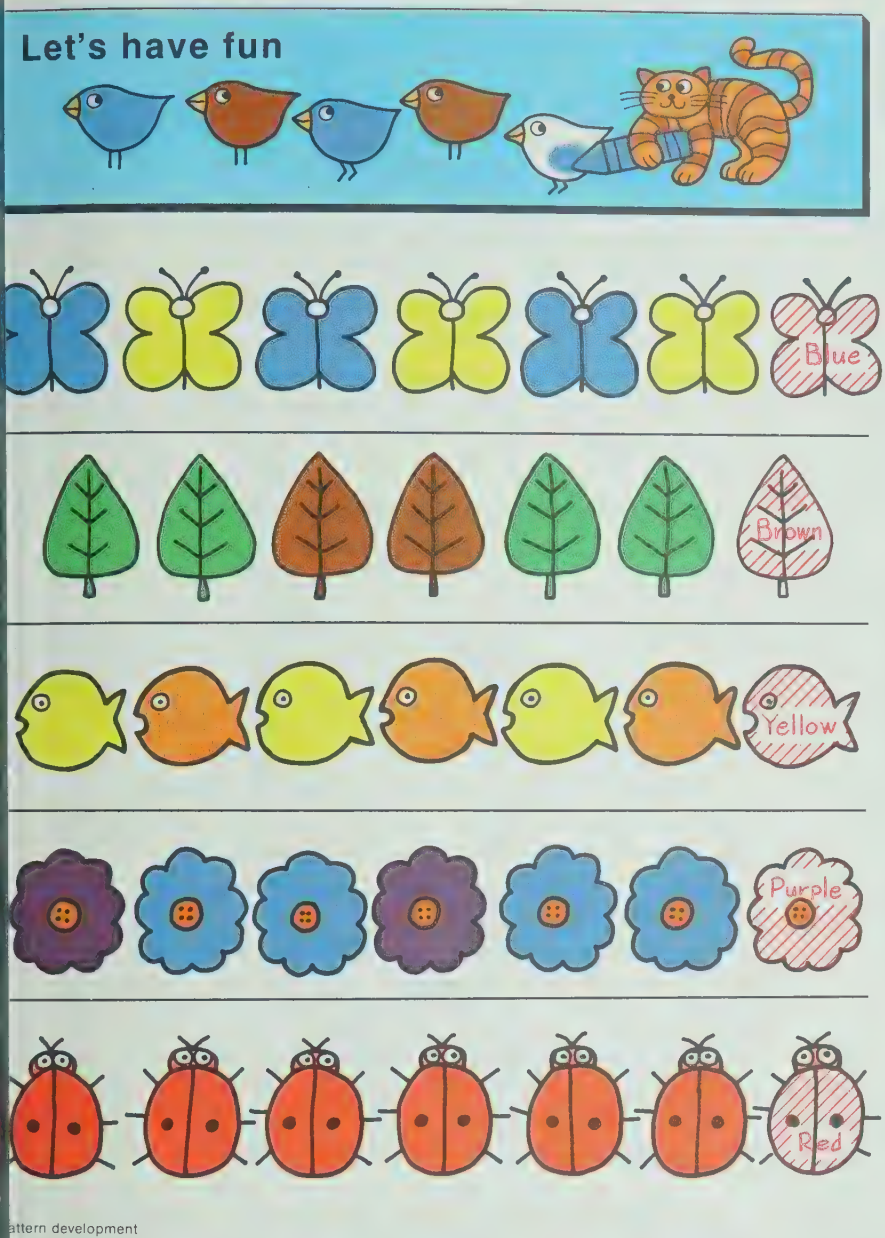
Objective

Given illustrations which show size discrimination, such as largest and smallest or longest and shortest, the child will be able to identify the objects pictured according to oral directions.

Page p-15 may be used as a review of the concepts presented on pages p-1 through p-14. Page p-16 is intended as a change-of-pace page. Such a page at the end of a module often introduces a topic that has not been treated in the module but does not need extensive development.

Pre-Book Activity

To continue the development of various types of discrimination, provide opportunity for work with different materials and for work with various quantities. For example, you might use this time to help the children measure quantities of sand, water, and clay. For each material, have two large plastic containers, one holding the material and the other to be used by the child as a receptacle. Then put a set of measuring containers with each material. Ask the children simply to experiment and explore using these containers to pour the materials. It would also be helpful to review with the children the relational terms which they will use on page p-15 and which they have used in the module. For example, show the children two pieces of string obviously different in length and ask the children to explain how they can describe the difference in the two pieces of string, stressing the use of the words shortest and longest. Point out objects in the classroom to review the terms higher, highest; lower, lowest; taller, shorter; smallest, largest.



Teaching
Page p-16

You might find it helpful to prepare for this page by allowing the children to string beads. Watch carefully to see whether they develop patterns on their own. If they string the beads with no apparent pattern, ask them to try stringing another set according to a pattern, such as only beads that are alike or perhaps two kinds of beads in an alternating sequence. When you direct the children to page p-16, call attention to the set of birds at the top. Point out the coloring pattern used. Then discuss the pattern used for the row of butterflies. Explain that they should complete the pattern by coloring the last object. Similarly in each of the following rows, they should color the uncolored items so that each row's pattern is completed. Help those children who have difficulty, but avoid giving specific answers.

Resources for Active Learning

Developmental Math Cards, "Patterns," A^{3}13; "Patterns with Tiles," A^{3}19, Addison-Wesley

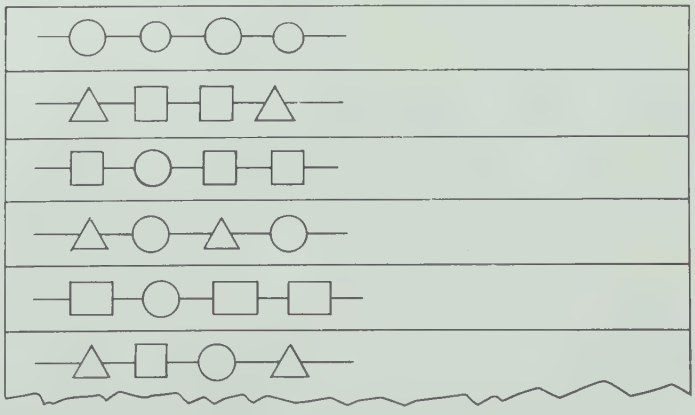
Mathex: Matching and Graphing No. 1, "Patterns," pp. 12–13, Encyclopaedia Britannica Educational Corp.

Math Workshop: Games and Enrichment Activities, "Pattern Activities," pp. 8–9, Encyclopaedia Britannica Educational Corp.

Follow-up

To give children more practice in discovering patterns, provide each group of three children with shoestrings and boxes of objects: buttons, lengths of paper or plastic soda straws, perforated seeds, spools, paper or cardboard disks, and macaroni, as well as conventional wooden beads. Urge each group to devise a secret pattern and to string enough objects to have the pattern repeat at least twice. Then knot a small piece of bright colored yarn on each shoestring and pass the string and box of objects on to another group. Each group should determine the pattern devised by the first group and complete it by stringing two more repeats of the given pattern.

You might also provide additional practice in completing patterns by duplicating worksheets like the one below. Allow children to complete their patterns in any way they choose, so long as they show a pattern.



ORANGE MODULE, UNIT P

Geometry and Classification

Pages p-17 to p-32

General Objectives

To provide pre-number experiences with classification as a readiness for developing number concepts

To develop the child's ability to classify according to function or use

To develop the child's ability to classify according to external attributes such as color, size, and shape

To develop the child's ability to separate a set into two subsets and to think of a set separated into a subset and its complement

The emphasis of this module is on classification experiences. Research of Piaget and others has suggested that pre-number experiences with classification are of extreme importance in forming the foundation for concept development. Crucial to any concept learning is the ability to select attributes and classify objects or entities that have an attribute in common. According to Piaget, this ability is often not attained by the five-year-old child, but appropriate experiences at this level can enrich the child's environment and enable him to begin to classify according to his own choice of attributes.

Mathematics

In this module the children will be given opportunities, first, to focus on similarities and differences (a prerequisite for classification) and then to classify according to function (or use) external attributes such as what they do or where they live, or color, size, or shape. Another aspect of classification which is important is the ability to separate a set into two subsets. Yet another significant ability to be fostered is the ability to think of a set separated into a subset and its complement. It is here that the embryonic idea of *negative* emerges. Beyond this, the child should have experience sorting objects into more than two subsets. The child should also be able to categorize by focussing on conjunctive attributes, that is, two attributes at once. These beginning experiences form the core of the activities in this chapter. An overriding goal is that of bringing the child to the level of ability in classification at which he is able to choose the distinguishing attributes and classify according to those attributes.

Teaching Orange Module, Unit P

Materials

cutouts of counters: 5 small blue circles, 5 red circles (2.5 cm in diameter), 5 small blue squares, 5 red squares (2.5 cm)

pictures or toys representing horses, dogs, cats, automobiles; trees, flowers, rocks, leaves; insects, birds, fish, houses, buildings, and so on

play sets for doctor and nurse

fire trucks, tools, comb, brush and mirror

colored beads or counters or small squares of paper

colored objects (anything available)

cutouts of triangles, squares, and circles of at least two sizes

paper bags

yarn, two long pieces of different color

Vocabulary

belong	differences	likenesses	small
blue	inside	outside	square
circle	large	red	yellow
color			

The most useful material for making classifications is a very familiar object, the box. The box is a physical object with which the child is well acquainted. It is very natural for the child to think about separating objects and placing them in boxes. In fact, the physical act of placing in boxes is easily transferred to a mental thought process of placing in boxes; hence, the box is a valuable aid in preparing children for the experiences provided in the lessons. Actual boxes should be used in the classroom to provide sorting experiences for the children.

Lesson Schedule

The amount of time needed for the activities suggested in this module will vary greatly from child to child and from group to group. Some children will need many classification activities before they will gain much benefit from the work on the printed page. Therefore, time should be given to such activities even though there may be days when the booklet pages are not used at all.

Evaluation of Progress

A child's work on the modular pages will often serve as an evaluation instrument. However, a clearer understanding of the child's ability to classify will come from daily observation and conversation about a classification activity in which he is involved.

Resources for Active Learning

General Activities

Attribute Games and Problems, Webster, McGraw-Hill
Developing Number Experiences, Kit A, "Classification

Games and Activities," Teachers' Guide, pp. 1–40, Holt, Rinehart and Winston

Developmental Math Cards, ". . . Scales Balance?" A$^{2}_{2}$6; "Sorting Objects," A$^{1}_{2}$7; "Sorting Tasks," A$^{3}_{2}$11; "Building with Cuboids," A$^{2}_{2}$14, Addison-Wesley

Elementary School Science, Primer, Teachers' Edition, "Matter," T66–T88, Addison-Wesley

Mathex: Matching and Graphing No. 1, "Sorting," pp. 1–4; Measurement and Estimation No. 5, "Thickest . . .," pp. 3–6, Encyclopaedia Britannica Educational Corp.

Nuffield Project: *Mathematics Begins* 1, "Sorting . . .," pp. 10–26, Wiley

Workjobs, Classification Activities, pp. 68–107, Addison-Wesley

Manipulative Devices

Basic Shapes Set (Educational Teaching Aids; Math Media; Responsive Environments Corp.)

Classification; Number Measurement and Space Kit (Learning Research Assoc.)

Cubēduc (Educational Teaching Aids)

Foundations for Mathematics (Teaching Resources)

Geo Shapes (Scott Foresman)

Parquetry Pieces and Patterns (Ideal; Teaching Resources)

Poleidoblocs Set G (Responsive Environments Corp.; Selective Educational Equipment)

Commercial Games

Connect (Responsive Environments Corp.)

Shape Analysis Matching Game (Math Media; Responsive Environments Corp.)

(For additional suggested manipulative devices and commercial games, see the introductory notes for the Yellow Module and the Red Module of Unit P.)

Activities for Continued Use

Module 2 provides children with many important experiences with classification. Activities which develop understanding of words such as the following are also helpful: *not, and, all, some, or.* As children work through the activities, give them language models to help them express the activity of classification with which they are involved. As an example, notice the questions which are used in the following activity, which deals with the ideas of "all" and "some," that is, with class inclusion. Young children will usually need many chances to physically manipulate objects as such questions are being asked.

Materials: 10 toy horses, 2 toy cows, or the equivalent
Procedure: Distribute the toys and ask the children to describe the kind of animals the toys represent. Help them verbalize that some are horses and some are cows. Ask, "Are there more horses than there are cows?" Again, give those who need it a language model to help them verbalize their response. Then ask the children to put all the toy animals into one bunch. Now ask, "Are all the toys animals?" "Are the horses animals?" "Are the cows animals?" "Are there more horses, or are there more animals?" The younger, less mentally developed child may respond that there are more horses than animals, or that there are more animals than cows. He will benefit from other physical activities with other groups and subgroups of materials. You might also ask which bunch is bigger, the bunch of cows, or the bunch of animals; the bunch of horses or the bunch of animals?" Also ask them to put all the animals that are *not* horses together; all the animals that are *not* cows together. If the toy horses are different colors, such as spotted or brown, you might ask them to put all the brown horses together; all the brown animals together. In each case, help children verbalize their response.

Other sets of materials may be used similarly, such as flowers: red roses, yellow roses, and daisies; or wooden beads: red beads and blue beads. Attribute pieces may also be used. In each instance, keep one subgroup obviously greater than the other; for example: 10 red roses, 2 yellow roses, and 5 daisies. For further reference, see Copeland, *How Children Learn Mathematics,* pages 33–39, and Lavatelli, *Piaget's Theory Applied to an Early Childhood Curriculum,* pages 91–95.

Investigation
Page p-17

After the child has received his counters and the page, ask these questions as you point out the brown and the orange rugs on the page: "How would you place your counters in the trays on the brown rug?" "How would you place your counters in the trays on the orange rug?" Be sure the children understand that they should place all of their counters first on the brown rug. It is possible that the children will notice in the brown section that one box is colored red and the other box is colored blue. If they do and if they observe such classification from the demonstration art at the top of the page, they are likely to place the red counters in the red box and the blue ones in the blue box. This investigation will enable you to note which children seem naturally able to classify the figures according to color and which seem to classify more at random. Be ready to accept other interesting means of classification.

Direct children's attention to the bottom section of the page by asking, "How would you place your counters in the trays on the orange rug?" Again, allow for a variety of methods of classifying. Here, since one box is square and one box is circular, it is possible that children will match the shapes of their counters to the shapes of the boxes. When they have finished, have a brief discussion of how the children classified their shapes.

Purposes

To introduce the child to simple classification
To introduce classification according to color
To introduce classification according to shape
To introduce classification according to size

This lesson introduces the child to the classifications which will be developed more specifically in subsequent lessons. It also provides an opportunity for you to evaluate a child's ability to classify, thus enabling you to better plan the activities for the remainder of this module.

Preparation

Materials

5 each blue circles, red circles, blue squares, red squares (NOTE: These materials should be saved for use with the investigation on page p-33.)

Prepare the following counters for each child: 5 blue circles, 5 red circles, and 5 yellow circles (2.5 cm in diameter); also, 5 blue squares, 5 red squares, and 5 yellow squares (2.5 cm). Be sure to save these counters for later use.

Encourage the children to discuss briefly the counters. Stress phrases such as "square ones," and explain that the "round ones" are called "circles." Then introduce the *Investigation.*

Discussion
Page p-18

This page may be treated as a separate lesson or may be included in the same activity just described in the investigation. Be sure that each child can identify each toy. Give directions like these: "Pretend the top toy box is yours. Write your name or make a special mark on it with your crayon. The other box is for a friend. Which toys will you put in your box? Which toys will you give to your friend? Draw lines from the toys to the boxes to show this." Be sure the child realizes that he is being asked to select some of the toys for himself and draw a line to his box and also that he is being asked to give some of the toys to a friend and draw a line from those toys to that box. The objective is to get the children used to sorting and to thinking about sorting by putting the objects in a box. The notion of putting toys in a box is a very natural one. It is easy for children to make the transition from the actual box idea to the picture of the box.

Following the child's activity it would be helpful to ask the children to tell which toys they chose and why. It should be emphasized that there is certainly no right or wrong answer for this page, and it is designed to provide discussion and to provide a beginning experience in classifying or sorting.

Follow-up

It would be helpful to extend this lesson by allowing the children to participate in an activity suggested by the discussion for page p-18. For example, provide the children with some large boxes into which they can put toys that are in the classroom. If these materials are already organized in this manner, simply use the actual classroom situation to extend the discussion. Be sure to respect any original suggestions children offer concerning different manners of classification. For instance, rather than classifying toys according to interest for a boy or a girl, they might be classified as large or small and placed in the boxes accordingly. Or, they might be classified in terms of those that roll and those that do not; all those that roll or have wheels might be placed in one box and the others might be placed in another.

Teaching
Page p-19

Point out the set used in the demonstration. Discuss why the bird has been circled. Then discuss the sets in each frame. Ask the questions, "Which is different and does not belong with the others? Why?" "Why do the other things belong together?" Encourage the children to explain the reasons for their choice of the item that does not belong in the set. Also, help them observe the manner in which the items of a set are alike. Point out in the first frame, for instance, that all the items except the fork are used in painting. (Children might benefit from the use of a stiff card as a place marker to help them focus on each separate frame.) Direct the children to mark with an X or a check mark each item that does not belong. Provide extended experiences with actual sets for any children who find this page difficult.

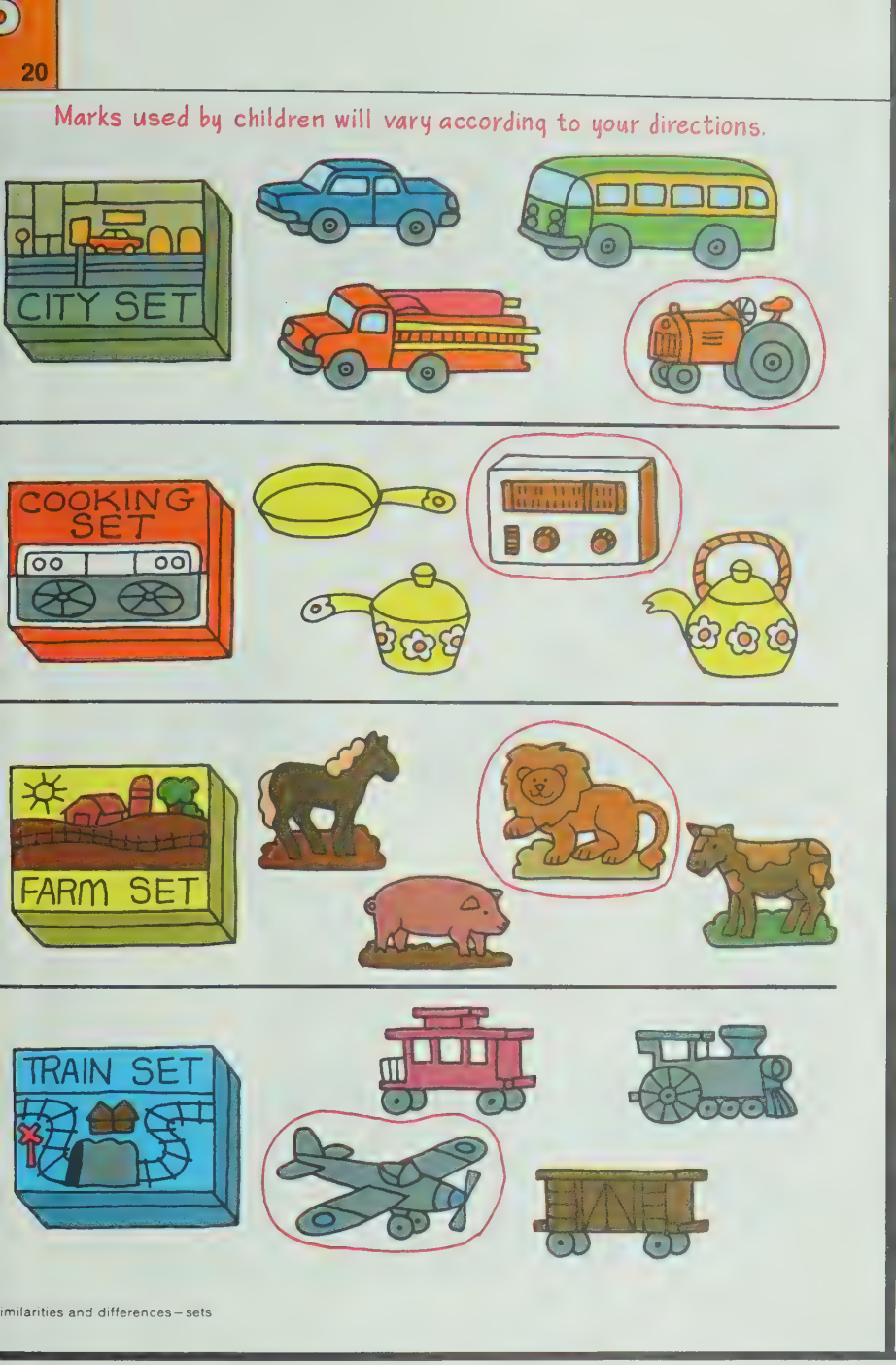
Objective

Given a set of clearly related objects, the child will be able to identify an object which does not fit in the set.

The idea of "belongs to" is closely related to concepts developed in the study of sets. This lesson provides an opportunity to discuss the similarities and differences among objects which belong or do not belong to a particular set.

Pre-Book Activity

Capitalize on any situation in the classroom which might be used to show how some objects are related. For example, if children have a playhouse which they use, place a large box of playground equipment inside it and ask the children if they can see anything in it that does not belong. Then discuss why the box of playground equipment does not belong. Or, display one ball and three different kinds of blocks. Discuss the way these objects are related and ask the children which one is different from all the rest. Display pictures of other sets of objects, such as a rabbit, a horse, a dog, a cat, and an automobile; or a tree, a flower, a rock, and a leaf. In each case, discuss with the children how one object differs from the others in the set. If sets similar to those shown on the page are available, it would be helpful to actually have the set on a table and place in it something that does not belong in the set. For example, collect all the tools together in a set and have a ribbon or a comb in the tool set and ask children to explain why the ribbon or the comb does not belong in the tool set. Many such experiences with actual sets would be helpful.



Teaching
Page p-20

The development of this page could be similar to that for page p-19.

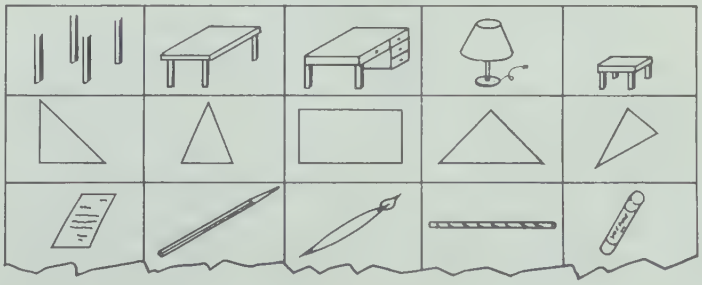
You will probably want to discuss each scene shown at the left of the frame to be sure that the children understand each category that is depicted. Only then will they be able to make a valid judgment concerning the item that does not belong in the set.

Follow-up

Supply the children with pages from toy catalogues. Ask them to cut out objects and paste on a piece of paper three objects which belong to a set and one which does not. For example, a child might cut out some dolls and some fire trucks and then place three dolls and one fire truck on a set. You might also encourage them to circle the object which they have placed there which does not belong to the set.

Another possible activity is that of picture dominoes. Large dot dominoes or picture dominoes provide fun and continued opportunities for discrimination activities. To make a worthwhile classroom game, select four to six categories, such as fruit, flowers, vegetables, trees, and seeds, and paste seals or pictures on either end of small index cards to make picture dominoes.

You might also duplicate worksheets to provide further pencil and paper discrimination activities. Urge the children to use the picture clue to find the object or objects that are different. If they are very good at finding differences, you may wish to leave out the clue at the beginning of each row and let them discover the likenesses and differences for themselves. Such an activity worksheet might be similar to the following.



Teaching
Page p-21

Give the children an opportunity to talk about the objects that they see on the page. The children should naturally think of things that you eat and things that you wear. If they do not, however, ask them questions such as these: "What kind of sack is shown at the top?" "Where would the sack at the top be used?" "Where would the box at the bottom have come from?" "From what kind of store?" and so on. Help the children to understand that the shopping bag at the top could be the container for the food items. Suggest that they draw lines from any object which they think would go into the shopping bag to the bag, and draw a set of lines to the box for items they might purchase at a department store.

Objective

Given two or three subsets related by external attributes, the child will be able to classify the objects in the sets by either their use or their habitat.

Pre-Book Activity

Discuss with the children different objects in the classroom and decide what these items are used for. When their use is determined, a label might be put on them. For example, a label on which you have printed the word "writing" might be put on anything used for writing or coloring. A label reading "sitting" might be put on things used to sit on, and a label reading "teacher" might be put on things which the teacher uses. Objects such as these might also be labelled by pictures: a picture of a teacher on everything that the teacher uses, a picture of a child on everything that the child can sit on, or a picture of a pencil and paper on everything that is used by the children for writing or coloring.

If you prefer a pre-book activity that is more suitable for small groups, you might adapt the follow-up suggested at the end of this lesson. For example, cut out some common objects from magazines or newspaper advertisements, paste them on cards, and ask the children to sort the cards into piles. One set of cards might include collections of chairs, sofas, etc. (things that they would sit on); another collection might include different kinds of tables or shelves (objects that you set things on). Other sets of cards could be made for food items and items that you drink. Groups of children might sort one deck of cards and then trade with another group of children and sort their group of cards.

assification by an external attribute

Teaching
Page p-22

Point out to the children the house, the tree, and the fish pond. Explain that each of the animals lives in one of these places. Help them realize that they should simply draw lines to show where each of the animals lives; that is, does it live in a tree, in a house, or in a pond? You might also suggest that children use different crayons for the lines to the tree, the house, and the pond.

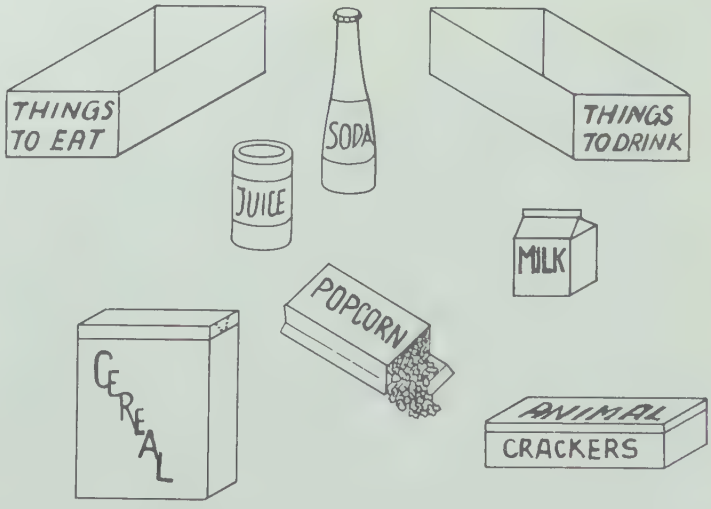
It would be helpful to make sure that each child knows the animals that are being classified. Allow for different classifications in this situation. For example, a snail might live in a house in an aquarium, and some snails might be tree snails, though most children will associate the snail with the pond.

When the children have finished, provide ample time for them to discuss their choices.

Follow-up

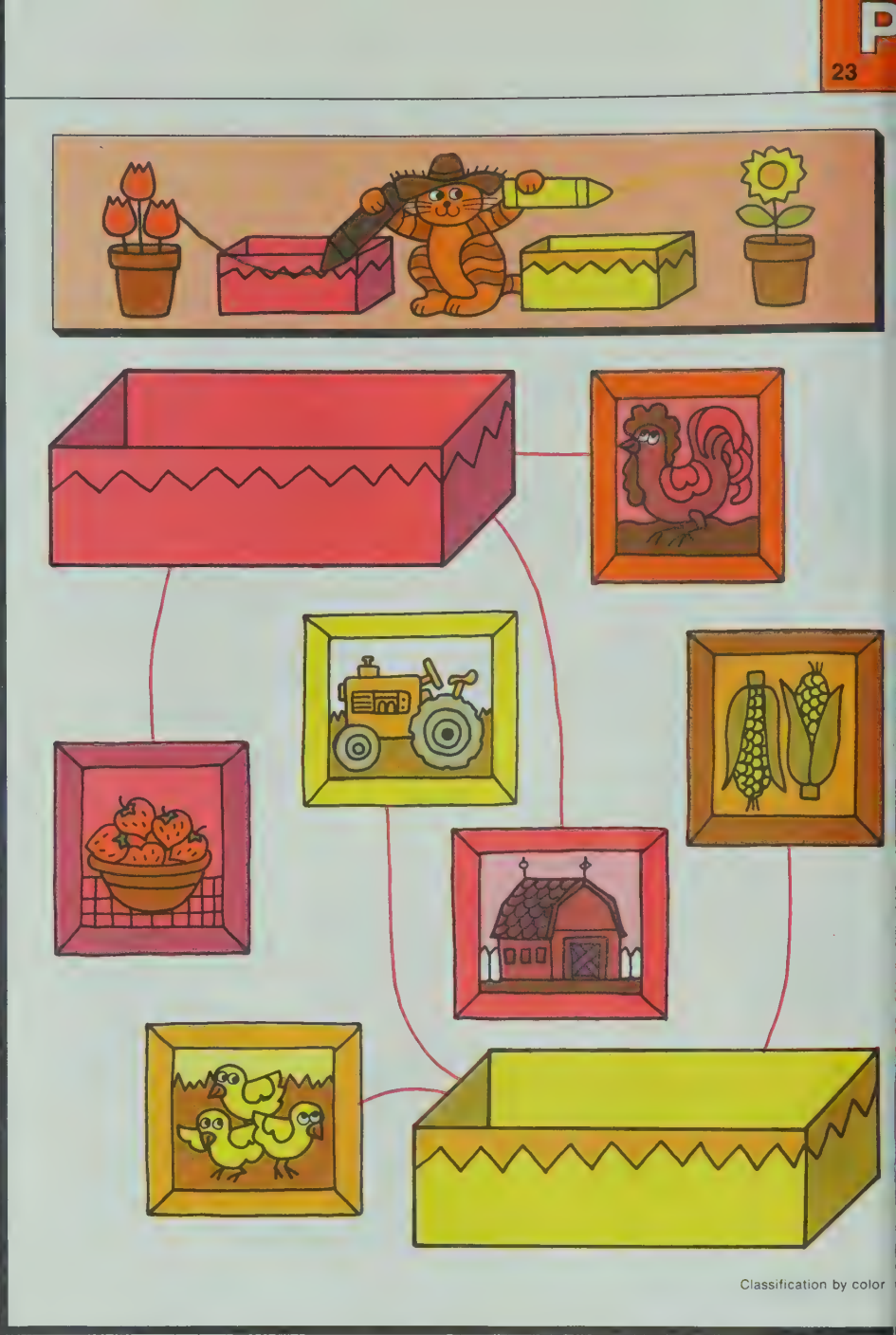
The following suggestions for items to classify might be handled by pasting pictures of these on cards or, if feasible, by grouping actual objects in boxes, as suggested by the illustration at the right.

1. *Things to sit on:* chair, sofa, stool, pillow
   Things to put things on: shelf, table, desk top
2. *Things to read:* book, cards, street sign
   Things to listen to: radio, record, chimes
3. *Things to eat:* apple, jello, cereal, sandwich
   Things to drink: juice, soda, water, milk
4. *Hard things:* nail, pencil, key, scissors
   Soft things: pillow, fur, whipped cream, velvet
5. *Rough things:* sandpaper, coarse wool, scouring pad
   Smooth things: silk, smooth paper, chalkboard, tiles



Teaching
Page p-23

Ask the children to identify by color the two boxes on the page. Ask: "What color is the box at the top?" "What color is the box at the bottom?" Give directions like these: "Draw a line to the red box from every object which you think should be put into the red box. Draw a line to the yellow box from every object which you think should be put into the yellow box. Be sure that every picture has been matched to one of the two boxes." Use the page as a basis for pointing out how things may be sorted according to color.



Classification by color

Objectives

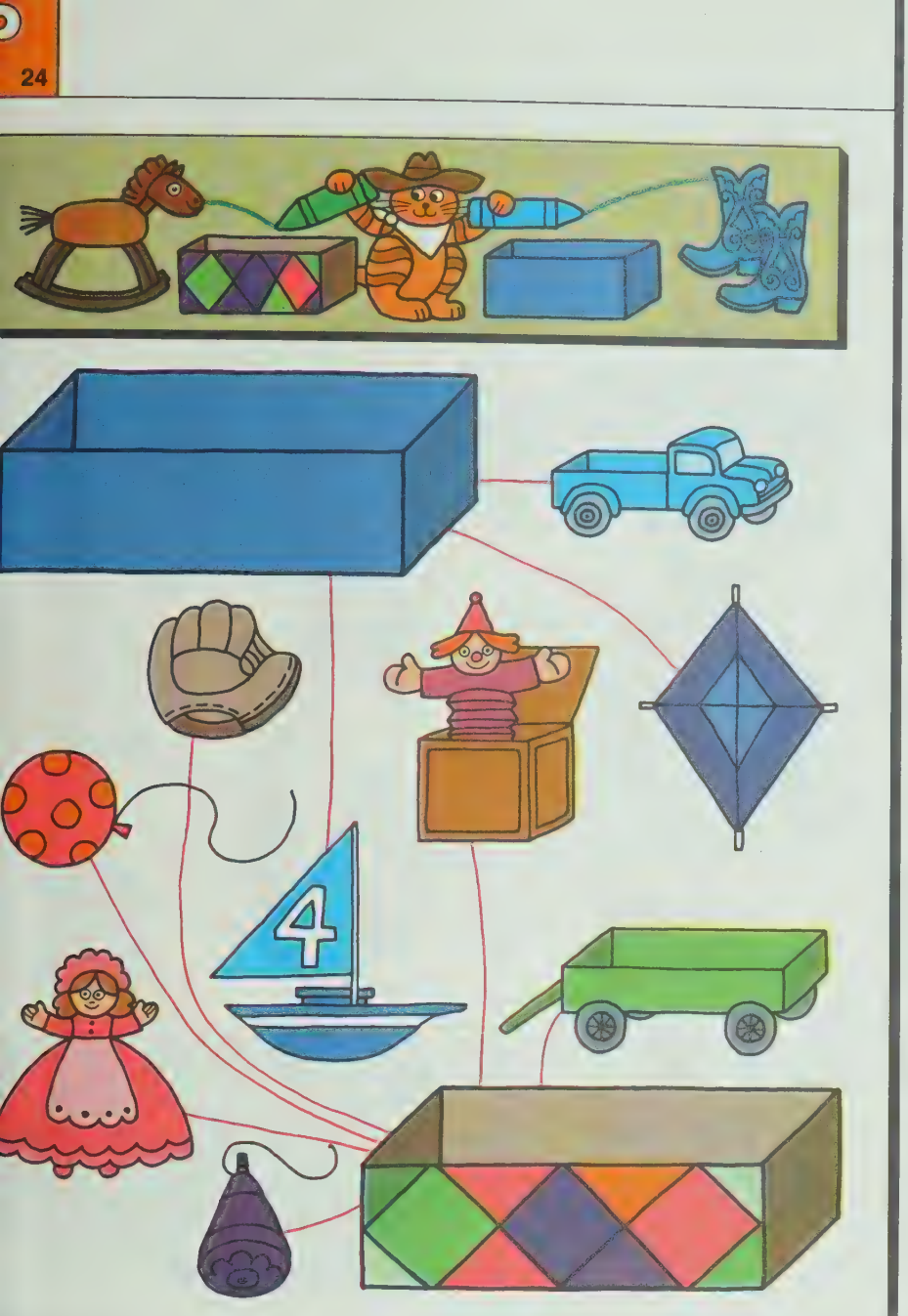
Given a set of objects of two different colors, the child will be able to form two subsets as he classifies the objects according to their color.

Given a set of objects of many colors, the child will be able to form two complementary sets, for example, the set of blue objects and the set of not-blue objects.

Although both pages deal with classification by color into two subsets, page p-24 treats a more sophisticated set concept, namely, complementary sets. Two sets are said to be complementary with respect to each other if their intersection is empty and their union is some given set.

Pre-Book Activity

Although many children by now will be able to recognize and name colors, there may be some children who do not yet possess this skill. Prepare some boxes colored on the outside or covered with different colored construction papers. Make one red, one blue, and, on the third, place a variety of construction papers to show a multi-colored box. Also prepare a collection of objects and a variety of other colored things, such as marbles, pieces of paper, pencils, crayons, checkers, pieces of ribbon or string, etc. Have the children sit in a circle, and place the color boxes in the centre of this circle. Have the objects placed in conspicuous places around the room. Ask the children to find a blue object and put the blue object in the blue box. Ask them to look for a red object and put that in the red box. Ask them to look



lassification by color

Teaching
Page p-24

On this page we ask children to distinguish between a set of blue objects and a set of objects that are many colors other than blue. Call attention to the box at the top which is colored blue. Then call attention to the box at the bottom pointing out that it is a "many-colored" box. Ask a child what box he would put the pink doll into. As you discuss the choices that the children make, use the expression, "not blue," for the pink toy, the green toy, and so on. Help the children see that besides simply describing the box at the bottom as a "many-colored" box, we can talk about it as the "not-blue" box. Again be sure that they have connected every object on the page to one of the two boxes. Discuss with the children the various colors of toys that are shown in the picture. After the children classify the toys, it would be helpful to allow time to discuss the reasons for the classification.

Resources for Active Learning

Early Number Multi-Group Lab, Activity Cards 17–19, Responsive Environments Corp.

Elementary School Science, Primer, Teachers' Edition, "Color," T69–T70, Addison-Wesley

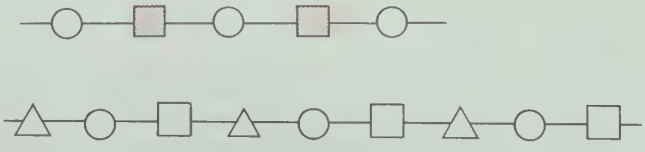
Sets, Numbers and Powers, Lessons and Games on Sets, pp. 72–79, Herder and Herder

for a green or yellow or black object to place in the multi-colored box. The object of this would be to help the children think about sorting objects by their color and to focus on naming and discussing the different colors.

Follow-up

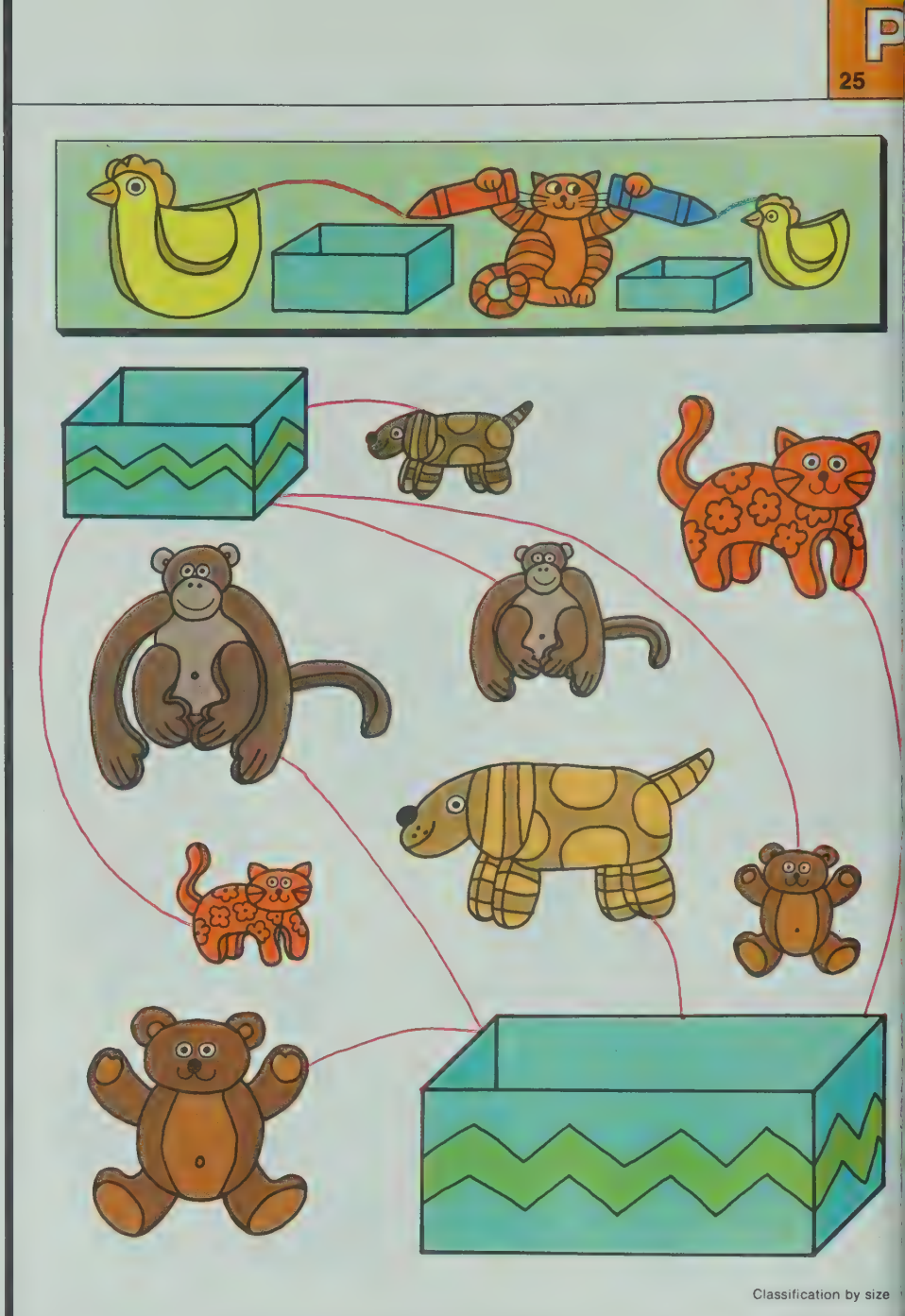
Various activities using patterns may be used to provide the children with more opportunity to work with a number of different colors. For example, provide each group of children with two shoestrings and a set of three or four different-colored sets of beads. Suggest that they use only two colors on their shoestring and form any pattern that they wish with the beads in their group. As the children work, point out the various patterns that may be made by using the same colors. For example, one child may alternate a red, a blue, red, blue, red, blue; another may have two red, a blue, two red, a blue; another may have three blue, a red, three blue, a red; and so on. Accept any pattern which a child can justify.

You might extend this activity by having the children only partially fill their shoestring. Then, when they have finished enough of the pattern, they can knot the string and pass it to the next child and ask him if he can put on the next correct bead. Other coloring activities which use a variety of more unusual colors would also be helpful. For example, help the children mix various paints to create colors.



Teaching
Page p-25

Call attention to the demonstration art at the top of the page. Ask children to explain why the cat is putting the smaller chicken into the smaller box. Help them identify the boxes by size. Then point out the two boxes on the page. Ask: "What objects would you put in the smaller box?" "What objects would you put in the larger box?" "Draw a line to show where you would put each object."



Classification by size

Objectives

Given a set of objects of two different sizes, the child will be able to classify the objects according to size: smaller or larger.

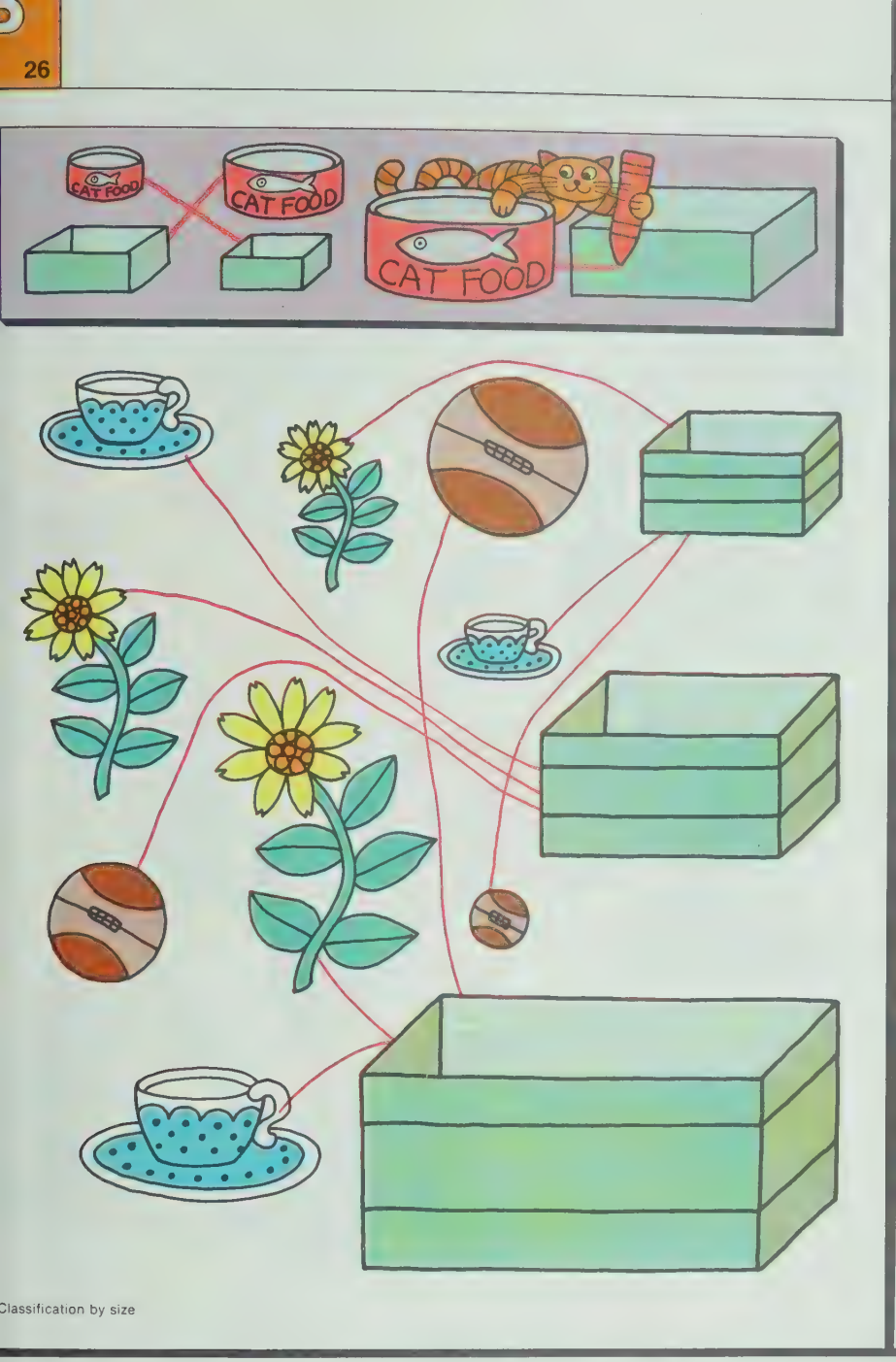
Given a set of objects of three graduated sizes, the child will be able to classify the objects into three subsets: small, medium, or large.

Both of these pages deal with classification by size. It is recommended that many activities with actual objects precede the specific work on each page.

Pre-Book Activity

Give each group of two or three children a collection of objects and two boxes, one labelled *large* with a picture of a large toy and the other labelled *small* with a picture of a small toy. Ask the children to put the objects of their collection into the two boxes according to size. Use different-sized objects such as balls, marbles, blocks, artificial flowers, pencils, boxes, pie tins, refrigerator containers, and so on. You might suggest that for each object they ask themselves: "Is this large or small?" "Is there another object I can use to help me tell?" For example, if there are a large marble and a small marble, these might be placed into the large box and the small box respectively, even though the large marble might be quite a bit smaller than some other large objects. However some children might choose to compare both the large and the small marble to something else and put both marbles in the small box.

It will also be helpful to use flannelboard cutouts to show small, medium, and large figures. Stress the words "small," "medium," and "large."



Teaching
Page p-26

Develop this page in a manner similar to that used for page p-25. Again use the demonstration art as an introduction. Instruct the children to draw lines showing into which box they would put each object. Remind them that they should classify these objects according to sizes. You might use the words "small," "medium," and "large" as children discuss this page.

Resources for Active Learning

Developmental Math Cards, "Ranking." A‡8; "Number Path," A‡18, Addison-Wesley

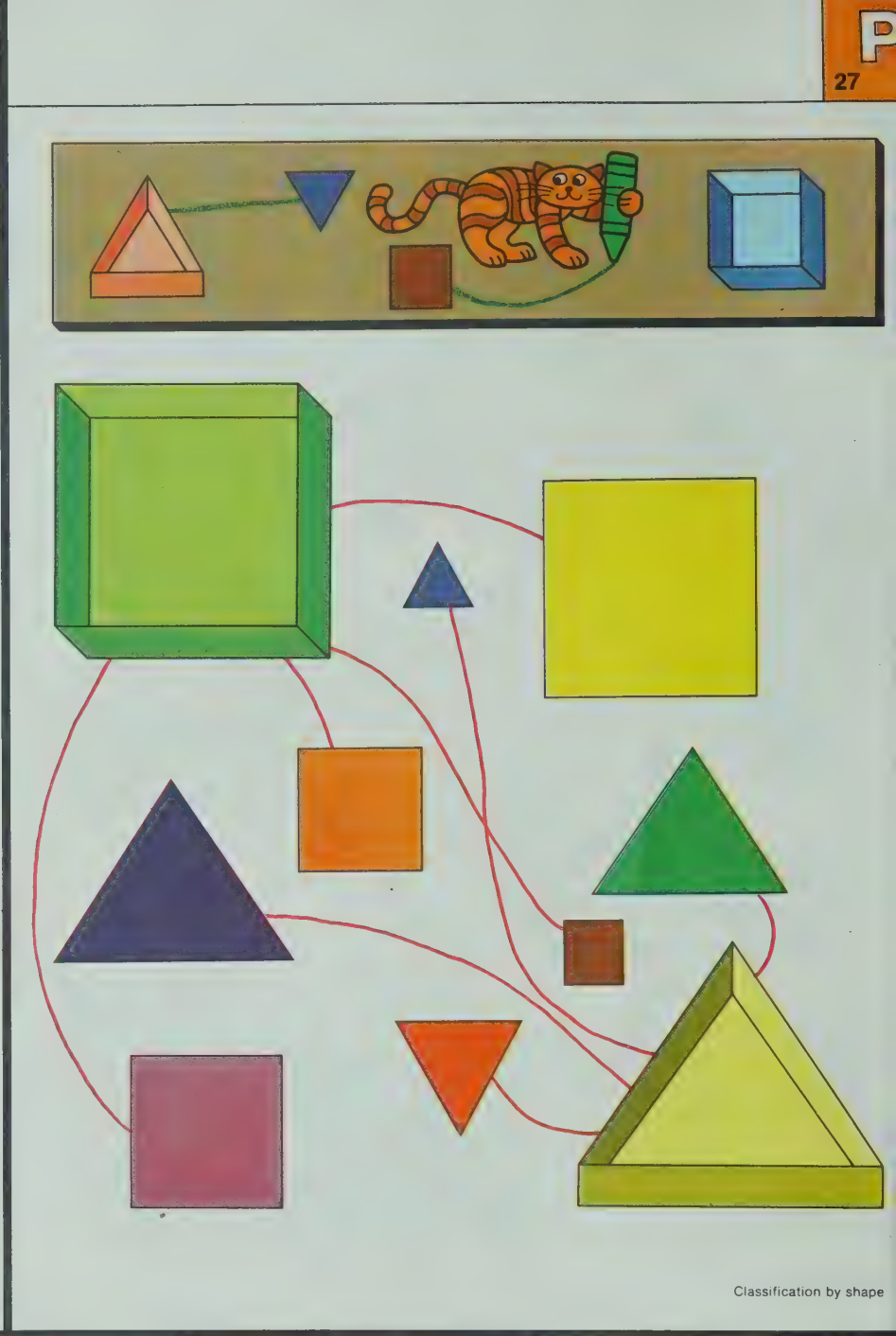
Workjobs, "The Screw Game," pp. 46–47, Addison-Wesley

Follow-up

Give the children a large piece of newsprint and guide them in folding it into thirds; then suggest that they draw a small object, a medium-sized object, and a large object, in that order, on each of the three sections of their paper. Cutouts from magazines or newspapers would also provide opportunities for cutting out large objects and small objects which might then be classified either on a bulletin board display or by using a large box and a small box. If children are writing alphabets, you might also ask them to classify small letters and large letters.

Teaching
Page p-27

Use the demonstration art to explain to the children that the triangle has been matched to the triangular box and the square to the square box. Point out the two boxes on the main part of the page and ask the children to draw lines to show into which box they would put each shape.



Classification by shape

Objectives

Given sets of large and small triangles or squares, the child will be able to classify them according to shape.

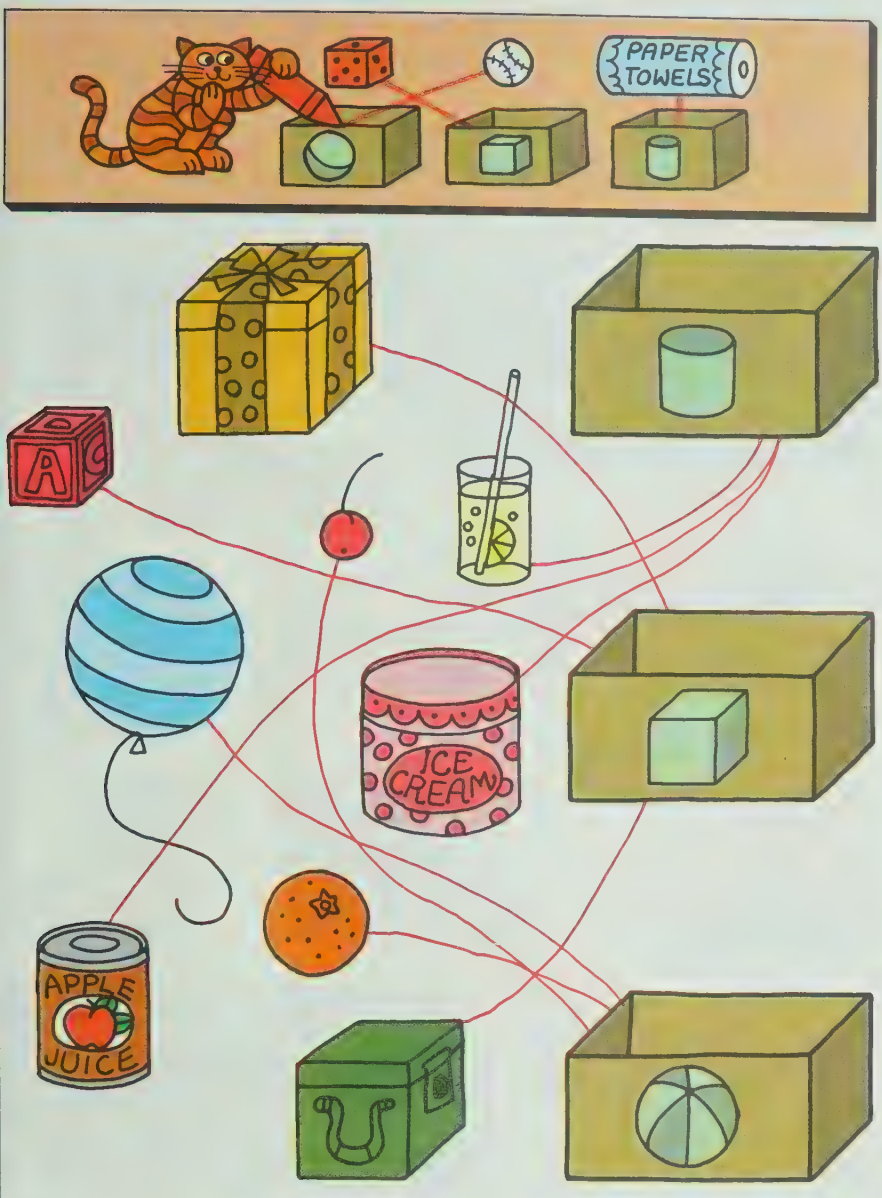
Given sets of solid figures that are cubes, spheres, or cylinders, the child will be able to classify them according to their shape.

Activities with attribute pieces (available from school suppliers) would be excellent accompanying material for this lesson. Sets of solid figures are also desirable. Children will benefit from extended pre-book experience with these materials.

Pre-Book Activity

Prepare cutouts of triangles, squares, and circles of various sizes. Label three boxes with pictures of triangles, squares, and circles. Set these boxes in the centre of the room. Give sets of cutouts to each child. Ask the children to put them into the boxes so that the shape that they put into the box matches the label on the box. For a group activity, give one set of cutouts to each group of three children, and then ask each child to find the shape which belongs in one of the boxes.

A class activity or large-group activity to prepare for page p-28 would be to fill a shopping bag with various items which suggest cylinders, spheres, and rectangular prisms, such as spools of thread, balls, frozen juice cans, cereal boxes, etc. Then have a child come forward and remove an object from the bag. Have the children discuss the different things that they notice about the object, such as whether it has straight lines, is round, etc. Then guide the children in sorting the items into three groups according to shape.



Classification by shape

Teaching
Page p-28

If you followed the suggestions in the pre-book activity, relate them to the demonstration art on this page. Discuss with the children how the objects are being sorted. For example, stress that objects which are round are being put into the box labelled with a round object. Similarly, the paper towel roll has been put into the box labelled with a can since the towel roll may be said to be shaped like a can. Direct the children to draw lines to show into which box they would put each shape. Remind them to use the picture on each box as their guide.

Resources for Active Learning

Developmental Math Cards, "Shape Corner," A[3]5; "Find Shapes . . .," B[3]4, Addison-Wesley

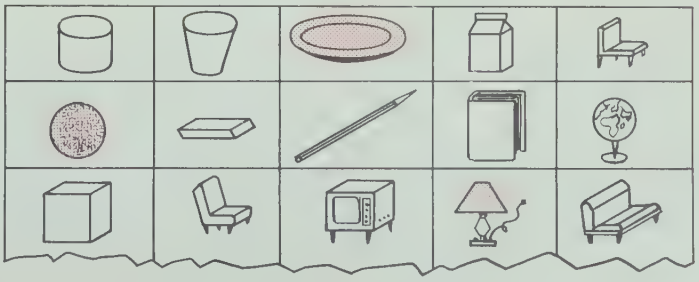
Elementary School Science, Primer, Teachers' Edition, ". . . Shapes," T71–T76, Addison-Wesley

Mathex: Geometry No. 4, "Games with Solid Shapes" and "Three-Dimensional Models," pp. 3–6; "Recognition of Shapes and Games," pp. 16–19, Encyclopaedia Britannica Educational Corp.

Follow-up

Put triangles, squares, and circles or models of cylinders, spheres, and boxes into bags. Ask a child to reach in without looking and, just by feeling, describe one of the shapes that is in the sack. The difference between this and the activity suggested in the pre-book activity is that in this activity a child merely reaches in and feels an object; he does not look at it. These "feely" bags might be used simply with the cutouts that were prepared for the pre-book activity. Children might do this in pairs if they only put three or four items in each bag. Then they can ask their partner to reach in and, just by feeling, describe one of the items in the bag. Since there are only three or four items in the bag, the partner should be able to tell if the child is correctly describing one of the items in the bag.

If the attribute pieces are available, many children will benefit from working with them as suggested in the booklets which accompany most such materials. Also, magazine cutouts of objects that have the shape of the three solid objects shown on page p-28 might be interesting for some of the children, but fewer children will be successful in classifying pictures. A worksheet such as the following might also be helpful.



Teaching
Page p-29

Children will need your guidance in working with this page. At least two different classifications are possible: shoes-hats or yellow-blue. Ask the children to draw lines showing how they would sort these items, that is, into which boxes they would put the items. After the children have attempted to classify the objects, ask them where they put their objects and have them give reasons for their choices. This discussion should bring out at least two possibilities and, hopefully, will provide a broader view of the notion of classification. Note that some children may even suggest a classification based on whether the item is considered men's or women's apparel, and you should by all means be ready to encourage any other such reasonable suggestions.

Choosing attributes

Objectives

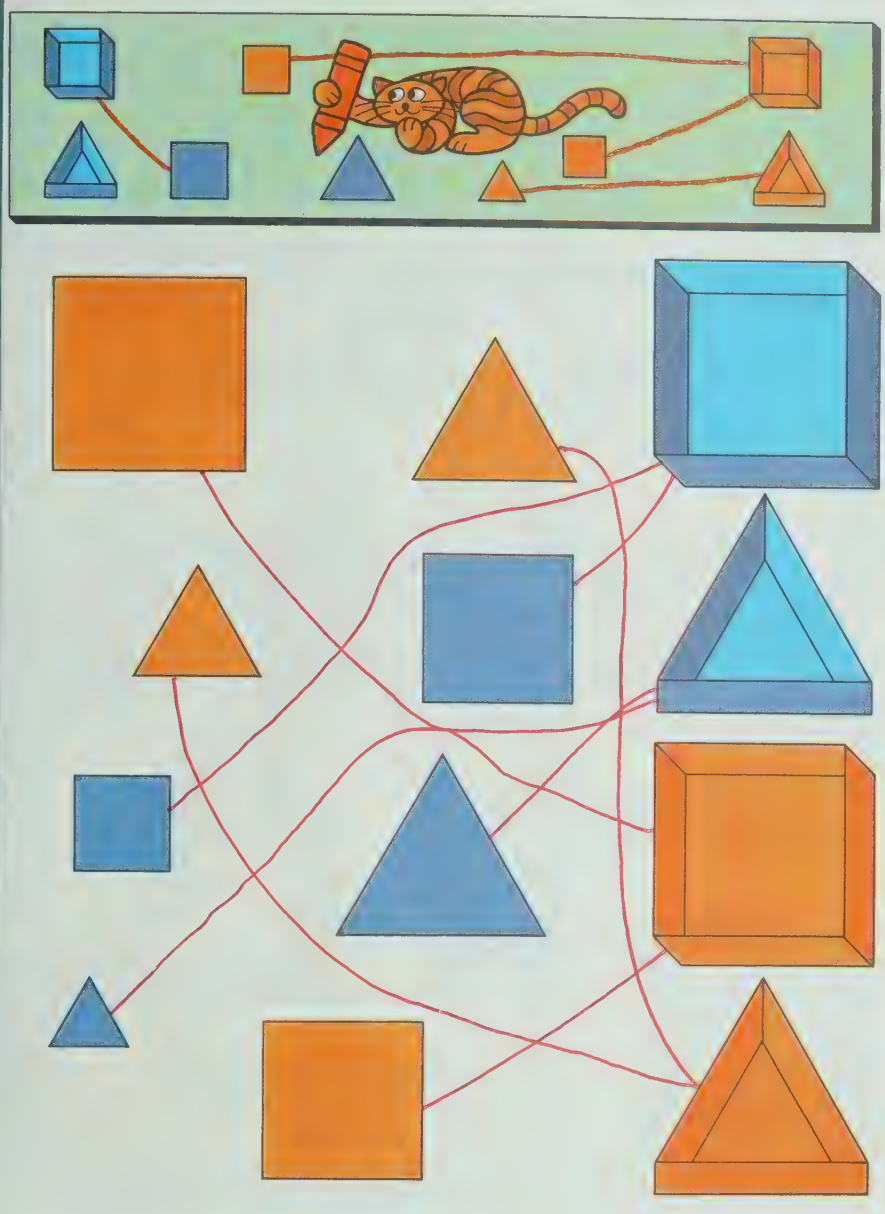
Given an appropriate set, the child will be able to choose a particular classification and use it to sort out the objects.

Given objects which can be classified by color and shape, the child will have the opportunity to classify these objects by using both attributes.

This material should be adapted to the ability of the children. Not all children will be able to cope with the material on these pages, for it concerns more difficult levels of classification. On page p-29 children are encouraged to identify an appropriate attribute to use for classification. On page p-30 they are to classify objects according to two attributes—color and shape.

Pre-Book Activity

Cut two pieces of yarn of two different colors, long enough so that two children may stand in a circle formed from each. Ask a girl to stand in one yarn circle and a boy to stand in the other. Next, rearrange the circles of yarn so that they overlap, forming an intersecting loop large enough for both children to stand in. Then ask each child to stand in the intersecting area if he is a member of the class. Help the children in the class discuss what has happened. Point out that both children are still in the circle that they first stood in, but now they are also in the intersection of both circles because both of them are members of your class. Depending on the ability of the children, you might modify this activity by using geometric cutouts with yarn to show classification by two attributes.



Classification by color and shape

Teaching
Page p-30

This page provides an opportunity for children to classify on the basis of two attributes. Many young children, however, will find it difficult to classify using two attributes simultaneously. Hence, you might first ask the children to find the two containers that are the same color as the geometric figure being considered. Then, after they have found the matching colors, ask them to find which of the two containers has the same shape as the figure and draw the matching lines accordingly. For example, call the children's attention to the large square at the upper left and ask, "In which color boxes would you put the large square, the blue boxes or the orange boxes?" After they have agreed that the figure should go into one of the orange boxes, ask them which one of the two orange boxes they would put the figure in. Finally, direct them to draw a matching line to show this. Some children may be able to simultaneously classify and match the figures with the appropriate containers without these specific directions.

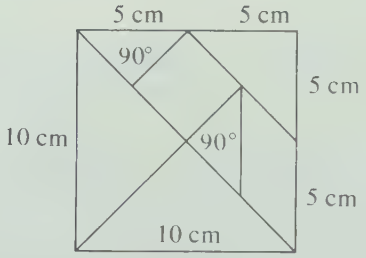
Resources for Active Learning

Tangrams Unit, Webster, McGraw-Hill
Tangrams: 330 Puzzles, Dover Publications
Think and Color, "Matrices," pp. 38–82, Educational Science Consultants

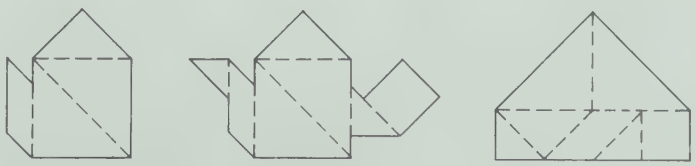
Follow-up

To give the children further opportunity to work with geometric shapes, you might prepare a set of tangram outlines on pieces of heavy cardboard, approximately 10 cm by 10 cm. Put the outline of a tangram puzzle on each piece of board. Then prepare a set of tangram pieces which the children can move around on the outline board until they find the pieces which fit within the outline. (See illustrations at the right.) If this is the children's first experience with tangram pieces, it would be helpful to have them begin by using only 3 or 4 pieces and try to fit them within simple outlines requiring fewer pieces.

For children who are capable, further exercises using intersecting loops of yarn and logical block figures would be valuable enrichment for this lesson.



Tangram Pieces

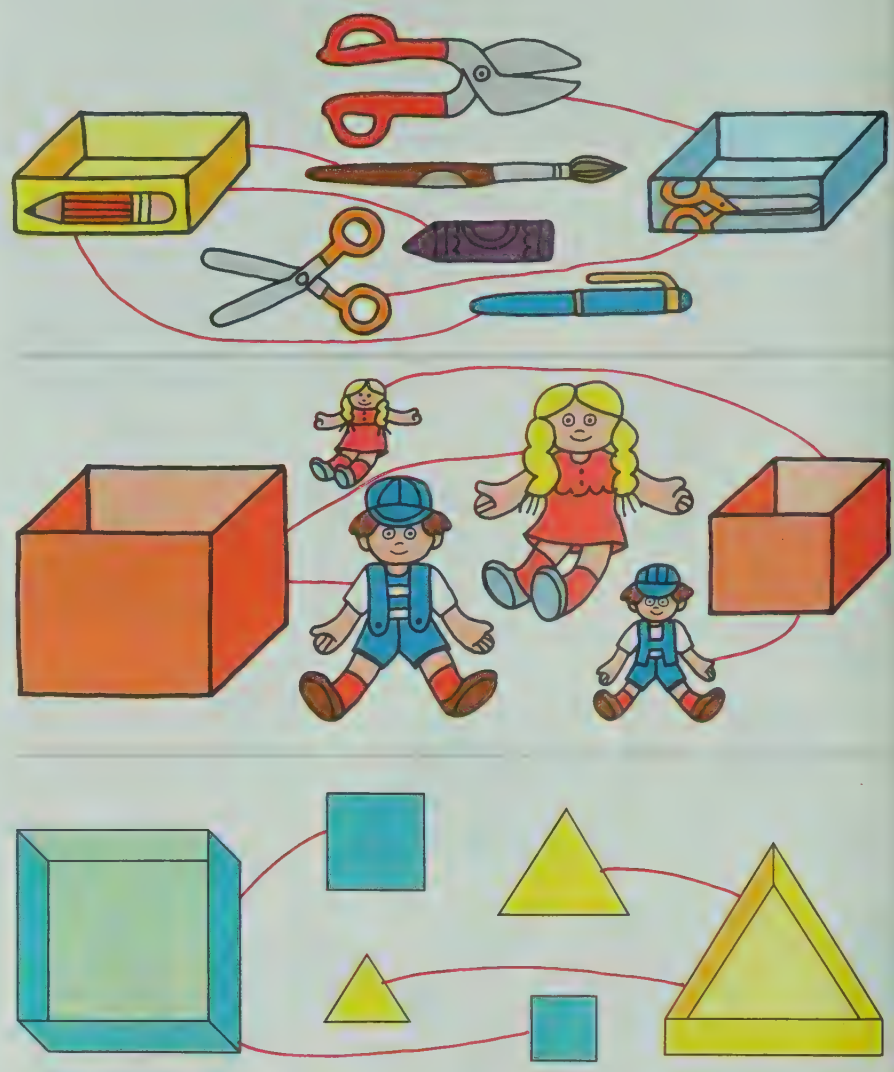


Sample Outlines

Teaching
Page p-31

Direct the children to look at the first frame. Explain to them that there are two boxes and there are objects which should be put into the two boxes. They should draw a line from each object to show which box they would put the object in. You might point out that one of the boxes is for writing and one of the boxes is for cutting. In the second frame, again, point out that there are two boxes, that one is large and one is small, and that here they should draw a line from each item to one of the boxes to show whether they think it belongs in the small box or the large box. Finally, in the last frame, they should match the objects according to their color and shape. Notice that here they are only classifying by two attributes, but intersecting sets are not involved. Thus, they may classify by color or they may classify by shape.

Show you know



Module review

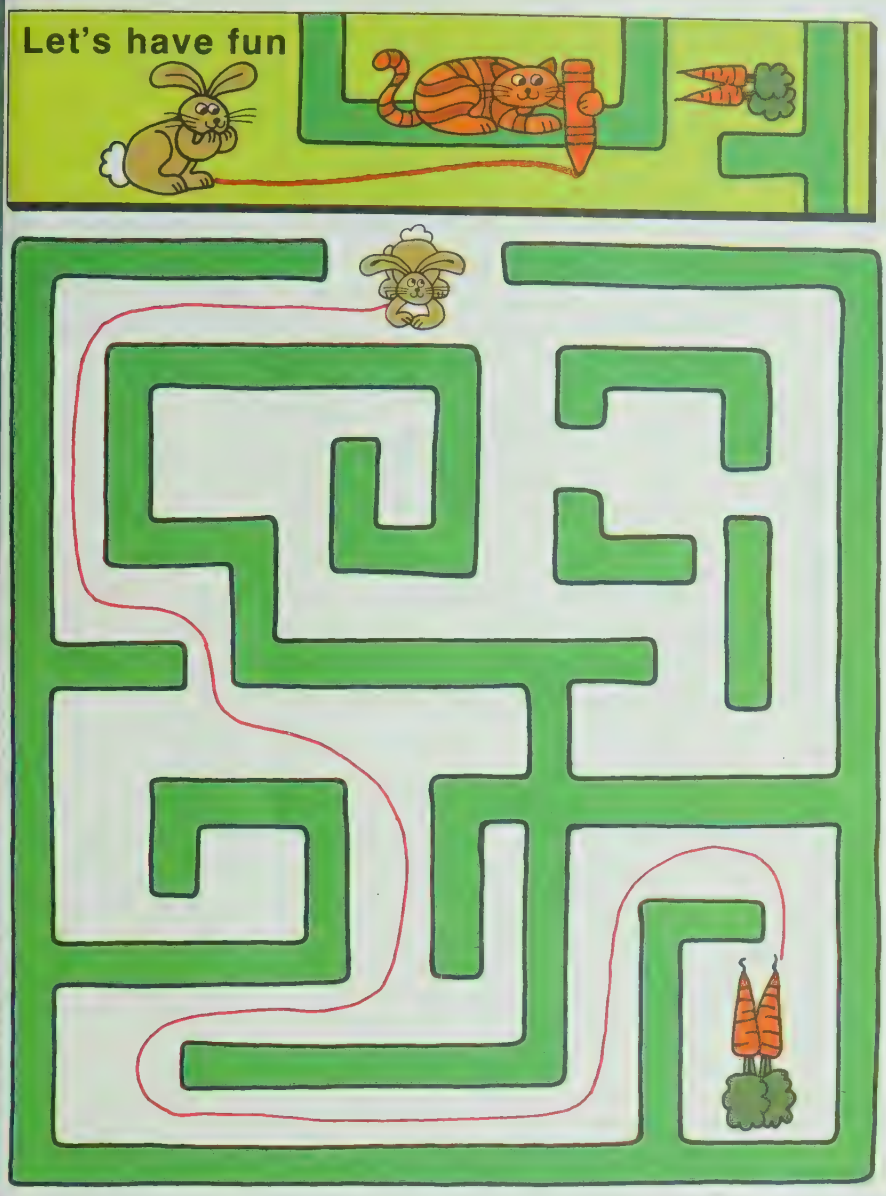
Objective

The child will show his ability to work with the concepts presented in this module.

Page p-31 may be used as a review page. Page p-32 treats no new concepts and should be presented with a light touch.

Pre-Book Activity

It would be helpful to review the classifications studied throughout this module. To do this, classify things in the classroom according to some of the categories children are already familiar with. You might use such categories as "larger or smaller," "round or straight," "used for writing or not used for writing," "orange or not orange," and so on. You might also use categories such as "inside the classroom or outside the classroom," "home or school" and have the children suggest the objects to be classified, even though some of the items are not in the classroom.



Change of pace

Teaching
Page p-32

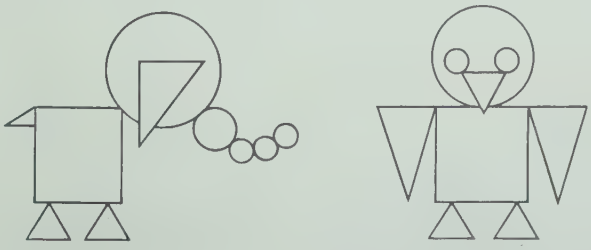
This page should be treated with a light touch since it is a page for the children to have fun with; however, they will need help in understanding the directions. Some children may use the pencil to show all the paths they try; that is, they may start drawing a line, find that they have come to a dead end, and then retrace their drawing, showing the line as it goes back to another dead end, and finally to the carrots. This method is acceptable, but you might suggest to the children that, once they have found the correct path, they draw it with their crayon. If children have retraced their paths, you will at least have a method for finding out how readily the child was able to perceive the path without trial and error. Remember, however, that this page should be treated as an enjoyable experience for the children.

Resources for Active Learning

Mathex: Geometry No. 4, "Designs and Shapes," pp. 15–16, Encyclopaedia Britannica Educational Corp.

Follow-up

Children might enjoy constructing a figure from several geometric shapes. Give each child an assortment of small squares, circles, and triangles which you have cut from construction paper. Also provide sheets of paper on which the children can form and paste a figure.



Sets and Matching (pre-number activities)

Pages p-33 to p-48

General Objectives

To further develop the idea of sets
To compare sets by visual inspection
To introduce the concepts of more and less
To introduce the idea of one-to-one matching
To introduce the concept of equivalent sets
To lead toward an understanding of cardinal number without using number names and numerals

In this module, ideas vital to a clear understanding of the concept of number are introduced. The fact that each set has associated with it a concept called number is familiar to most children. To strengthen this familiarity, we present pairs of sets to be compared, first for the case in which one set has more objects than the other, then for the case in which both sets have the same number of objects.

After an investigation and discussion lesson which introduces these concepts in general, children will be asked to determine by simple visual inspection which of two sets has more objects. The sets to be compared should differ enough in the number of objects that the children can easily tell whether each set has more or fewer objects than the other. The next concept developed, more difficult than simple comparison, is that of two sets having the same number of objects. The idea of one-to-one correspondence between pairs of sets is developed carefully by means of matching activities. When this is established, we present pairs of sets for which it is not immediately obvious which set has more objects, so that the children must match objects in order to discover which set has more.

Mathematics

Several significant mathematical concepts are embodied in this unit. These ideas include sets, subsets, one-to-one correspondence, equivalent sets, cardinal numbers, and order. We list here some basic descriptions and definitions of these concepts.

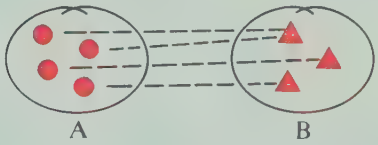
A *set* is a group or collection of objects considered as a single entity.

The objects contained in a set are called *elements* or *members* of the set.

A *subset* of a given set is a set that contains no element which is not also in the given set.

Every subset of a given set is a *proper subset* except the given set itself.

The sets are in *one-to-one correspondence* if for each element of one set there is exactly one element of the second set, and for each element of the second set there is exactly one element of the first set. Children show this relationship between sets by drawing matching lines between the objects of the sets. It is important that you are careful to see that two objects of one set are not matched with one object of the other set.



Notice in the illustration that each object of set A is matched with exactly one object of set B. However, the illustration fails for the second part of the definition, because one element of B is matched with two elements of A. Therefore, in this example, sets A and B are not equivalent to each other.

When two sets are in one-to-one correspondence, they are *equivalent*. It is essential that we introduce the word *equivalent* with respect to sets having the same number of objects. There is a strong tendency to call such pairs of sets "equal." But in this unit, we restrict the use of the word *equals* to mean *exactly the same as*. Observe that under these conditions, to say that $A = B$ for sets A and B means that set A is exactly the same as set B. That is, A and B are different names for the same set. The idea of equal sets is useful to express a change of notation. For example, the union of sets A and B can be written $A \cup B = C$. This simply means that C is another name for the set $A \cup B$. Equality of sets can also be used to express answers. For example, to find $A \cup B$ for sets A and B, where A is the set $\{1, 2, 3\}$ and B is the set $\{3, 4\}$, write $A \cup B = 1, 2, 3, 4$. This means that the set $A \cup B$ is the set $\{1, 2, 3, 4\}$.

A *cardinal number* is a class of equivalent sets. For example, the cardinal number 2 is the class of all sets which are equivalent to the set $\{a, b\}$.

If a set from cardinal number a has a proper subset which is equivalent to a set from cardinal number b, cardinal number a is *greater than* cardinal number b.

Teaching Red Module, Unit P

Materials

blue, red, and yellow circles
blue, red, and brown squares
buttons, bottle caps, bread tabs, or plastic disks in an envelope, one per child
crayons

envelopes, one per child
felt objects and geometric shapes
flannelboard
overhead projector (if available) and transparencies
paper bags
pipe cleaners
red and blue circles or other counters
sets of small objects for matching (checkers, plastic animals, cars, airplanes, ships, and so on)
yarn

Vocabulary

equivalent sets	less	one-to-one matching
fewer	lesser	rectangle
fewest	matching lines	set
greater	more	triangle
greatest	most	
least	one more	

All the words in the vocabulary list need not be introduced. The list contains those words most likely to arise in class discussion. Many of the words are essential for communication of this subject matter.

Lesson Schedule

You will probably want to spend from one to two weeks on this unit. Keep in mind that it is important that the work in the unit results in the children's understanding of the ideas underlying the one-to-one matching of the objects in two sets. The children should be able to demonstrate with matching lines that two sets have the same number or that one set has more than another.

Evaluation of Progress

While it is true that many of the children at this level will also be able to give the number of a set containing less than ten objects, we feel that it is important to emphasize that they are to make these comparisons without using number names or counting skills. In this module, we are building foundations for number concepts. For example, we lead toward the concept of cardinal number by demonstrating one-to-one matching between pairs of sets. If the objects in two sets can be matched one-to-one, they are, in fact, equivalent sets and hence belong to the same class or have the same number. Careful emphasis on ideas of this kind forms important background for a thorough understanding of number concepts. In this module, avoid the use of counting skills and naming the number of a given set.

Resources for Active Learning

General Activities

Early Number Multi-Group Lab, Activity Cards 1–4, Responsive Environments Corp.
Math and Measure unit, Webster, McGraw-Hill
Mathex: Geometry No. 4, "Space and Volume," pp. 9–10; Measurement and Estimation No. 5, "The Same Amount," pp. 8–13, Encyclopaedia Britannica Educational Corp.
Nuffield Project: *Mathematics Begins* 1, pp. 27–31, Wiley

Manipulative Devices

Beads and String (CCM School Materials; Childcraft; Ideal)
Cogno-Board (Teaching Resources)
Design Cubes (Creative Playthings; local supplier)
Geo Blocks (Selective Educational Equipment; Webster, McGraw-Hill)
Mosaic (Creative Playthings; local supplier)
Pattern Blocks (Selective Educational Equipment; Webster, McGraw-Hill)
Sorting and Order Kit (Mafex)

Commercial Games

Lotto Games (Milton Bradley; Responsive Environments Corp.; school supplier)
Spot the Set (Selective Educational Equipment)

(For additional suggested manipulative devices and commercial games, see the introductory notes for the Yellow Module and the Orange Module of Unit P.)

Activities for Continued Use

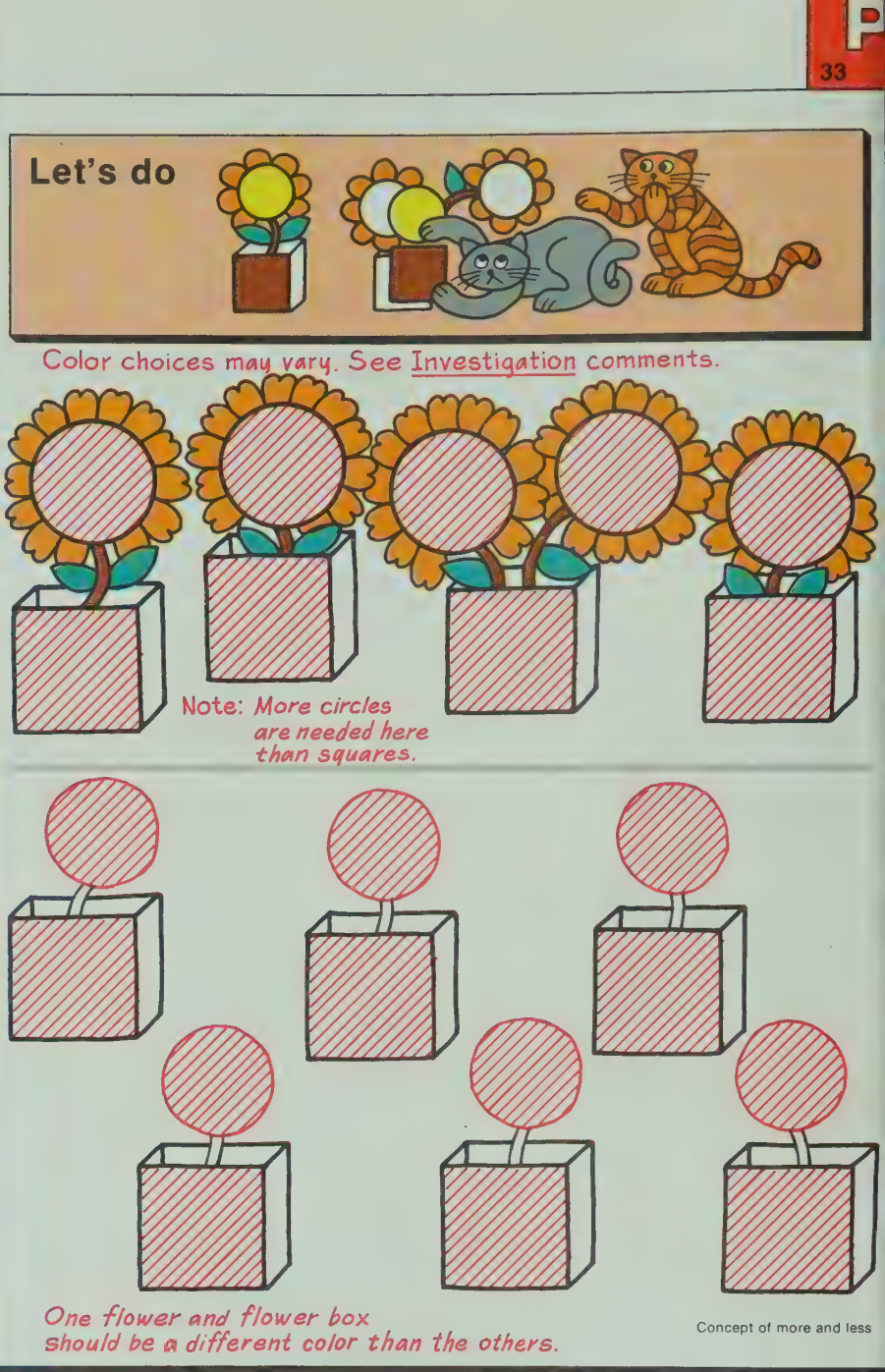
This module introduces the child to a type of pre-number experience essential to his concept development, namely, one-to-one correspondence. Supplemental activities such as that suggested here are also recommended. Not only will they provide further pre-number experiences for the child, but also they will provide you with a basis for evaluating a child's level of maturity with regard to number concepts. For further discussion on the importance of such activities, see Lavatelli, *Piaget's Theory Applied to an Early Childhood Curriculum*, pages 105–113.

Materials: 9 paint jars; 9 paint brushes
Procedure: Line up the paint jars, and give the brushes to a child (3 or 4 other children might observe). Ask the child to match each paint brush to a paint jar. Ask, "Are there as many brushes as there are jars?" Then take the brushes and place them together beside the line of jars. Again ask, "Are there as many brushes as there are jars?" Also ask, "Have any jars been added? Have any brushes been taken away?" If the child responds correctly continue this same questioning after you group the brushes into a bunch secured with a rubber band. Encourage the child to give reasons for his response and help him verbalize it. If he continues correctly, he is able to conserve number and needs few other such activities. But if he answers incorrectly, then he is confusing the quantity of space used for the brushes with the number of brushes and needs many activities similar to this. Similar activities can be conducted using other pairs of complementary sets such as paper dolls and doll dresses; cups and saucers; dolls and chairs; balls and bats; pencils and pieces of paper; milk cartons and children; and so on.

Investigation
Page p-33

Give the children instructions such as: "Choose one color of circles to place on the centre of each flower. Choose one color of the squares to place on the flower boxes. Color the flowers to match the circles you have chosen. Color the boxes to match the squares you have chosen. Does each flower have a circle? Does each flower box have a square? Did you use just *as many* circles *as* you did squares? Did you use *fewer* squares than circles?" Be sure the children observe that there is *one more* flower *than* flower boxes.

For the bottom section of the page, the children should pretend that they are planting a flower in each box. They can do this by placing a circle above the flower stem and coloring each flower the color of the circle. Then they should match a square with each flower box. Ask questions such as, "Do you have enough circles (squares) so that all the flowers (flower boxes) can be the same color?" Notice that since there are only five of each color, children will have to use another color to form their sixth flower. Thus, we may say: "There are *more* flower boxes *than* there are flower circles of the same color." Or, "There are *fewer* yellow circles than there are flower boxes." Or, "There are *not as many* brown squares *as* there are flower boxes."



Purposes

To continue to develop the idea of sets
To introduce the idea of a quantitative relationship between sets

This lesson is intended to serve as an introduction to the gradual development of one-to-one matching. It provides you with an opportunity to introduce phrases such as *more than, fewer, less, not as many as.*

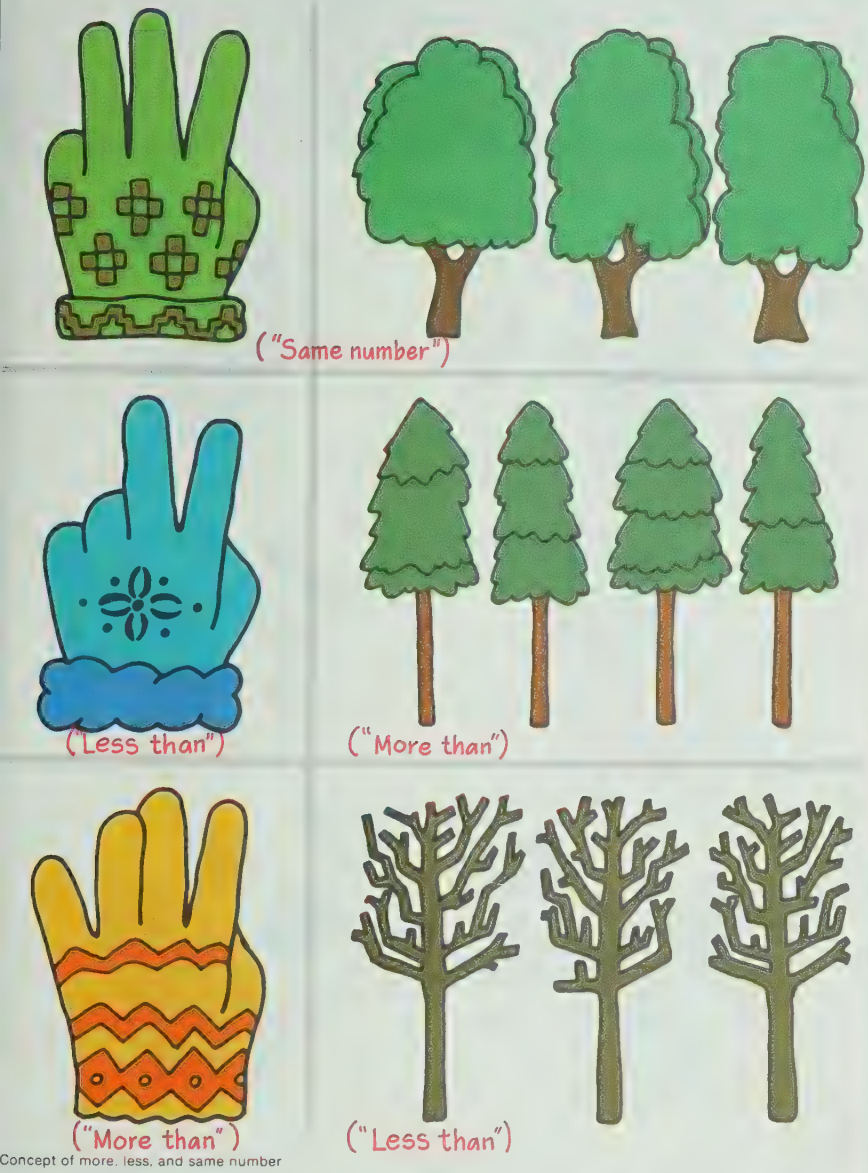
Preparation

Materials

cutouts: red, blue, and yellow circles; red, blue, and brown squares
crayons

Distribute the cutouts prepared for use with p-17. To be sure children are identifying the colors correctly, you might ask all of them to hold up the red circle or the blue circle or the brown square and so on; however, children will be able to choose what colors they wish for this investigation, so it is not essential that they be able to identify all the colors.

Let's talk



Concept of more, less, and same number

Discussion
Page p-34

The illustrations on this page provide a basis for discussion of the *more than* concepts. Use the phrases suggested in the investigation, such as *more than, fewer, less, as many as, not as many as*. The intent here is to develop the basic ideas of *same number, more, less,* and *matching*. Number names will be developed in the following module. If children refer to number names, accept their contributions to the discussion, but do not emphasize them. In particular, ask questions such as: "Do the fingers show *how many* trees there are in the first section?" "Are there the *same number* of trees as there are fingers?" "Do the fingers show *more than* how many trees there are?" "Do the fingers show *less than* how many trees there are?" "Can you show *how many* trees there are with your fingers?" "Do you have to use *more than* the hand in the picture has used?"

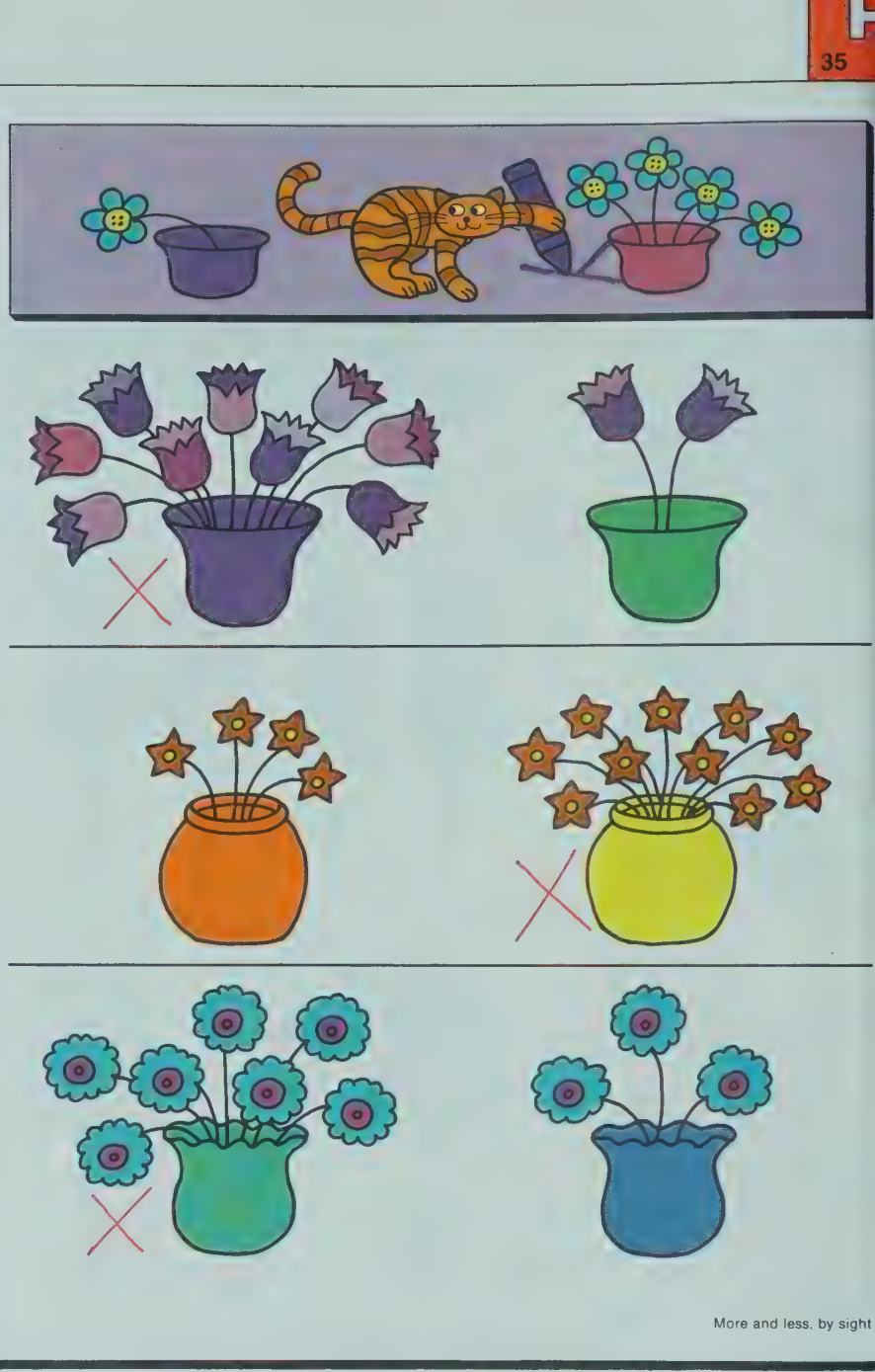
Follow-up

Display various demonstration sets one by one. For each set, ask the children to show with their fingers how many there are in each set. Keep in mind that the sets should have few members in them (less than five objects).

Children would also benefit from a matching activity such as putting on clothes. Children can put on an old shirt and match buttons to buttonholes or an old jacket and match hooks to eyes or snaps to snaps, and so on. In all such activities, point out that each button has a buttonhole, that each hook can be matched to an eye, each snap has a partner, etc.

Teaching
Page p-35

Call attention to the illustration at the top of the page. Point out that the cat is putting a mark beside the vase that has *more* flowers. Children will need oral instructions for the entire page. It would be helpful to refer to the sets by the colors of the vases. For each frame, ask the children to mark the set that has *more*.



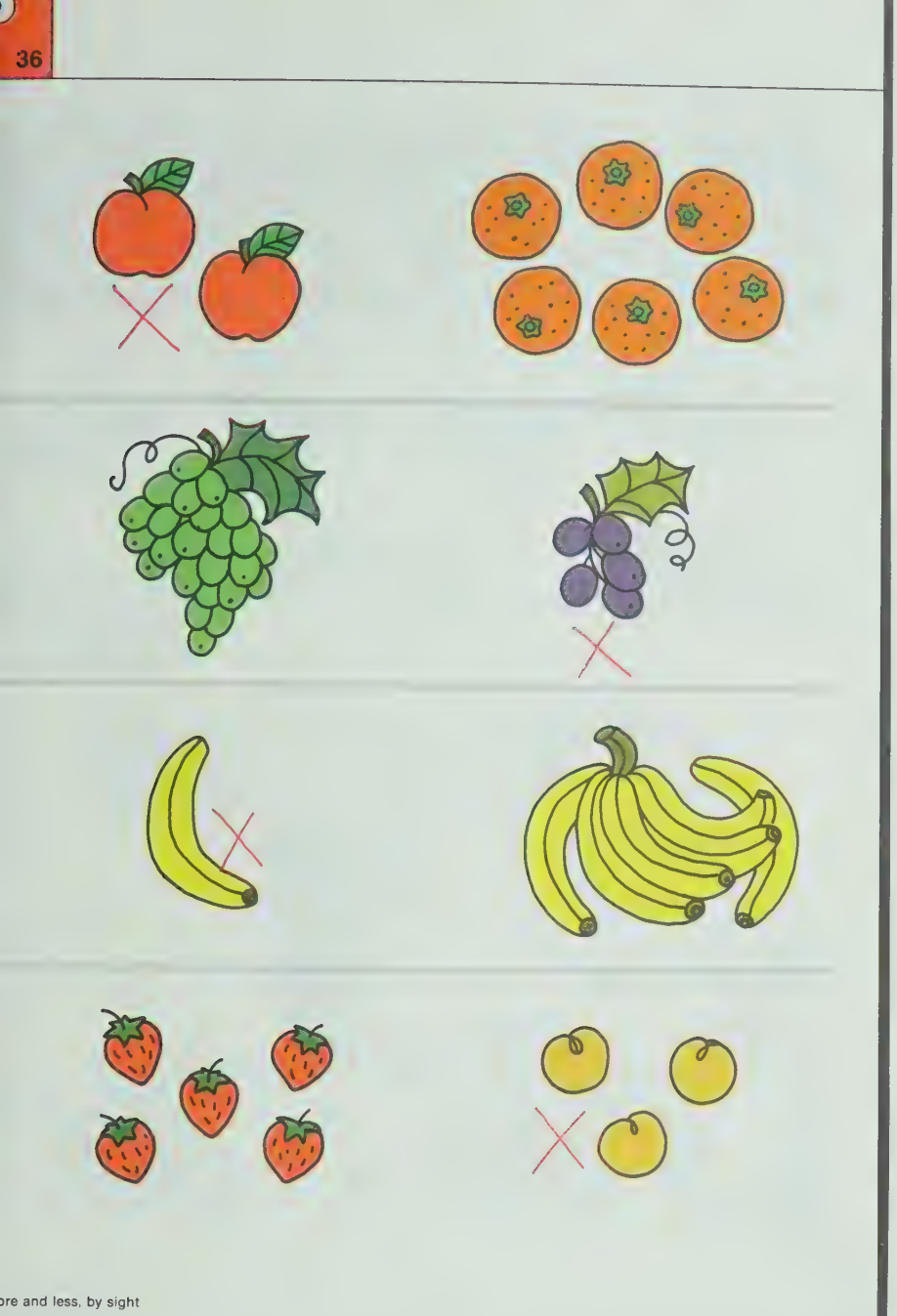
Objective

Given sets whose number of elements may be compared by sight, that is, without drawing matching lines, the child will be able to tell which set has more or fewer by visual inspection.

The intention of this lesson is to develop the *more than* and *less than* concepts with sets which children should be able to compare simply by visual inspection.

Pre-Book Activity

Prepare two fairly large paper bags. Fill them with previously cut out 2-by-30-cm strips of construction paper. Fill one of the bags to overflowing. In the other bag, only put a few strips of these same pieces of construction paper. These paper bags may be used in a variety of ways. First, ask the children which bag has more strips; it will be obvious to them that the one with the overflowing strips has the most strips. Next, bundle the strips in the overflowing bag into fairly neat groups held together by elastic bands or paper clips. Return these to the paper bag, and again ask the children which bag has more strips. Children who think that the bag containing the loose, bulkier strips contains more strips than the other will need further experiences of this type. Then unband the same strips and rearrange them so that it becomes less and less obvious which bag contains more strips. Keep the paper bags and use them in following lessons. Their use in this lesson should be mainly to stress the *more than* concept.



Teaching
Page p-36

Use the first frame to discuss which set has *more*. In the second frame, talk about bunches of grapes, again asking the children which set, the green or the purple, has more. In the next set, stress that one banana is *less than* the bunch of bananas put together. Finally, ask the children, "Are there more strawberries or peaches?" On this page, children should mark the set with the fewer items. In your discussion of the page, use the phrases *more than, less than, fewer than, not as many as.*

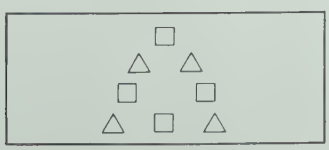
Resources for Active Learning

Early Number Multi-Group Lab, Activity Cards 6–11, Responsive Environments Corp.

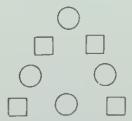
Workjobs, "The Pegboards," pp. 36–37; "The Block Patterns," pp. 40–41, Addison-Wesley

Follow-up

Children will benefit from various matching activities. For example, prepare a variety of colored cards, each showing a pattern which children can match by using actual attribute pieces or other card materials. (See the sample card below.) Ask the children to repeat the pattern exactly but to replace every square with a circle, and every triangle with a square.



Sample Pattern



Child's Pattern

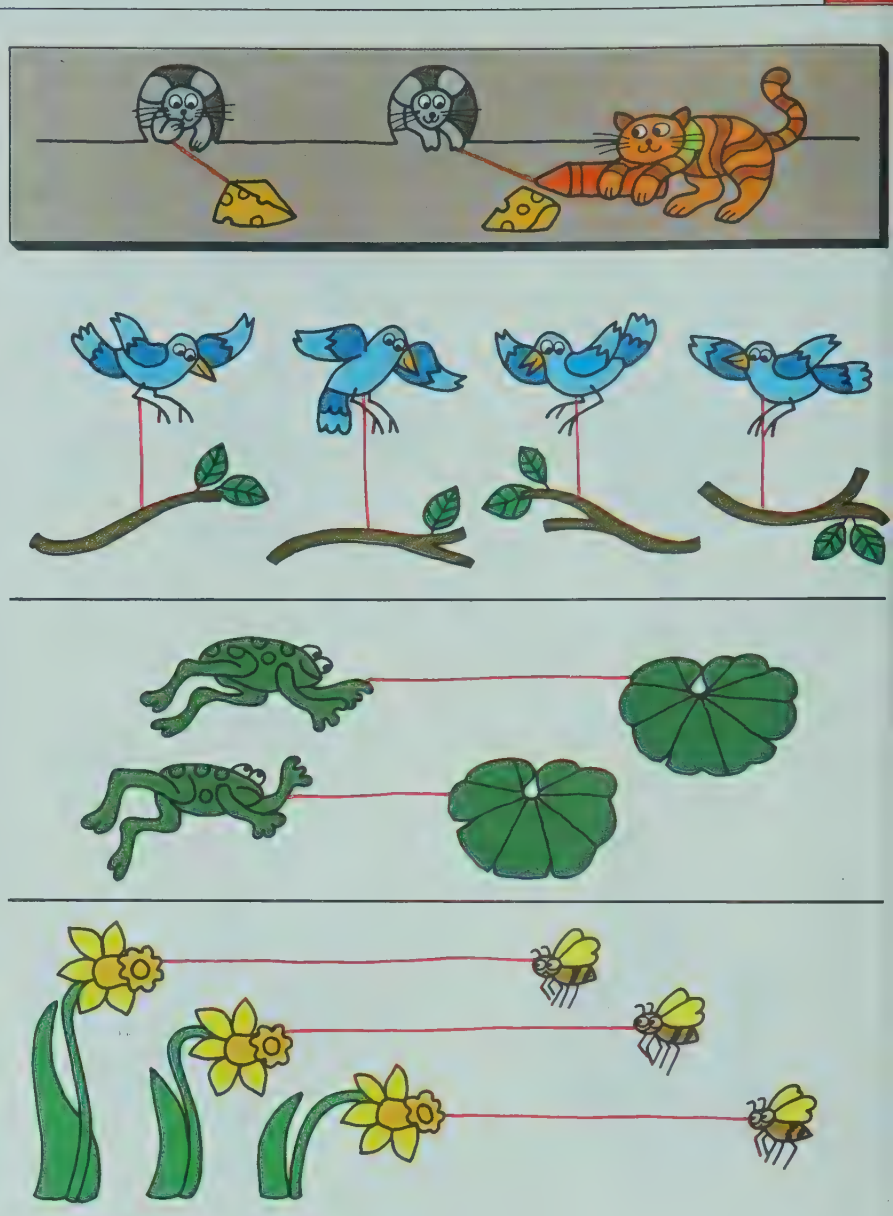
If pattern or other block materials are not available, individual small cards can be made. These sets of small cards might contain approximately twelve each triangles, squares, and circles. Of each set, six could be red and six blue. Then, when you prepare your set of master cards, show patterns such as three red triangles, or a red triangle, blue circle, and another red triangle. Children then should try to match the pattern on the master card with their sets of squares, circles, and triangles.

Mathematics

The pairs of sets on these pages are compared strictly by visual inspection. Precise mathematical concepts do not stand out clearly at this time, but this initial stage leads to the idea of one cardinal number being greater than another. When a child says that one set has more objects than another, he is claiming that the cardinal number of one set is greater than the cardinal number of the other set.

Teaching
Page p-37

Point out to the children the illustration at the top of the page and use it to introduce the page. Stress in your introduction the same-number concept, then call the children's attention to the birds and branches. Remind them that they can show that the number of birds is the same as the number of branches if there is a branch for every bird. Point out that they should draw lines to match each bird with one branch. Observe that the matching does show that the number of birds is the same as the number of branches. Avoid using specific "number names" in reference to these sets. If a child remarks that both sets have four objects, agree with him, but emphasize only that the matching lines show us that the sets have the same number. Next have the children look at the frogs and lily pads. Tell them that you want them to show that there are as many frogs as there are lily pads and that they can show this by drawing matching lines. When they have drawn the lines, observe again that the matching lines show that the number of frogs is the same as the number of lily pads. Ask the children to match one-to-one the two sets in the last row, observing that the matching lines demonstrate that the number of bees approaching the flowers is the same as the number of flowers.



One-to-one matching

Objective

Given equivalent sets, the child will be able to find that each set has the same number by matching objects in one set to objects in the other.

This lesson introduces the one-to-one matching technique used to determine if sets are equivalent. Here the sets should contain relatively few items, so that the matching is obvious. Sets containing more than five items should be avoided.

Pre-Book Activity

It would be possible to use the paper bags again as part of your pre-book activity; however, other demonstration sets could be substituted. This time use strips of a different color for each bag. Prepare the bags so that each bag contains the same number of strips. Ask the children to find which bag has more strips or if the bags contain the same number. To encourage discussion of how to do this, you might have one child come up and pass out the strips from one bag, and a second child pass out the strips from another bag. If your strips were red and blue, then every child who has a blue strip should become the partner of a child who has a red strip. Help the children see that as long as each child has a partner, there are just as many red strips as blue in the two bags. Use other appropriate pairs of sets on a flannelboard or chalkboard to develop this idea of matching objects one-to-one. For example, put three objects on each side of the flannelboard and demonstrate the matching with strands of yarn. Then have children do the same with other equivalent sets. Notice that this activity introduces matching actual objects one-to-one.

One-to-one matching

Teaching
Page p-38

Call attention to the frogs and insects. Tell the children that they are to draw matching lines to show that the number of frogs and insects is the same. Continue with similar directions for the dogs and bones in the second frame and the bananas and monkeys in the third frame. For each frame, emphasize that the children can determine whether these two sets have the same number by using matching lines.

Resources for Active Learning

Early Number Multi-Group Lab, Activity Card 5, Responsive Environments Corp.

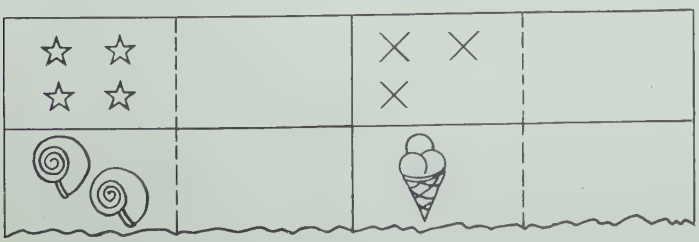
Mathex: Matching and Grouping No. 1, "Matching," pp. 5–6, Encyclopaedia Britannica Educational Corp.

Math Workshop: Games and Enrichment Activities, "Matching Activities," pp. 9–10, Encyclopaedia Britannica Educational Corp.

Think and Color, "Matching," pp. 83–87, Educational Science Consultants

Follow-up—"Cut-and-Paste Sets" I

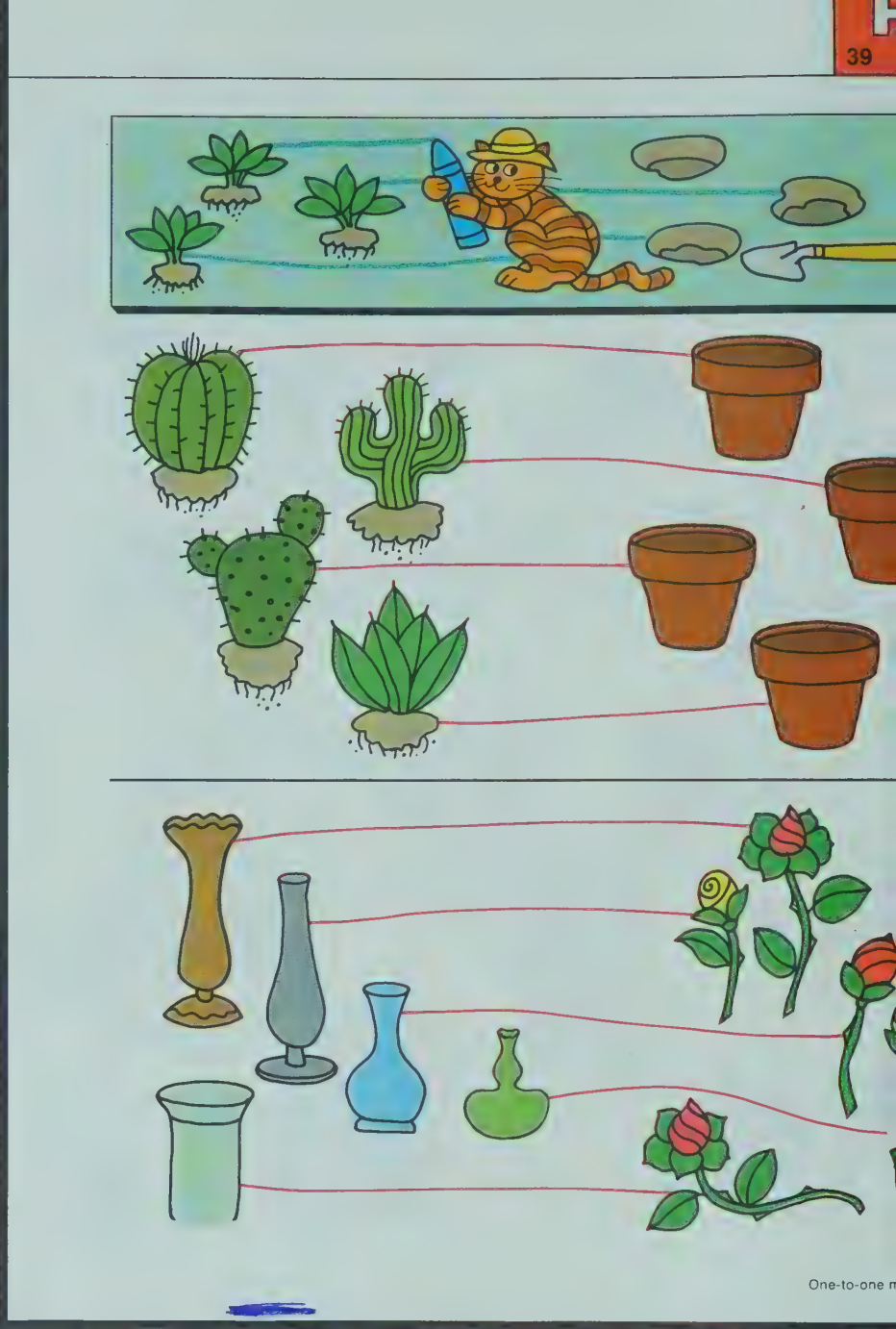
Give the children worksheets similar to the one below. Supply each child with scissors, paste, and an old magazine. Direct the children to find and cut out one set that has more objects than each set on their sheets and to paste it in next to the given set. If some children cannot manage the cutting, give them each an envelope of pictures from which to choose the larger sets.



To emphasize that matching will tell us if two sets are equivalent, even if the number of each set is unknown, display a large handful of pipe cleaners and a handful of cotton balls or the like and ask the children if there are enough cotton balls to make each pipe cleaner into a doll. Or use masts (coffee-stirrers) and sailboats, straws and milk cartons, or whatever is available in large quantity and has meaning for the children. Have the children match the sets and decide which set has more or fewer, or whether there is the same number in each set.

Teaching
Page p-39

Use the demonstration art at the top of the page as an introduction. Ask the children to look at the set of plants and flower pots. Point out that the one-to-one matching will show whether the two sets have the same number of objects. Ask the children to use their crayons to draw matching lines to show that the sets have the same number. Then call attention to the next row, which contains the set of vases and the set of flowers. Tell the children that they should match these two sets one-to-one to show that each has the same number. After they have drawn the matching lines, ask them how they can tell that the two sets have the same number of objects.



Objective

Given equivalent sets, the child will be able to show that they are equivalent by drawing one-to-one matching lines.

The emphasis in this lesson is essentially the same as in the last; it continues to stress the idea of equivalent sets and to further develop recognition of equivalent sets by one-to-one matching. Notice here, however, that the matching is a little bit harder and the categories are more clearly grouped into sets.

Pre-Book Activity

It would be helpful to introduce this lesson by using chairs and children in sets to show a one-to-one matching. For example, set up six chairs and ask four children to come up and sit on the chairs. Ask the questions: "Does every chair have a child sitting on it?" "Does every child have a chair to sit on?" Help the children conclude that there are more chairs than children. Then ask two other children to come up and sit on the chairs so that each chair has a child sitting on it. Again, ask: "Does every chair have a child sitting on it?" "Does every child have a chair to sit on?" Here, stress with the children that since every chair has a child sitting on it—since the answer to both questions is "yes"—then there are as many chairs as there are children, and we say that the set of chairs has the *same number* as the set of children. You might also introduce the term *equivalent sets* by saying that these two sets are called equivalent sets. Finally, ask three more children to come forward, and ask them if they can sit on one of the chairs.

Teaching
Page p-40

Observe with the children that the squirrels and the bears look as if they are holding something. In the top section, point out that this squirrel is holding an acorn drawn with a dashed line. Explain that in this dashed space they should complete the drawing and color the acorn. Then, next to each of the other squirrels they should draw another acorn so that each squirrel will have one acorn. Stress that when they have done so there will be the same number of acorns as there are squirrels. Give similar directions for the set of bears in the lower section of the page. Some of the children may count objects in order to complete this exercise correctly. This is not harmful so long as you do not try to explain it for the entire class. For those children who do count, stress the fact that they could have done the exercise without the counting.

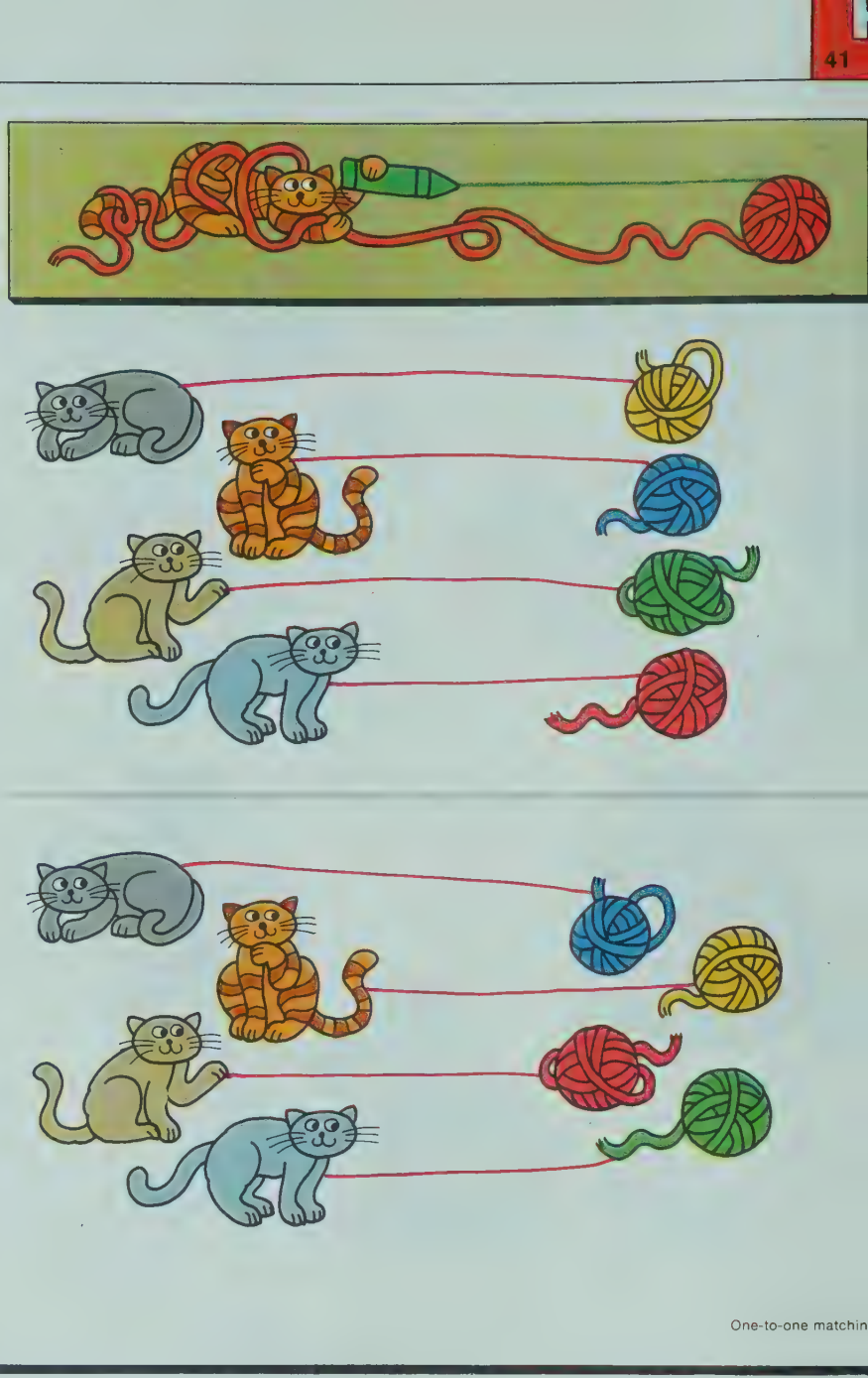
Again, ask: "Does every chair have a child sitting on it?" "Does every child have a chair?" Point out that here there are fewer chairs than there are children. There are more children than there are chairs. Give the children other opportunities to match sets in class demonstrations. It would be helpful to supply each child with an egg carton of paper or felt shapes. Then you might ask the children to do the matching in their seats as you place sets on the flannelboard for them to imitate. For example, place a set on the flannelboard and have the children show a set that is equivalent to it at their seats. After they have selected their set, you should have them use pieces of yarn to match the objects to be sure they are correct. At this stage it is important to be sure that each time they compare sets they actually show the matching. Use other classroom opportunities as they present themselves for one-to-one matching.

Follow-up

A matching game between two small groups may help some children improve their understanding of matching sets. The game requires an old folding checkerboard, two boxes of small objects, such as checkers, for matching, and some yarn for matching lines. Other materials adaptable to this game are a piece of railroad or tag board folded or taped down the middle, and felt, rubber, or plastic toys. Each group should choose an undisclosed number of objects from its "magic box" and give the objects to the player who has the first turn. Then as the game leader says either *more* or *less*, the players should place the sets chosen by their groups on their respective sides of the board. Let the children take turns in participating in the matching to determine which set is more and which set is less.

Teaching
Page p-41

Use the cats in the first frame to introduce this lesson. Explain that each cat should be connected to a ball of yarn. Direct the children to draw matching lines. Explain that they should do the same thing in the bottom frame. Point out that in both the top and bottom sections, each kitten has a ball of yarn even though the balls of yarn are in different positions.



Objective

Given sets which are equivalent, but not obviously equivalent, the child will be able to show that they are equivalent by drawing matching lines.

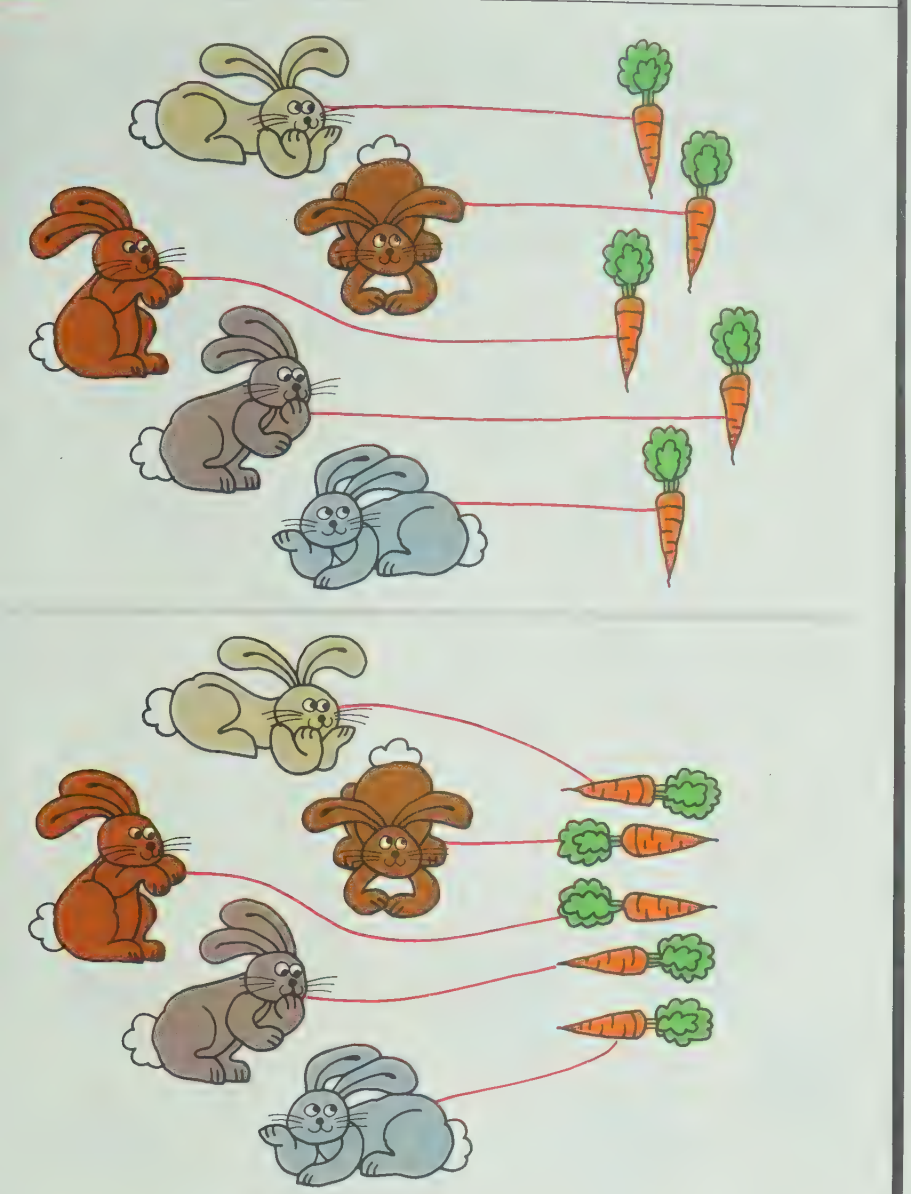
This lesson is related to the "conservation of number" task described by C. P. Dienes in *Building Up Mathematics* (Teachers' Bibliography). Children often do not distinguish between the position of objects and the concept of how many. Drawing matching lines, to see that each item of one set can be matched to an item in the other no matter what its position, should be helpful.

Pre-Book Activity

Materials

red and blue circles or other counters
5 pieces of yarn or string, or crayons

Distribute a set of at least ten counters to each child. (The five red and five blue circles which were made for p-17 would be appropriate.) Also, give each child five pieces of yarn or a crayon and a piece of newsprint. Guide them in folding the newsprint in half. Direct the children to take a red circle and a blue circle and place one on each section of their piece of newsprint and use the yarn or crayon to show a matching line. Discuss the fact that there is the same number of counters on either side of the fold. Then direct them to take another counter in each hand and put one on each side of the fold. Again, have them make a matching line between the counters. Point out that the counters need not be lined up side by side. Use examples to show that two counters on one side may be close together while two counters on the other side may be farther apart, but the counters in both sets can still be matched one-to-one.



e-to-one matching

Teaching
Page p-42

This page is simply an extension of page p-41. Again explain to the children that they should draw matching lines so that each rabbit has a carrot. Stress that matching lines may be drawn from the carrot to the rabbit in both instances, even though the carrots are positioned differently in the two sections.

Resources for Active Learning

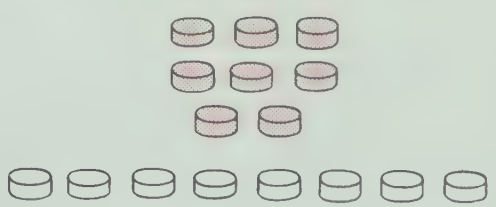
Mathex: Measurement and Estimation No. 5, "Conservation Tests," pp. 7–8, Encyclopaedia Britannica Educational Corp.

Follow-up

As independent enrichment for children who understand matching and the terms greater and greatest, give each child distinctively marked manila envelopes containing 15 or more items. Supply yarn for matching and ask the children to find whether the sets are equivalent and, if not, which has the *greatest* number.

It would also be helpful to give children tasks for the conservation of number. (See Copeland's *How Children Learn Mathematics,* pp. 61–62 ff., in the Teachers' Bibliography.) As you have the children individually perform the tasks, line up a set of red and a set of blue counters for the children to match.

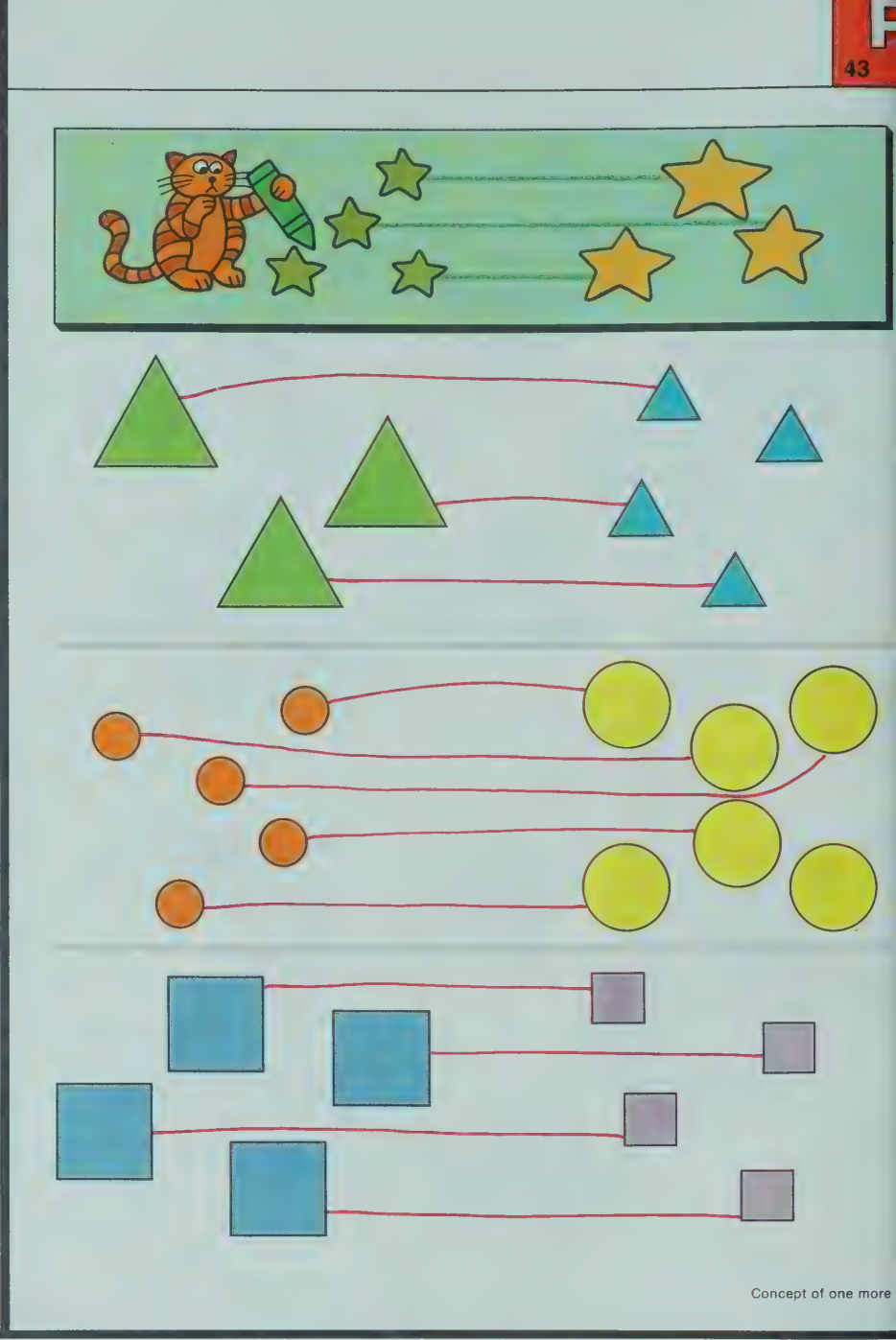
Ask, "Are there as many red counters as there are blue counters?" Assuming a child's response is "yes," bunch the red counters increasingly closer together, leaving the blue counters in a line.



Ask, "Now are there as many red counters as there are blue?" A child who responds "no" is probably not yet able to separate the concept of how many from his perception of the position of the counters. Further experiences of matching, particularly with complementary sets, should be provided for such a child.

Teaching
Page p-43

Discuss the demonstration art at the top of the page. Point out that the cat can draw no more matching lines even though one set has one more star. Use the expression "one more than" to describe how many small stars there are compared to how many big stars there are. Then call attention to the first frame. Direct the children to draw matching lines. Stress that only one matching line is to be drawn from each large triangle to a small triangle. Continue to give similar directions for the remaining frames. Ask the children which pair of sets have the same number. After children agree that there are as many small squares as there are large squares, use the other frames to develop the one-more-than concept. Emphasize that there is one more small triangle than there are large triangles and one more large circle than there are small circles.



Concept of one more

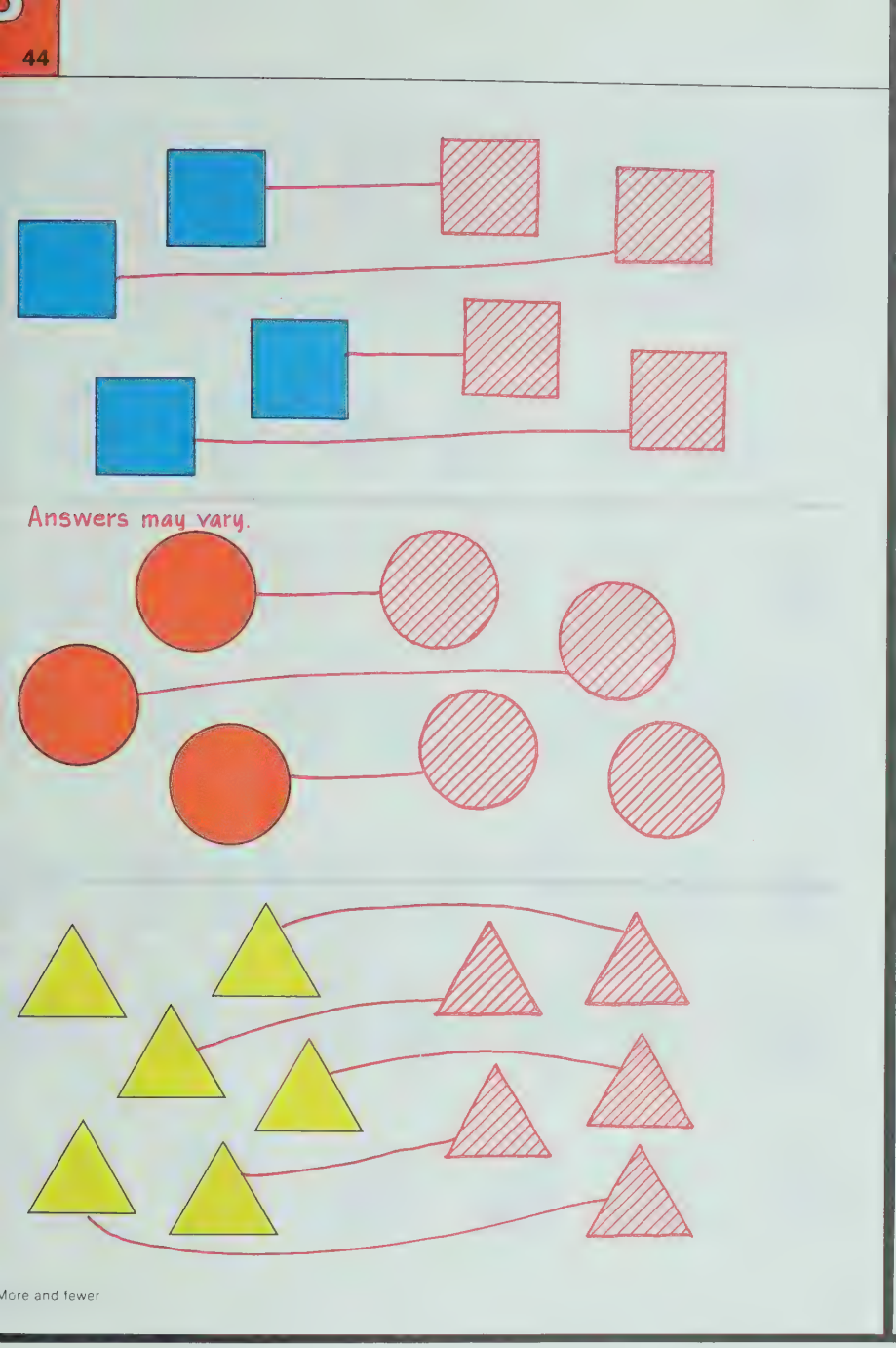
Objective

Given appropriate pairs of sets, the child will be able to identify those which have the same number and those which demonstrate the more than *and* less than *concepts.*

This lesson begins to extend the idea of one-to-one matching to the concepts of *more than* and *less than.* Here, children not only identify equivalent sets but also compare sets that include the *one more than* concept.

Pre-Book Activity

On the flannelboard exhibit a set containing three or four objects. Have a child show a set containing the same number of objects as the given set. Ask another child to use yarn to show matching lines. Then have a third child put one more object in the second set. Stress the idea that one set has exactly one more object than the other. Repeat this activity with several sets. If you prefer that each child work with objects, distribute buttons, washers, or counters. Form a set on the flannelboard and ask the children to form a set with their objects which has one more than your set. Children might use yarn to encircle their sets. Repeat this activity with another set. Also, ask children to form a set that has less than your set. It would be helpful to have one child show such a set on the flannelboard for all to see.

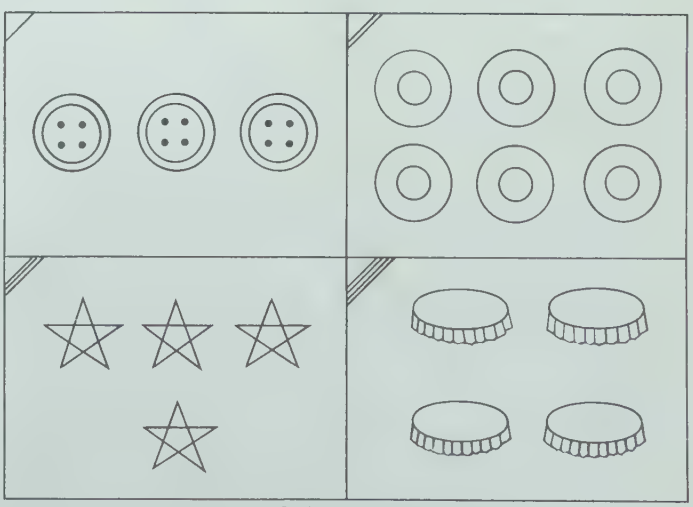


Teaching
Page p-44

On this page children are given a chance to draw their own sets. It is important to give careful directions for each frame. In the first frame, explain that they should draw a set that has the *same number* as the set of squares. In the second frame, they should draw a set that has *more* objects than the set of circles. Finally, in the last frame, they should draw a set that has *fewer* members or *less* than the set of triangles. To conclude the lesson, you might also have them draw matching lines for the objects in each frame to prove that they have done it correctly.

Follow-up – Set Designs

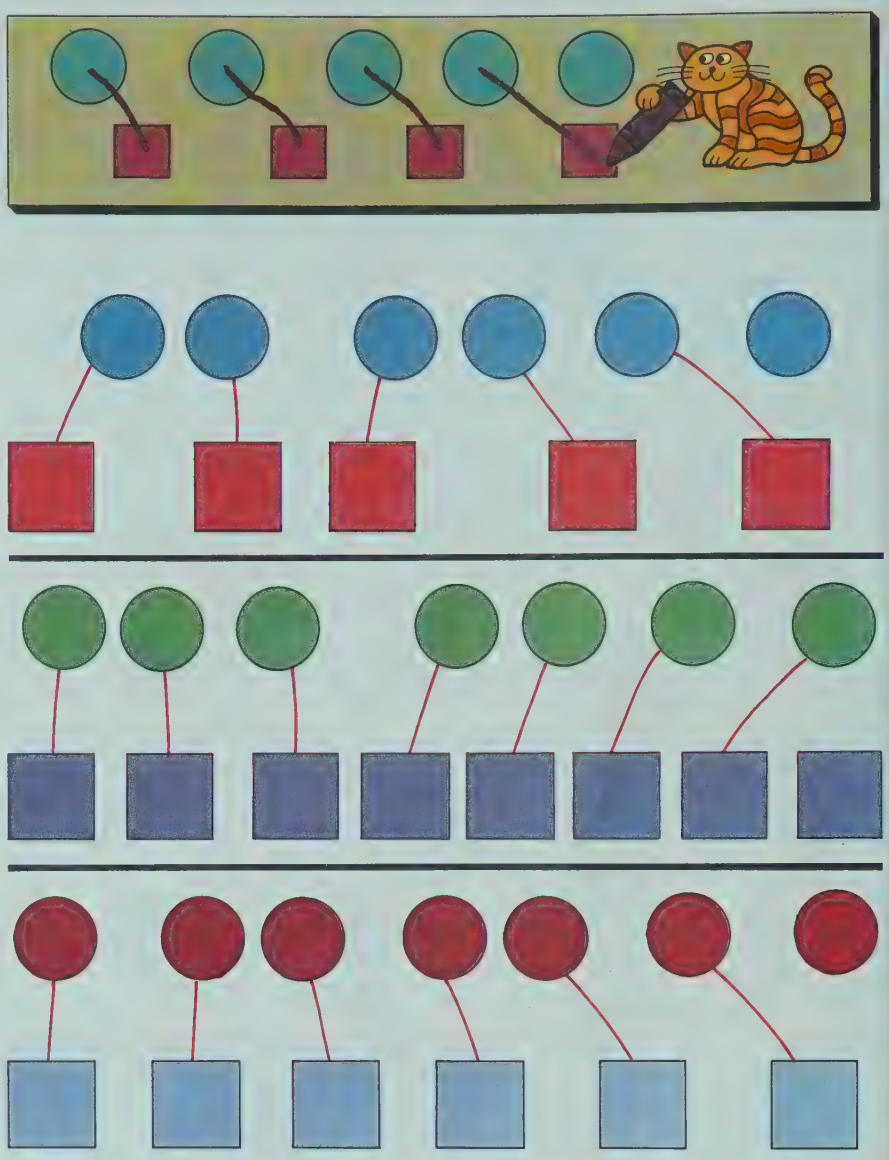
To provide more practice in making sets with *more* or *less*, give each child a large sheet of newsprint. Direct them in folding it twice in half to make four sections. Give each child an envelope of buttons, plastic disks, or bottle caps. Then, using yarn, divide the flannelboard into four sections. Put a different colored felt stripe in each corner to mark the region for the children. Have the children place corresponding crayon marks in each of the four sections of their papers. Place three felt regions in the upper left frame of the flannelboard as you identify it by its colored stripe for the children. Tell the children to choose a set greater than your set and to glue it on their paper in the proper frame. Continue identifying sections by color, placing objects, and asking the children to choose sets of more or less to glue on their papers.



Child's Paper

Teaching
Page p-45

Use the art at the top of the page to introduce the page. Ask the children whether there are more circles or more squares and encourage them to give a reason for their answer. Direct the children's attention to the set of circles and squares in the top frame. Ask them to draw matching lines to show which set has one more than the other. Elicit the comment that the set of circles has one more than the set of squares. Next, have the children match the circles and squares in the second frame. Again ask them to find which set has more than the other. In the third row, again have the children match the set of circles and squares to find which set has more. Stress the idea of one set having one more than the other, and the idea that matching lines help them realize this.



Concept of one more

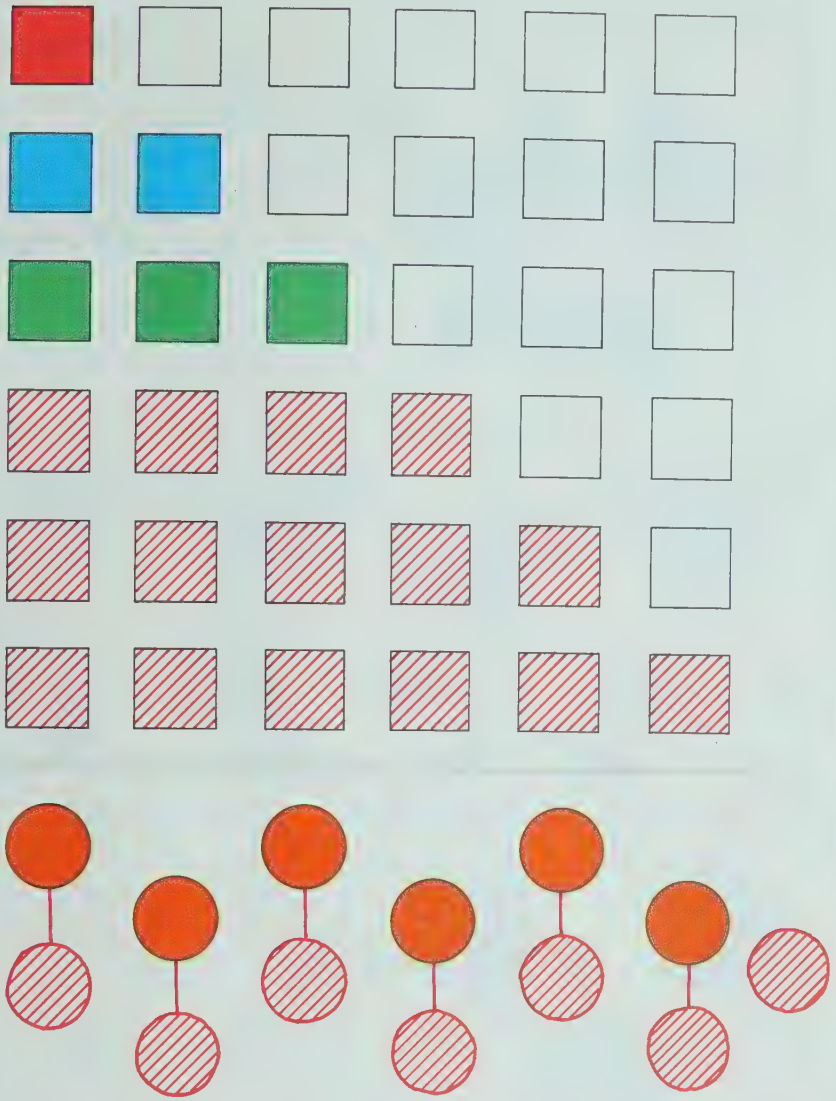
Objective

Given appropriate pairs of sets, the child will be able to find those sets which have one more than the other given set.

Matching lines are again the chosen technique for helping children identify a set that has *one more than* another.

Pre-Book Activity

On the flannelboard show at least three rows of cutouts of different shapes, with each row having one more than the previous row. Use yarn to show the matching lines between the sets. Then use these sets as the basis for discussion of comparisons between them. For example, ask, "Which set contains more?" while pointing to the top row and the bottom row, the middle row and the bottom row, or the top row and the middle row.



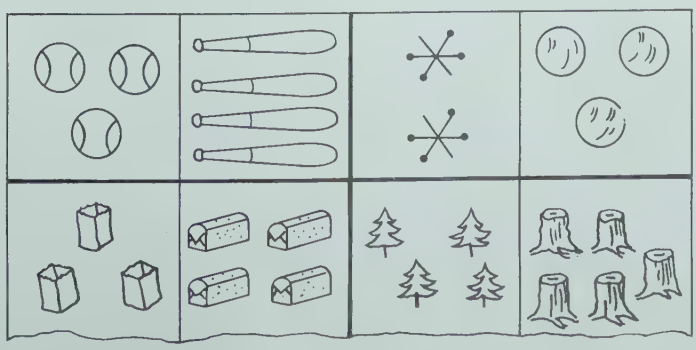
Concept of one more

Teaching
Page p-46

This page provides further practice with the one-more-than concept. Include directions such as: "In each row color one more than is colored in the row before. Choose your own color, but make each row a different color, if possible." For the bottom section, give the directions that the children should draw and color any set they want that is one more than the set that is shown. They should then draw matching lines to be sure that the set that they have drawn contains one more object than the set that is shown.

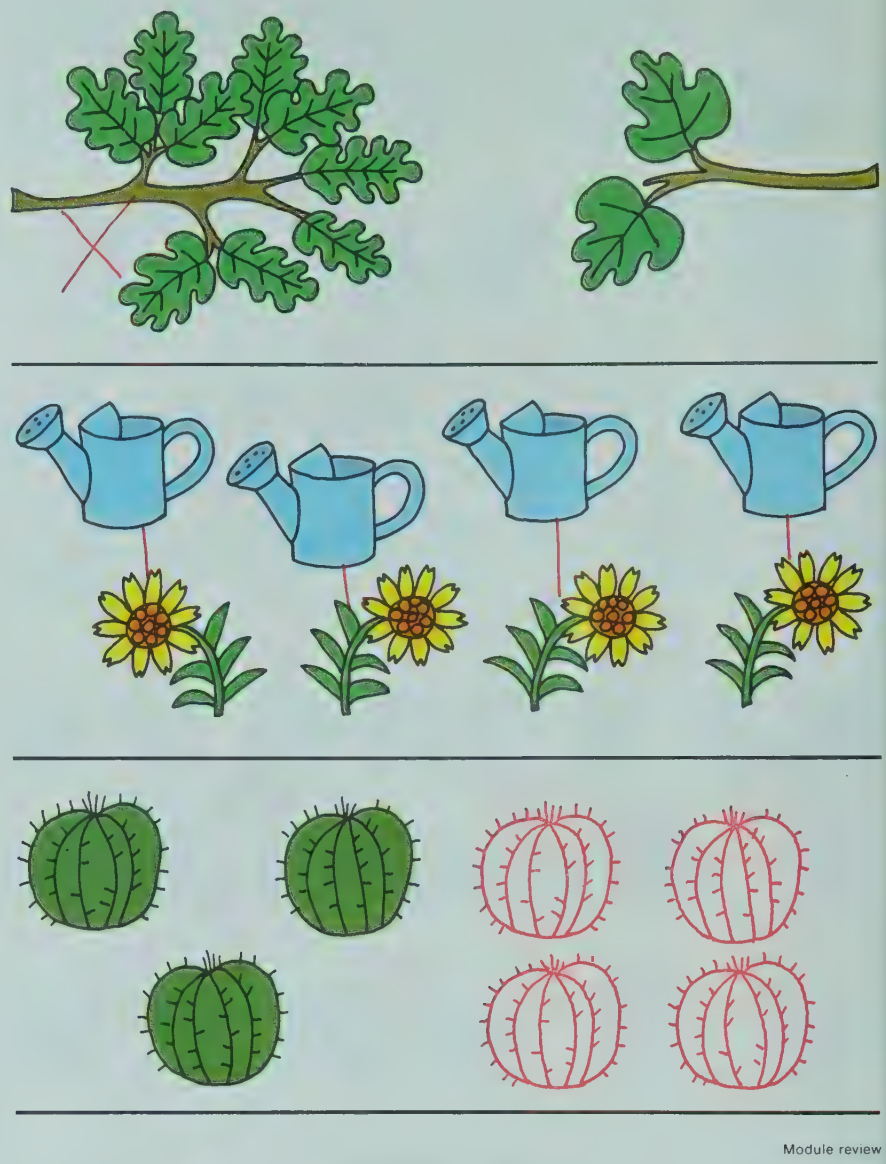
Follow-up

To provide more work with matching and the "one more" idea, duplicate a worksheet similar to the one below. Direct the children to draw matching lines and then color the set having one more.



Teaching
Page p-47

Each section of this page reviews the *more than* concept in a distinct way. In the first frame children should be able to identify the branch that has more leaves simply by visual inspection. Direct the children to mark with an X the branch that has more leaves. In the middle section, ask the children to draw matching lines to show one-to-one matching. Then discuss with them whether or not there are more watering cans or flowers and encourage them to give a reason for their answer. In the bottom section, ask the children to draw a set that has one more than the set pictured. The child should feel free to draw any objects he chooses. However, you may want to suggest that he draw another set of plants, or simply a set of circles or squares.



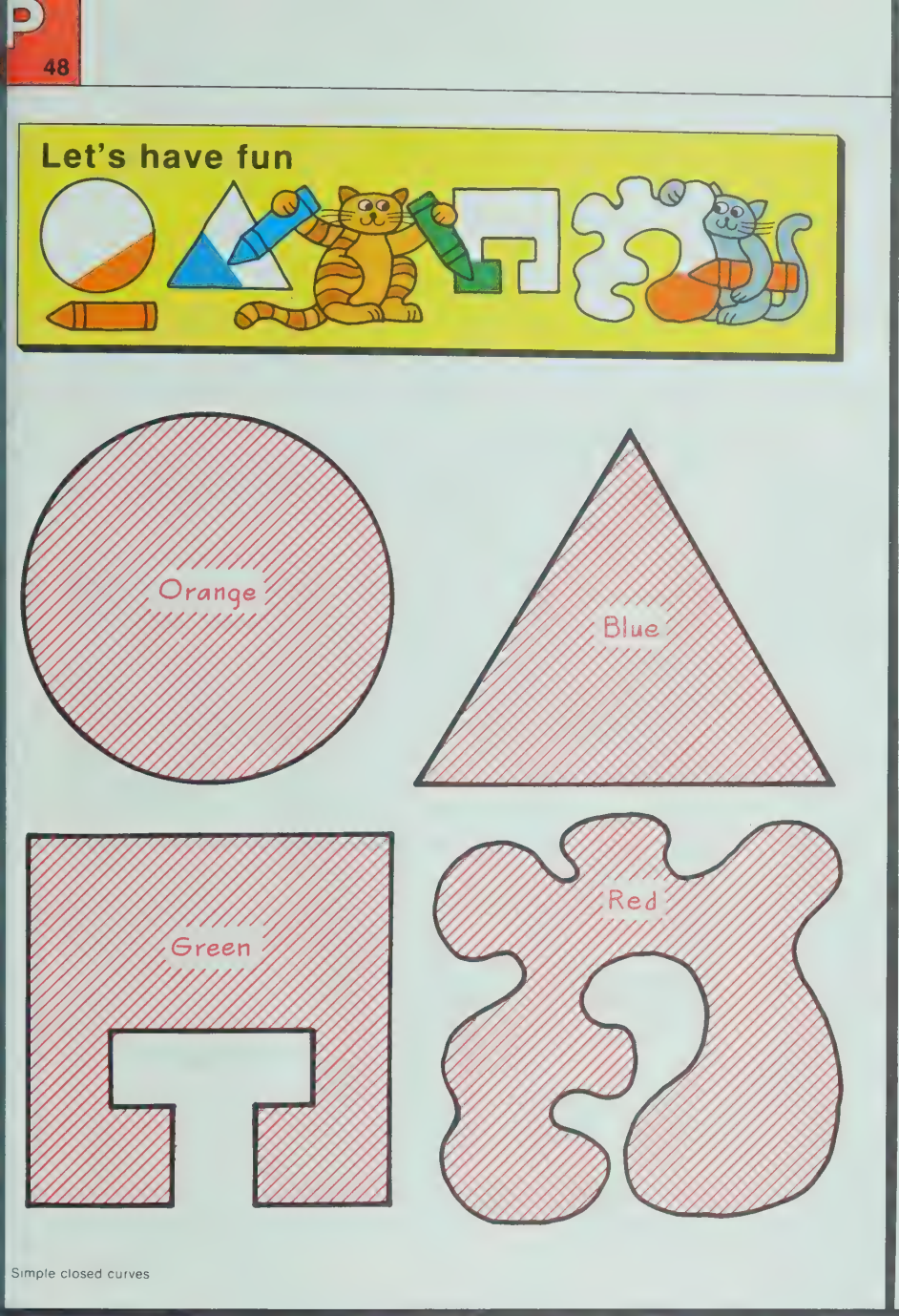
Objective

The child will demonstrate his ability to work with the concepts presented in this module.

This lesson may be treated simply as a summary or review page. Daily observation of children's understanding should serve as your evaluation. We do not intend that your evaluation of a child's number readiness skills be based solely on his performance on a page intended as a review page.

Pre-Book Activity

Prepare for this pair of pages by giving children opportunity to compare sets that are not equivalent. Since it is more common for two arbitrarily chosen sets to be nonequivalent than it is for them to be equivalent, more natural demonstration techniques are possible. For example, ask all the children to be seated and then take a seat yourself. It is likely that there will be some empty chairs in the room. Ask the children to look around and decide whether there are more chairs or more people in the room. If there are empty chairs in the room when everyone is seated, they should observe that there are more chairs. Because everyone is seated and there are chairs left over, they have matched a subset of the chairs with the set of all the people in the room. If you do not have the same number of girls as boys in your room, have the girls and boys pair off to see which set has more. Follow this type of activity with demonstrations on the flannelboard or chalkboard.



Teaching
Page p-48

Introduce this page with a discussion of the figures shown at the top. Explain to the children that they should color the inside of the figures according to the colors used at the top. Thus, the inside of the circles should be colored orange, the inside of the triangle should be colored blue, and so on. Treat this page with a light touch even though it uses the one-to-one matching concept developed in the module.

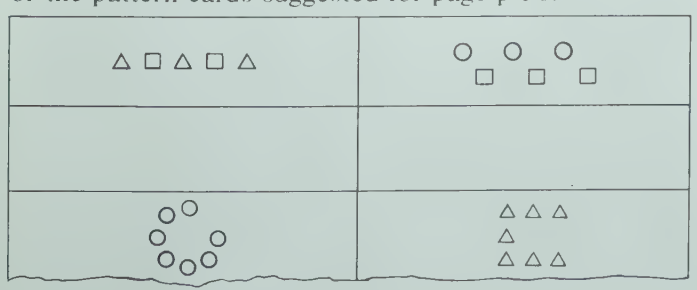
Resources for Active Learning

Mathex: Matching and Graphing No. 1, "Sorting and Matching Games," p. 7, Encyclopaedia Britannica Educational Corp.

Follow-up

Duplicate a worksheet such as that shown below. Notice that patterns of squares, triangles, and circles are used. Ask the children to copy the pattern and to continue it by using one more than the number of objects shown.

You might prefer that the children use pattern blocks or the pattern cards suggested for page p-36.



YELLOW MODULE, UNIT R

Numerals and Numbers 0–5

Pages r-1 to r-16

General Objectives

To introduce the numbers zero through five and their numerals

To continue stressing the idea of equivalent sets

To develop understanding of number concepts

To stress the idea of cardinal number

The idea of a number associated with a set of objects and the concept of two sets having the same number were introduced in the third module of Unit P. In this module, we capitalize upon this introduction to name the numbers zero through five. Children are given sets of five or fewer objects and are expected to identify the cardinal number of the set and the numeral for that number.

A significant feature of the unit is the introduction of the number zero. An important objective is to teach children that zero is a number that has equal status with all the other numbers. Frequently in the past, the number zero and its important arithmetical properties have been slighted or ignored. Zero has been treated primarily as a placeholder or simply as a symbol in the place-value system. In this series the role of zero in the place-value scheme is not formally introduced until Book 1, though the numeral 10 is included in the Primer program.

Most children are able to look at sets containing four or fewer objects and recognize, without counting, the number of the set. Taking advantage of this fact, we can present the idea of the cardinal number of a set in its purest form. We allow the children to sense that a number concept is associated with each set and that the counting process does not determine this number. However, counting should be used to introduce the number five.

Mathematics

The principal emphasis in this unit centres upon cardinal numbers. Earlier, we defined a cardinal number as a class of equivalent sets. This is a formal expression of how people think about numbers. To illustrate, consider page r-6. The fact that the four fingers and the four animals are equivalent sets suggests that they have the same number, an idea stressed in the last unit. The numeral 4 beside these sets names the number. The representations at the top of the page show only a few of the sets that belong to this class, cardinal number 4; other examples also appear among the sets in the lower part of the page, and the children are expected to find them.

Since the concept of number is abstract, the classes which we call cardinal number exist only in our minds. We could not show the class of all sets having, say, two objects. The best we can do is to show a few such sets, to describe the idea, and then to name the number. This is exactly what is done for the children.

It is sometimes extremely difficult to use the words *number* and *numeral* correctly. Remember that a numeral is a symbol (such as the character 5 or the word five) for a number. The number is an idea for which the symbol stands. However, there will be times when using the correct terms will be awkward.

As a rule of thumb for using the number-numeral terminology, use the word *numeral* whenever you are sure that you are referring to the symbol and not to the number. In all other cases, use the word *number*. However, keep in mind that it is better to abuse the language slightly than to make a major issue of it and confuse the children.

Observe that each cardinal number class, other than zero, contains many sets. For example, you can think of many sets containing one object or many sets containing 15 objects. But there is only one set containing no objects. Whether this set is described as containing all people over 6 metres tall, all living dinosaurs, or all four-sided triangles, each of these is the same set. That is, there is exactly one empty set, and the class named *cardinal-number zero* contains just one set. You can show several different illustrations of the empty set, but there is only one empty set. It is easy to confuse the concept of the empty set with the concept of the number zero. There are two different concepts involved: one is a set (empty, of course); the other is the number of the empty set.

Teaching Yellow Module, Unit R

Materials

bulletin board materials (pictures of sets, numerals, etc.)
coffee cans and lids
colored strips
discarded magazines or catalogues
display numerals, large size
envelopes
felt sets of geometric shapes and numerals
flannelboard
geometric shapes of construction paper, 10 per child
gummed seals (wildlife, stars, hearts, and the like)
objects for set demonstrations
paper bags
paper plates
paste
scissors
table tennis balls

Vocabulary

empty set	number	ring	two
five	numeral	three	zero
four	one		

In stressing the idea of equivalent sets and the idea of cardinal number, having group demonstrations with set materials may be more effective than having children work individually. However, the demonstrations should actively involve the children. The development of the idea of the empty set and the number zero is of particular importance in this unit.

Notice that we do not include the words *cardinal* or *whole* in the vocabulary list. It suffices to speak only of *number* with children of this age.

Lesson Schedule

You will find that most children will readily understand this material, and you can expect to move through it quite rapidly. Plan to spend enough time on various patterns so that all the children can, without counting, recognize a set of four or less, give its name, and recognize the numeral for it.

Evaluation of Progress

Although many of the children will be able to recognize the number of a set containing four or fewer objects, it is important to review the ideas formally for all children. Many children who can determine the number of a given set will need practice in identifying the numeral for a particular number. Page r-15 will help you evaluate the children's understanding of these concepts or provide you with a model for designing evaluation materials of your own.

Resources for Active Learning

General Activities

Early Number Multi-Group Lab, Activity Cards 12–15, Responsive Environments Corp.
NR Math Activities, "Shapes," Nos. 6171, 6181, Midwest Publications
Workjobs, Set Activities, pp. 130–179, Addison-Wesley

Manipulative Devices

Color Block Designs (Ideal)
Counting Objects (Responsive Environments Corp.)
Cubical Counting Blocks (Milton Bradley; school supplier)
Cuisenaire Rods (Cuisenaire Co.)
Hainstock Blocks (Creative Publications; Lakeshore)
Hundred Pegboard and Cylinders (Educational Teaching Aids; Mafex)
Pegboards (Ideal; Responsive Environments Corp.)
Seriation Kit (Learning Research Assoc.)
Unifix Material (Educational Teaching Aids; Math Media)

Commercial Games

Dominoes of all kinds (Hammett; Milton Bradley; Responsive Environments Corp.)
First Game (Educational Playsystems)
Hi-Spot (Educational Playsystems)
Puzzles (Childcraft; Ideal)
Smarty (Ideal; school supplier)
Tri-Aminos (Educational Playsystems)

Activities for Continued Use

As children progress into a study of number, they should also progress in the ability to order numbers. Activities in which the children order actual physical objects will be helpful during the three remaining modules of this book. The following are but two examples of the many activities which would be appropriate for children at this age level.

Materials: 10 sticks of increasing length
Procedure: Distribute a set of sticks to a child. Unless he begins to do so spontaneously, ask him to put them in order of size from shortest to longest. Observe whether he looks back and forth between the sticks he has already arranged and those remaining. The ultimate objective is for him to be able to judge not only that a given stick is longer than the one last positioned but also that it is the shortest such stick of those remaining to be placed. It may be necessary to ask pointed questions to help him; for example: "Is this stick longer than the one before it in the row *and* is it shorter than any other remaining stick?" Most children will be able to order the sticks by trial and error, but many will need activities of this sort to develop the ability to consider both relations (longer than the ones in the row and shorter than those remaining).

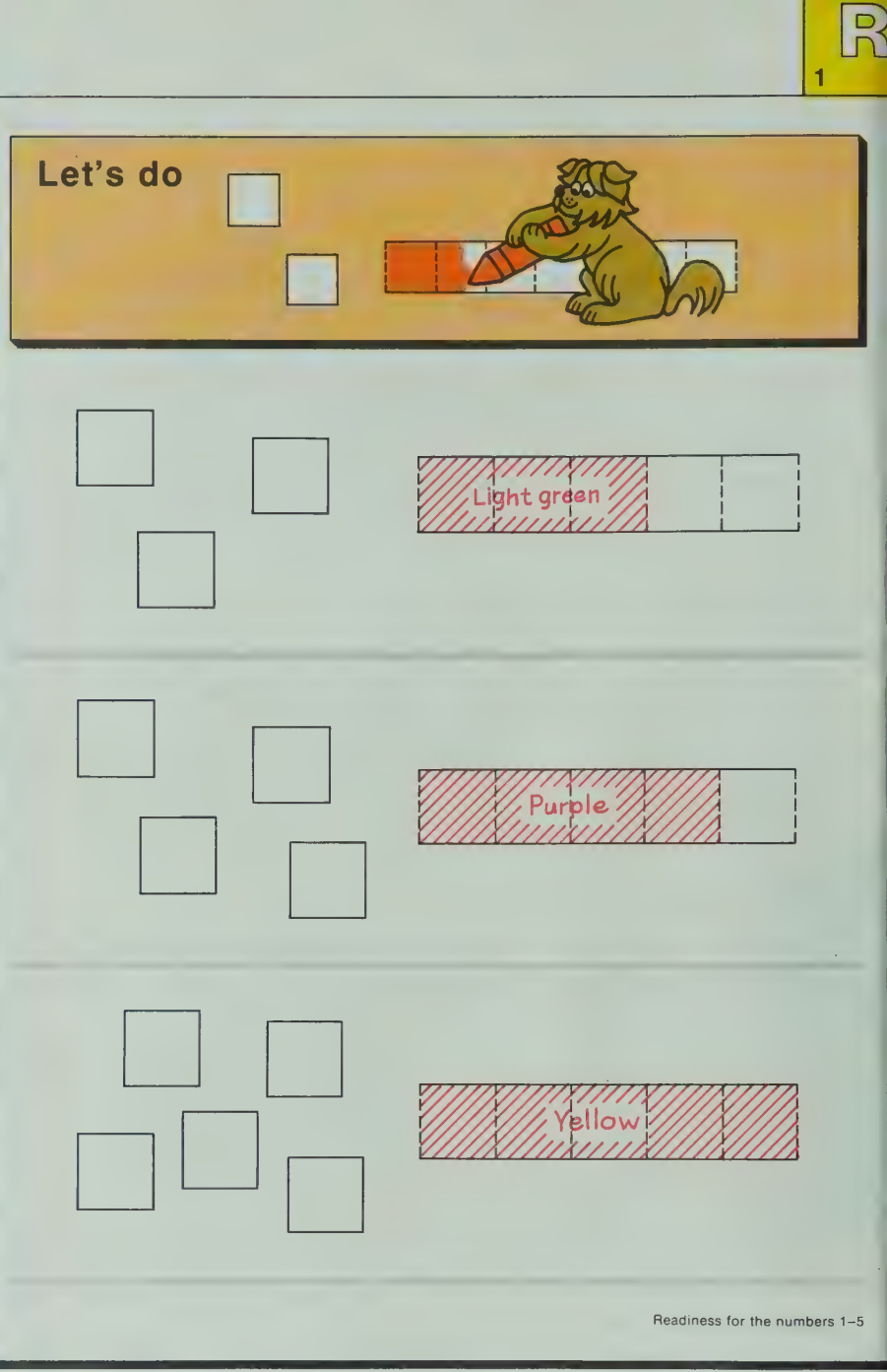
A second activity for the child who can order a set of objects is to order two sets side-by-side as in the following activity:

Materials: 10 pictures of flowers or artificial flowers of increasing size; 10 flower pots or vases of increasing size to match the flowers
Procedure: Distribute both sets of materials to a child or a group of two or three children. Ask them to arrange both flowers and vases from smallest to largest. Help the children arrange them so that they are in parallel lines but not matching in pairs. Then, ask them which vase should be used with which of the flowers. The advanced child will use counting to determine which pair matches, whereas the child still at the perceptual level will match the flower to the opposite or nearest vase. If the child matches the flower to the incorrect but opposite vase, spread out the vases to match the flowers one to one and ask him to point out again the vase for the flowers.

Investigation
Page r-1

Call attention to the illustration at the top of the page. Point out that if the individual white strips are placed together they form a train which matches the red strip. Thus the dog is coloring the matching spaces red. Direct the children's attention to the first frame. Explain that they should find a white strip for each strip shown on the left side of the frame. Next they should make a train with these strips in spaces outlined at the right. Then they should find the strip which matches, or is as long as, this train. Finally, they should color the spaces covered by their white strips the same color as the strip they found which matches this train of white strips.

Give similar directions for the next two frames. In general, explain to the children that they should match the pictured white strips with their white strips, make a train in the space provided, find the colored strip which is as long as their train and color the spaces covered accordingly. If the number names—one, two, three, four, and five—are introduced by the children, use them in a natural way, as convenient. But do not stress the number names; they will be introduced specifically later in this module.



Purpose

To provide a general introduction to the numbers 1 through 5

Preparation

Materials

strips: 1 white—1.5 cm by 1.5 cm; 1 red—1.5 cm by 3 cm; 1 light green—1.5 cm by 4.5 cm; 1 purple—1.5 cm by 6 cm; 1 yellow—1.5 cm by 7.5 cm.
envelopes (1 per child)

Children will need the colored strips for this investigation. Since continued work with these strips for prebook or follow-up activity is recommended, provide each child with an envelope, These strips will be needed again for the last module of this unit.

Encourage children to explore the strips you distribute. You might suggest that they line them up in order of shortest to longest. It would also be helpful to introduce the word "train" by asking the children to make a train with a red strip and a white strip, or with two red strips, (by sharing), and so on. If any have difficulty understanding how to make a train, help them place their strips horizontally end to end. In particular, ask them to make a train using white strips.

A "Train" of Two Red Strips

R 2

Let's talk

See Discussion comments.

Readiness for the numbers 1–5

Discussion
Page r-2

This illustration provides the basis for developing readiness for the numbers 1 through 5. The specific number names need not be introduced, but if they arise during the discussion, be sure that they are used correctly.

Lead the discussion so that the emphasis is on the one-more-than concept. Point out the various sets that appear in the picture. For example, talk about the pelicans perched on the pilings and the seagull flying above. Ask: "How many more pelicans are there than seagulls?" "Are there more fish shown than birds? How many more?" "How many pilings are left for the seagull to perch on?" Elicit from the children the fact that there is *one more* piling left; that there is *one more* fish than birds; and that there are *as many* pilings as there are birds.

Depending on the need of the children, you might extend this discussion to include sets that may be found around the classroom.

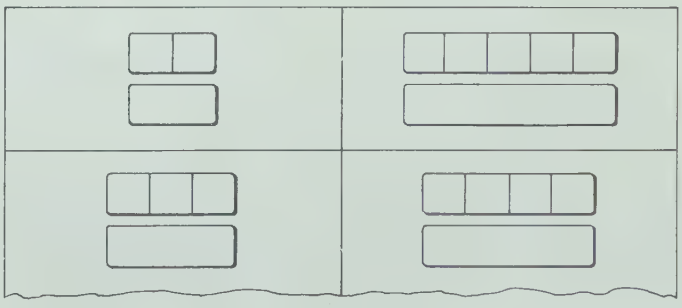
Resources for Active Learning

Math Activities, Activity 2/1, p. 25, Allyn and Bacon

For example, have the children find a red strip. Then, as a preparation for the text page, ask them to place as many white strips as they can on top of the red strip so that the white strips are in a train. Then introduce the text page.

Follow-up

Various activities may be used to give children further experience with the strips. For example, a worksheet such as the one shown at the right will help to familiarize them with the differences among the lengths of the strips. Instruct the children as follows: "One strip matches each frame. Find the strip and color the space to show the strip you found."



Teaching
Page r-3

Use the demonstration art at the top of the page as an introduction. Point out the various representations of one, namely, one finger, one chicken, and the numeral 1. Then explain to the children that they should identify those sets which have one member. Ask them to find all the sets that have exactly one member and mark each with a check (✓).

Objective

Given a set containing exactly one or two objects, the child will be able to identify the number of the set and the numerals 1 and 2.

Since most children are familiar with the numbers one and two, one of the main objectives of this lesson is to teach the numerals 1 and 2. The suggestions in the pre-book activity should help you with this objective.

Pre-Book Activity

Materials

paper for labels

To help children become accustomed to the numerals, it would be helpful to use them on labels throughout the room. For example, label such things as 1 flagpole, 2 erasers, 2 windows, 1 door, 2 bookcases, 1 calendar, 2 boxes, 1 drawer, 1 shelf, 1 paint jar, and so on. (You might also label items for the numbers 3 and 4 to use with the next lesson.) Have the children identify sets of one and two around the room. Point out the labels when appropriate. Then show sets of one and two on the chalkboard or flannelboard and have the children identify the numeral that belongs with the sets. As stated above, most children are familiar with these numbers, so do not over-prepare. You need not use the word "cardinal" in discussing these number ideas with children.

The number 2

Teaching
Page r-4

Again, refer to the illustration to introduce this page. Discuss the various representations for the number 2: 2 fingers, 2 chickens, and the numeral 2. Then, call attention to the sets on the page. Ask the children to mark with a check all the sets that have exactly two members.

Follow-up

After a walk around the school or playground, make co-operative charts of the sets of one and two that the children observed during their walk. Specific ground rules agreed upon beforehand (such as, not including people in the sets), may be necessary to sharpen the children's observation skills.

Free play with the strips is also appropriate to help children build concepts of the relationships among the strips. Encourage children to make any design or picture they choose. Also, suggest that they place strips in order of size to make a stair-step arrangement. No mention of number need be made. Children will benefit simply from the experience of exploring these numerically related strips.

Teaching
Page r-5

Introduce the number and the numeral three by a brief discussion of the demonstration art. Then explain to the children that they should find and mark with a check all the sets which contain three members.

The number 3

Objective

Given sets having exactly three or four objects, the child will be able to identify the numbers of the set and the numerals 3 and 4.

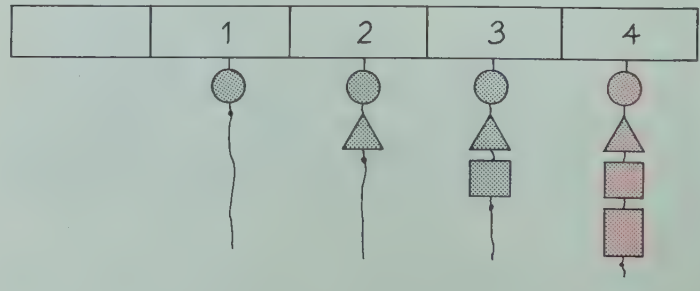
This lesson continues the development of cardinal number by presenting the numbers three and four. Note that counting is not necessary here since most children are capable of recognizing a set of four or less simply by visual inspection.

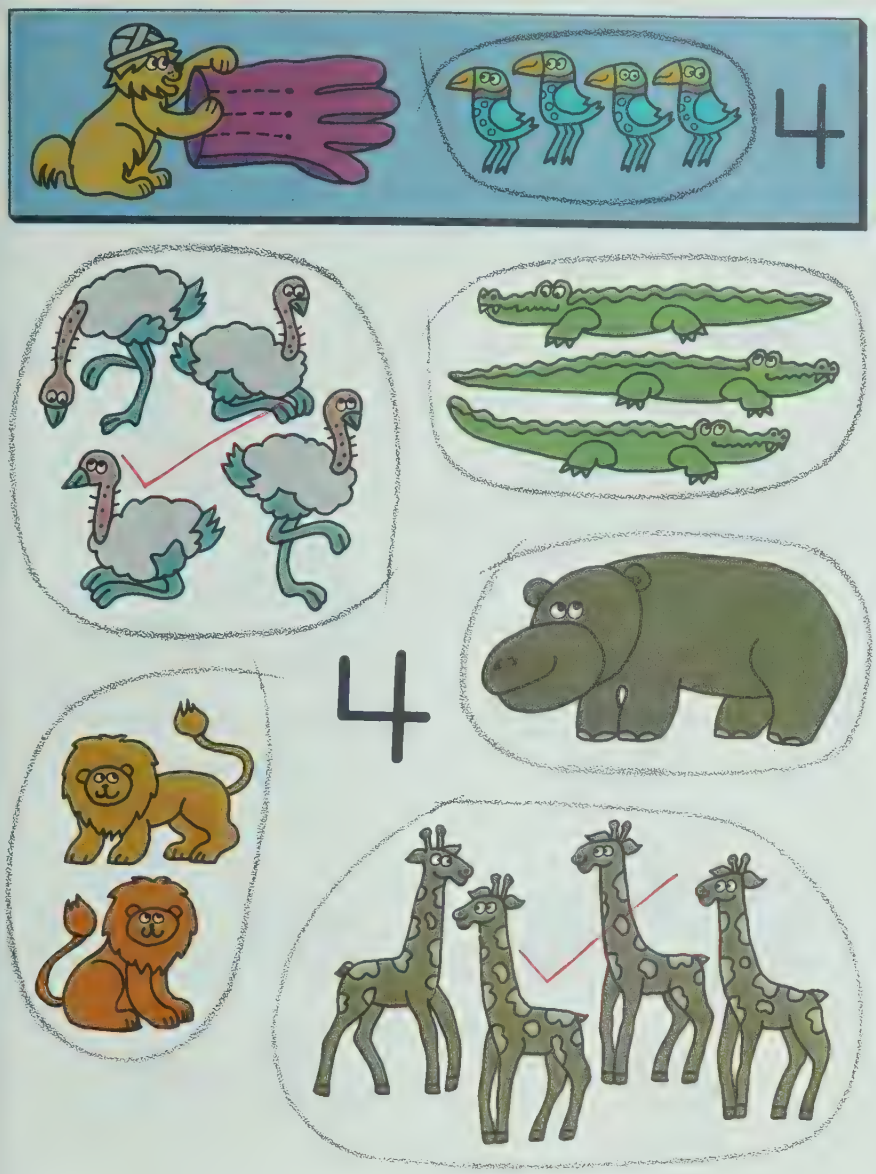
Pre-Book Activity

Materials

flannelboard

Have the children identify sets of three and four in the classroom. Then display sets of three and four objects on the chalkboard or flannelboard. Have the children tell how many are in each set and have them identify the numeral which names this number. Also, review the numbers one and two in the same way. Prepare a display board similar to the one shown below. Show only the numerals 1 through 4, leaving space at the left for 0. Later, when zero is introduced, you can add an empty string to demonstrate this idea.





The number 4

Teaching
Page r-6

Again use the illustration to discuss various ways of representing the number four. Point out the 4 fingers, the 4 birds, and the numeral 4. Continue with directions like those of the previous similar pages. Ask the children to mark with a check all the sets which contain four members.

Follow-up

Give each child an empty egg carton and a collection of assorted objects such as buttons, washers, paper clips, erasers, acorns, screws, bolts, nuts. You may want four or five children to share such an assorted collection. Then distribute cards on which you have printed one of the numerals 1, 2, 3, 4. When a child has received a card, its number determines the number of things he should put in each set he makes. Thus, a child who received a card numbered 3 would collect 3 buttons, 3 screws, 3 paper clips, and so on, placing his sets in the egg carton container. When he has finished, he can lean his card next to his egg carton for all to see.

Or, give the children large sheets of newsprint and crayons. Have them fold the paper to make four sections. Tell them to make sets in each section corresponding to the number shown in your model on the board and to color them accordingly. Sample board instructions might look like this:

4 (blue)	1 (green)
2 (yellow)	3 (orange)

Teaching
Page r-7

Encourage children to discuss the illustrations at the top of the page. Observe with them the different representations of the number five. Then ask them to mark with a check all the sets below which contain five objects. Remind the children that they might want to count the objects in the sets to be sure that they mark only those which have exactly five objects.

The number 5

Objective

Given a set having five members, the child will be able to identify the number of the set and the numeral 5.

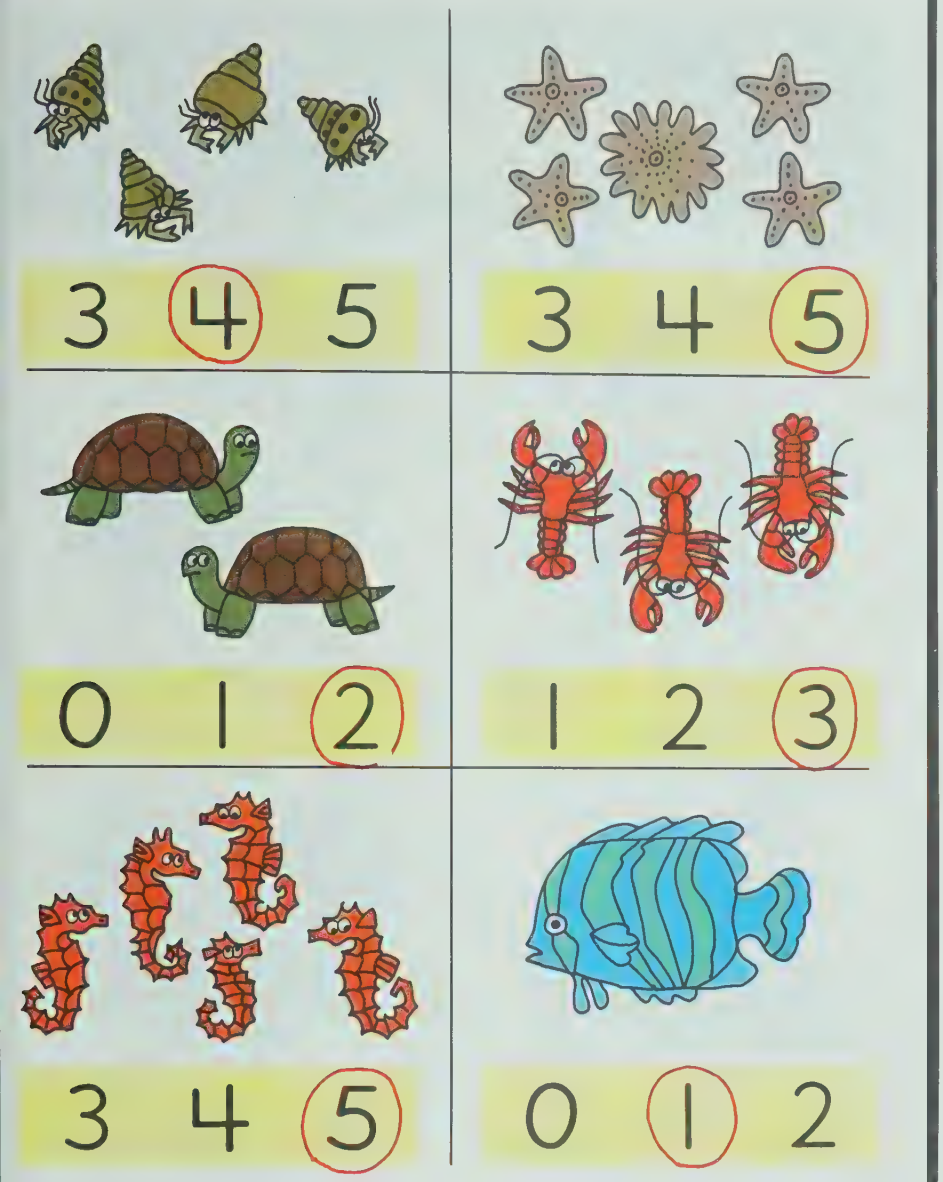
As sets of greater number are introduced, counting will become increasingly important. This will be more applicable in the following module, but most children will need to count a set as large as five to identify its number. Therefore, counting is stressed as a pre-book activity.

Pre-Book Activity

Material

objects for set demonstrations

Although many children already know that counting yields the cardinal number of the set, this idea should be stressed throughout this lesson. Use sets of three and four elements to demonstrate counting techniques. Count very slowly and touch each element as you name the number. Give the children an opportunity to count sets of three and four. Explain that, even though they can tell the number of these sets without counting, they will soon be working with larger sets whose number they will not be able to find unless they count. Display a set of four objects and have them count to four with you. Observe that there are four objects in this set. Now put another object with the set of four, count together to five, and ask the children how many objects there are in this new set. When someone concludes that there are five objects in the new set, explain that five is *one more than* four. Display the numeral 5.



Numbers and numerals 1–5

Teaching
Page r-8

Call the children's attention to the first frame. Point out both the set of four shellfish and the numerals 3, 4, and 5 in this frame. Explain that you want them to figure out how many objects are in this set and to circle the matching numeral. Give careful directions for each frame until you feel that the children are capable of completing the page independently.

Follow-up

This activity can be used here to develop the number concepts of 1–5 and later with zero and the numbers 6–10 as they are introduced in following lessons. Prepare a set of "listening cylinders." These may be made from babyfood jars, though 35 mm film containers would be ideal. Put an opaque piece of paper in the jar so that the view of its contents is blocked as much as possible. Prepare the jars by putting a certain number of items in each. Label the jars with letters or colors or animal pictures, so that children can discuss their choices. The child should take each jar individually, listen to it when he shakes it, and try to guess how many items are in it. Then place the jar near a numeral card of his choice, to show how many items he thinks are in the listening cylinder. In one container have one bean, in the next have two beans, and so on. (Later you can leave a container empty for zero). Have a child listen and try to guess the number of beans in each cylinder or jar and set the jar on top of a numeral card. The numeral cards can simply have a numeral printed on them, or they can show a single dot with the numeral 1 below it or two dots with the numeral 2 below them, and so on. After the child has guessed the number of beans he thinks is in the jar, he should open it to see whether or not his guess was right. If not, he should move the jar so it is near the correct numeral card. Other items you might use in the jars are pebbles, pins, or paper clips. When a child has finished, he should remove all the numeral cards from under the jars and place them in a pile ready for the next child to use.

Teaching
Page r-9

After using the art to introduce the page, tell the children that they are to draw a ring around the numeral which tells how many objects in each set. Have them do the first exercise, check the answer with them, and then instruct them to complete the page by themselves.

Numbers and numerals 1–5

Objective

Given pictures of sets having 1, 2, 3, 4, or 5 objects, the child will identify the number of the set by marking the corresponding numeral.

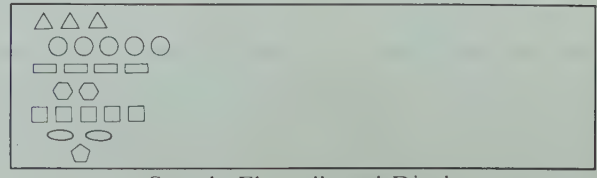
Although no effort is made to teach the kindergarten child to print the numerals, it is intended that he learn to recognize them. This lesson specifically treats this ongoing objective of developing the child's ability to identify numerals which correspond to the number of a given set.

Pre-Book Activity

Materials

flannelboard and felt numerals 1–5
sets of geometric shapes (or other felt objects)

Give the children an opportunity to identify sets of 1, 2, 3, 4, or 5 objects and the corresponding numerals. For example, before class begins, place on the flannelboard eight sets of geometric shapes with 1, 2, 3, 4, or 5 members in each set. During class, ask eight different children to come up, in turn, to choose the numeral that corresponds to the number of objects in each set. Then remove these sets and place the numerals 1 through 5 on the flannelboard and ask other children to place the correct number of felt objects next to the numeral.



Sample Flannelboard Display

Numbers and numerals 1–5

Teaching
Page r-10

Ask how many rabbits there are in the first frame at the top of the page. Tell the children to circle the numeral for the correct number in the set. Have them complete the page by circling the proper answer for each frame.

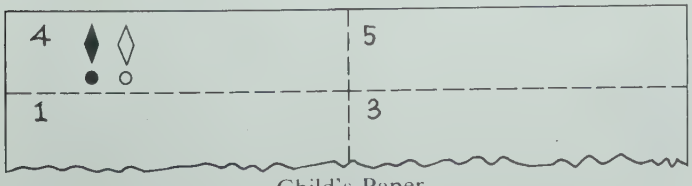
Resources for Active Learning

Math Activities, "Finger Count," p. 30, Allyn and Bacon

NR Math Activities, "Beginning Number Concepts," No. 2162, Midwest Publications

Follow-up

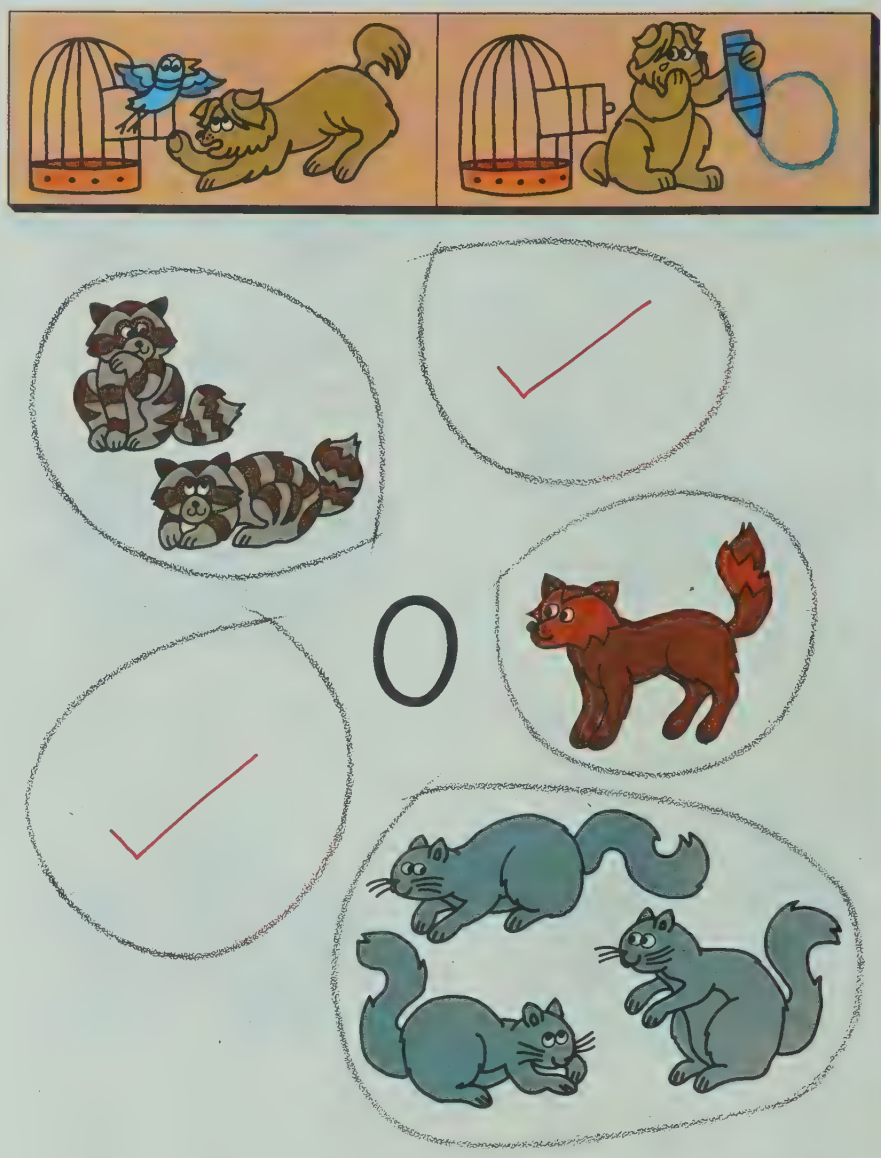
Give the children large worksheets on which the numerals 1 through 5 have been duplicated, or written with a felt marker, in each of four sections. Give each child scissors, paste, and bits of colored construction paper from the scrap box. Ask them to cut out and paste the proper number of shapes in each section. Shapes and colors can be any that strike the child's fancy, but he *must* paste the proper number of shapes in each section.



Child's Paper

Teaching
Page r-11

Use the empty bird cage at the top of the page as a basis for discussion. Stress that the bird cage has zero birds in it. Then explain to the children that they should study each ringed set shown on the page and place a check mark on all those which have zero members.



The number 0

Objective

Given a set having zero objects, the child will be able to identify the number of the set as zero and the numeral 0.

Zero is the only number which has exactly one representative set. There are many ways to describe the empty set. For example, you can talk about the set of all wild tigers in your classroom, or the set of all children in your room who are over two metres tall. Yet, there is one and only one set involved in these descriptions, namely, the empty set. The most important mathematical idea to stress in this lesson is that zero is a number; zero is the number of the empty set.

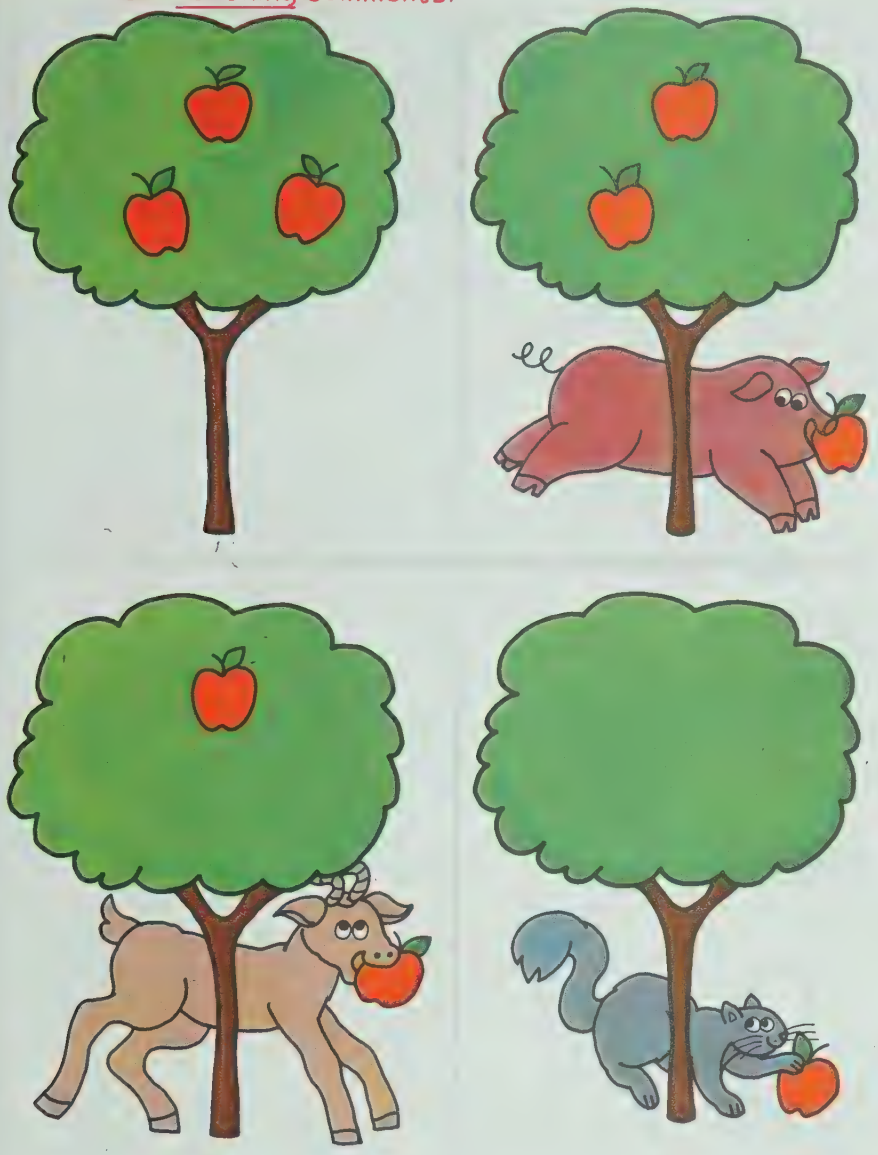
Pre-Book Activity

Materials

coffee cans with lids
table tennis balls (15)

Begin by asking such questions as, "How many of you are older than your father?" or "How many elephants are there in this room?" When the children naturally respond "None" or "There are no elephants," point out that the number zero tells how many elephants are in this set. That is, when a set contains no objects, the number of that set is 0. Use the term *empty set* freely. As a concrete example, prepare six coffee cans. Leave one can empty. In the others, place table tennis balls so that you have set of 0 through 5: zero in one can, one in another, two in another, and so on. Then show these cans, with

See Teaching comments.



The number 0

Teaching
Page r-12

Tell a story about animals taking apples from the tree. For example, you might develop the theme of Farmer Jones leaving his barnyard gate open and narrate the wandering of the pig and goat.

Use the sequential development of the four pictures to discuss the fact that there were three apples to begin with but zero apples after each animal in turn removed one.

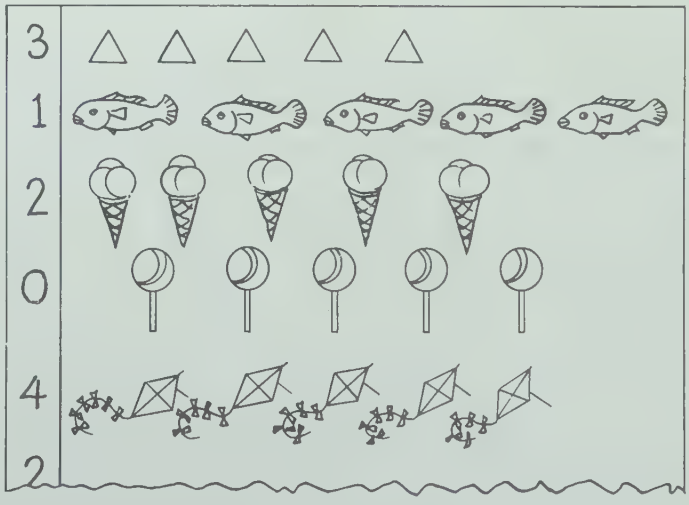
Resources for Active Learning

Mathex: Numeration No. 2, Activity 2, p. 6, Encyclopaedia Britannica Educational Corp.

lids on, to the children. Explain that you would like to play this guessing game with them: They should try to guess how many table tennis balls are in each can, as you shake each one. As children guess the number in each can, take the lid off and count to see if they are correct. If they say: "None" for the empty can, explain that the number in the empty can is zero table tennis balls and teach the numeral 0. Stress the fact that the symbol 0 represents the number of the empty set.

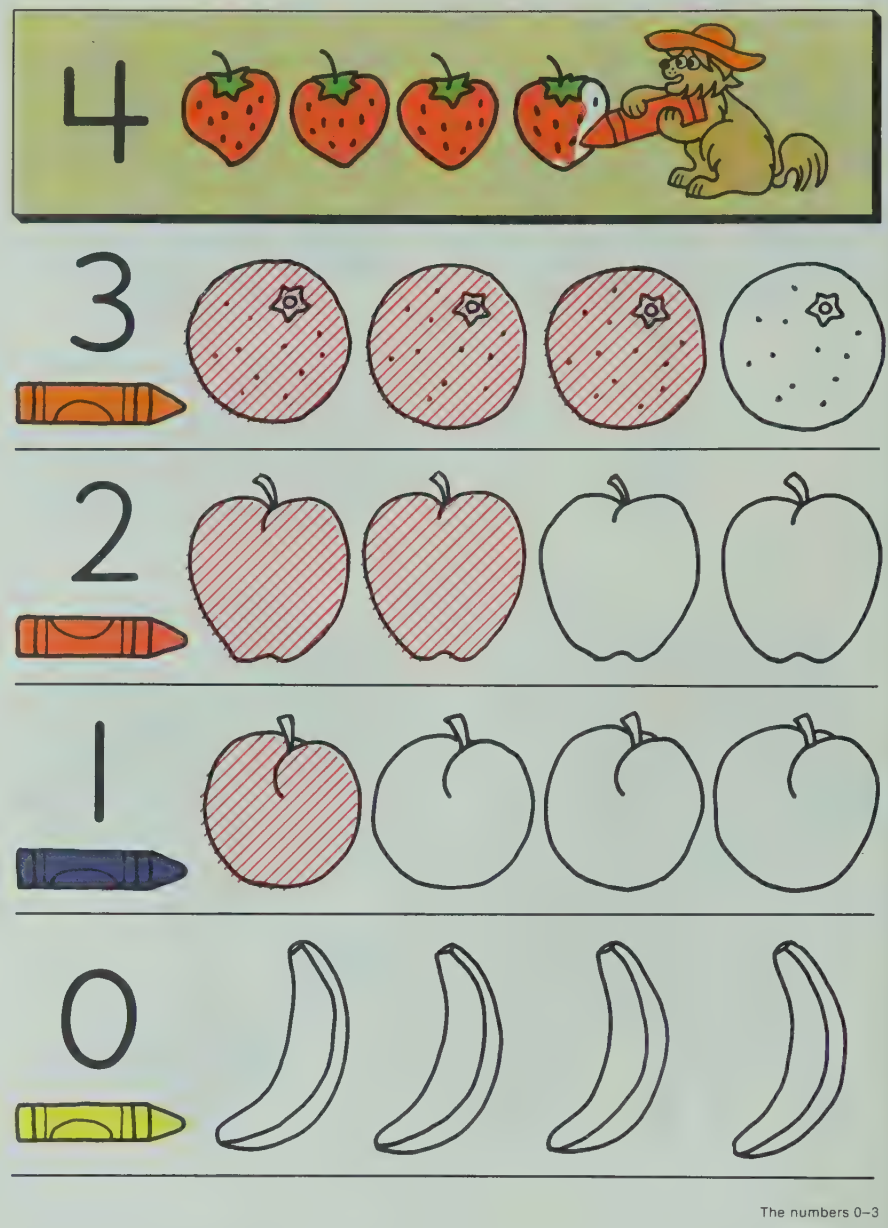
Follow-up

For more practice in identifying sets, give the children a worksheet similar to that shown at the right. Direct the children to color enough objects in each row so that the colored set matches the numeral at the left.



Teaching
Page r-13

Use the demonstration art to explain the directions for this page. Point to the numeral for each row and tell the children that the numeral shows how many pieces of fruit to color. Direct them to start their coloring on the piece of fruit next to each numeral.



The numbers 0–3

Objective

Given one of the numerals 0, 1, 2, 3, 4, or 5, the child will be able to identify it by coloring a corresponding set.

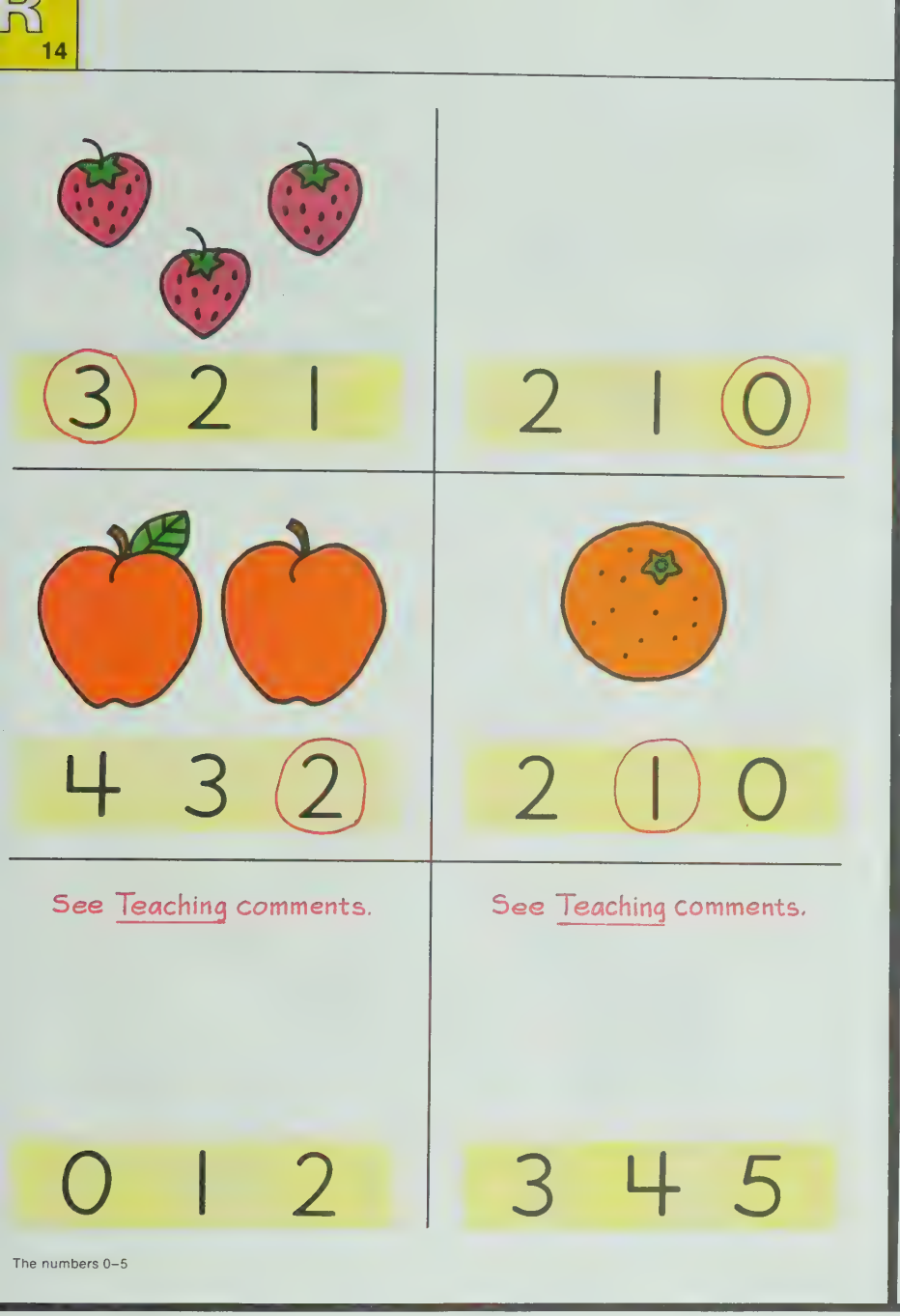
This lesson reviews all the numbers presented thus far in the text.

Pre-Book Activity

Materials

felt numerals, 0 through 5
strips: white, red, light green, purple, yellow (see p. 72)

Give the children an opportunity to identify the number of sets containing five or fewer objects. For example tell them to listen as you tap a drumstick on your desk, and to tell you how many taps they hear if you call on them. Tap three times, then two, four, and one. Ask a different child each time to tell you how many taps he heard. Or display felt numerals 0 through 5 on the flannelboard. Say, "I am thinking of the number *one more than* two. Who can show me the right numeral?" Continue giving numbers and asking the children to find the numeral that is one more, one less, or the same as the given number. You might also ask children to place their colored strips (made for R-1) in stair-step fashion to show the numbers 1 through 5. Help them order them correctly.



The numbers 0–5

Teaching
Page r-14

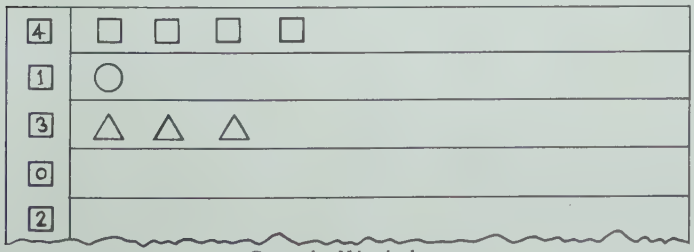
Tell the children that for each of the first four frames they are to draw a ring around the numeral which tells how many pieces of fruit are in the set. Have them do the first exercise at the top of the page. In the second frame (first row, right column) they should circle zero since there are zero pieces of fruit shown. Check their answers and then ask them to complete the page. Notice that the two frames at the bottom of the page are blank. Explain to the children that here they may draw in their own set and then circle the numeral to show how many they have drawn.

Resources for Active Learning

Math Workshop: Games and Enrichment Activities, "Pictures Suggest Number," pp. 12–13, Encyclopaedia Britannica Educational Corp.

Follow-up

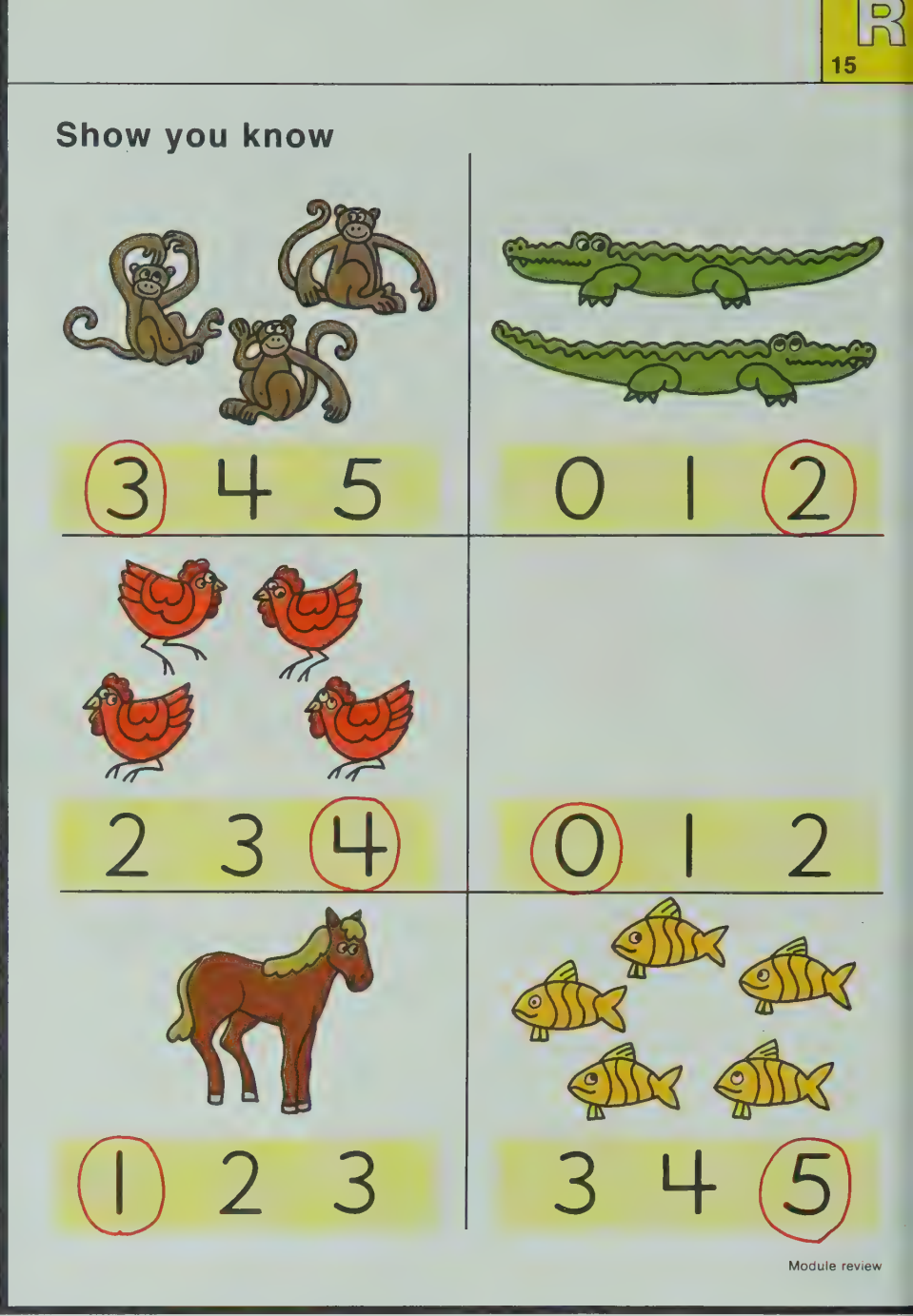
Give each child an envelope with 10 construction-paper geometric shapes, the numerals 0 through 4, paste, and a large worksheet made from newsprint or construction paper. Tell the children to paste down a numeral, and then paste down the proper number of objects beside each numeral.



Sample Worksheet

Teaching
Page r-15

Explain to the children that this page is similar to others that they have worked previously. Tell them that you would like them to figure out the number of each set and circle the numeral which matches the set.



Objective

The child will demonstrate his ability to work with the concepts presented in this module.

Both of these pages review the concepts presented in this module, but it is intended that page r-16 be treated with a light touch.

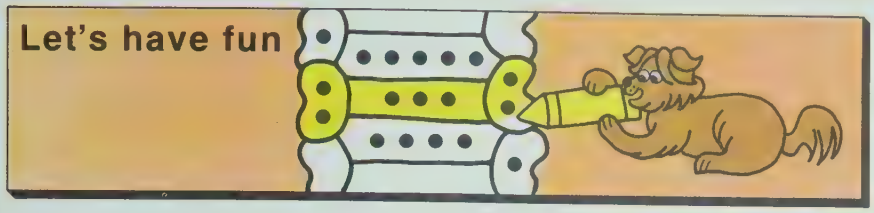
Pre-Book Activity

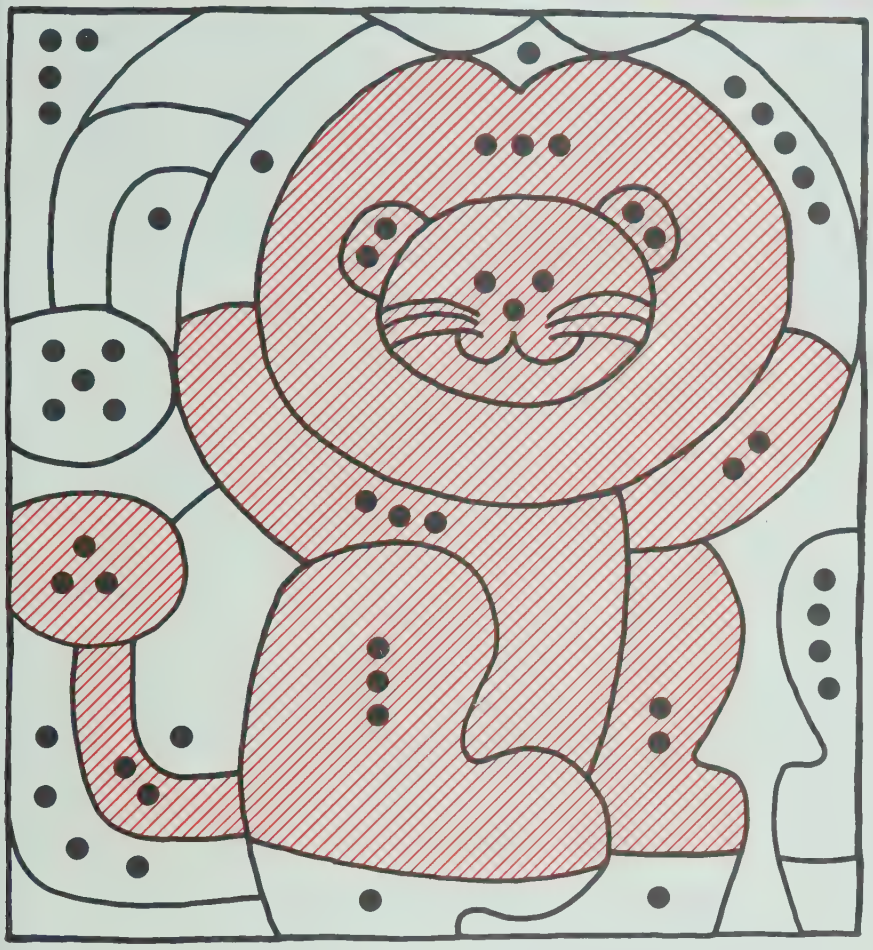
Materials

display numerals

You can conduct two types of activities as a review. Begin by saying a number from one through five and asking the children to find sets in the room which have this number of objects. The second type of activity is a reversal of the first. Mention a set and have the children tell the number of the set. For example, ask the children how many teachers are in the room, or ask them how many doors there are in the room.

During both types of discussions, you should continue to focus attention on the numeral for each of these numbers. For example, each time a number or a set is mentioned, ask someone to identify the numeral for that number from a display of numerals on the chalkboard.





Change of pace

Teaching
Page r-16

Explain to the children that this page consists of a hidden picture which will appear when they color the sections according to a particular rule. Tell them to color all the spaces where there are 2 or 3 dots with one color, and to leave blank or color very lightly all the spaces that have 0, 1, 4, or 5 dots.

Resources for Active Learning

NR Math Activities, "Patterns," No. 2963, Midwest Publications

Follow-up

This activity will show the children how to interpret the numerals they see on the board by pasting objects to match them. Each child will need five paper plates and collections of small pictures. Draw circles like paper plates on the chalkboard, and write the numerals from 1 through 5 in them. Ask the children to paste a set on each paper plate to match the numerals on the board. Let the children choose the objects, but be sure they make one plate for each numeral.

Collect the plates, shuffle them, and show them to the class, one at a time. Ask, "How many?" for each plate. Keep the pace as rapid as the children's responses will permit.

Circles on the Board

Individual Sets of Paper Plates

Numerals and Numbers 6–10

Pages r-17 to r-32

General Objectives

To introduce the numbers and numerals 6 through 10

To develop the counting technique to determine how many are in a set

The numbers six through ten are introduced by stressing that each is *one more than* the preceding number. Counting is introduced as the technique of determining how many are in a set. In the previous module the numbers one through four could be recognized by visual observation. But as the numbers increase counting must replace sight as a method of identifying the number of a set.

Mathematics

When we introduce counting, we encounter both the ordinal and cardinal concepts of number. You should recognize that a cardinal number refers to a quantitative aspect of a set of objects, whereas an ordinal number refers to the position of one object in an ordered set. Both concepts are important. Cardinal-number arithmetic is the mainstream of the mathematics presented in the elementary school. Ordinal numbers are brought in as an aid in shedding light upon certain topics in cardinal-number arithmetic.

Teaching Orange Module, Unit R

Materials

coffee cans, 9 (Smaller cans may be used if beans replace table tennis balls. See page r-25.)
colored paper cups, 10
counters, 9 per child
flannelboard and felt objects
numeral cards
paper plates
strips: 1 dark green—1.5 cm by 9 cm; 1 black—1.5 cm by 10.5 cm; 1 brown—1.5 cm by 12 cm; 1 blue—1.5 cm by 13.5 cm; and 1 orange—1.5 cm by 15 cm.
table tennis balls, 36 (Large beans may replace these.)

Vocabulary

compare	matching	seven
count	more	six
eight	nine	ten
less	number	

Since the strips will be used again later, keep them carefully in envelopes or other such containers. Activities with the strips may be extended, but it is not essential that children be able to associate a number with each strip. Counting activities should be provided in order for each child to count objects in a variety of sets. Twelve circular counters may be used for this purpose.

Lesson Schedule

Pace your treatment of these lessons according to the children's needs and abilities. Allow children many counting opportunities even if they seem able to easily identify the numbers from six through ten.

Evaluation of Progress

You can easily determine the ability of a child to recognize the number and the corresponding numeral of a set containing ten or fewer objects. Although a child's performance on the "Show You Know" page (r-31) might indicate his ability, your determination of his ability should also be based on daily observations.

Resources for Active Learning

General Activities

Developing Number Experiences, Kit A, "The Domino Games," pp. 42–44, Holt, Rinehart and Winston
Developmental Math Cards, "My Family," A19, Addison-Wesley
Early Number Multi-Group Lab, Activity Cards 20–22, Responsive Environments Corp.
Mathex: Numeration No. 2, ". . . Counting Process," pp. 1–3; "Basic Ordering Experiences," pp. 4–5; Encyclopaedia Britannica Educational Corp.
NR Math Activities, "Topology," Nos. 8513,, 8523, 8572, Midwest Publications
Nuffield Project: *Mathematics Begins* 1, "Ordering and Inclusion," pp. 32–36; "Towards Number," pp. 37–41, Wiley
Workjobs, Number Sequence Activities, pp. 182–191; Relationship Activities, pp. 214–223, Addison-Wesley

(See the introductory notes for the yellow module, Unit R, page 71, for suggested manipulative devices and commercial games.)

Activities for Continued Use

Graphing is a device used throughout the entire spectrum of mathematics. Simple graphing is helpful even at this early level. Children are made aware of certain relationships through pictorial representation and the graph serves as a basis for discussion of number and particularly of comparisons with number. The first graphing experience should be very simple; it should deal with only two alternatives. Consider the following example.

Materials: large chart paper; black crayon or pencil; paste or glue; squares of colored paper which children can use to represent themselves on the graph.

Procedure: Choose a yes-or-no question such as "Does your family have any pets?" Prepare the chart paper to show the following:

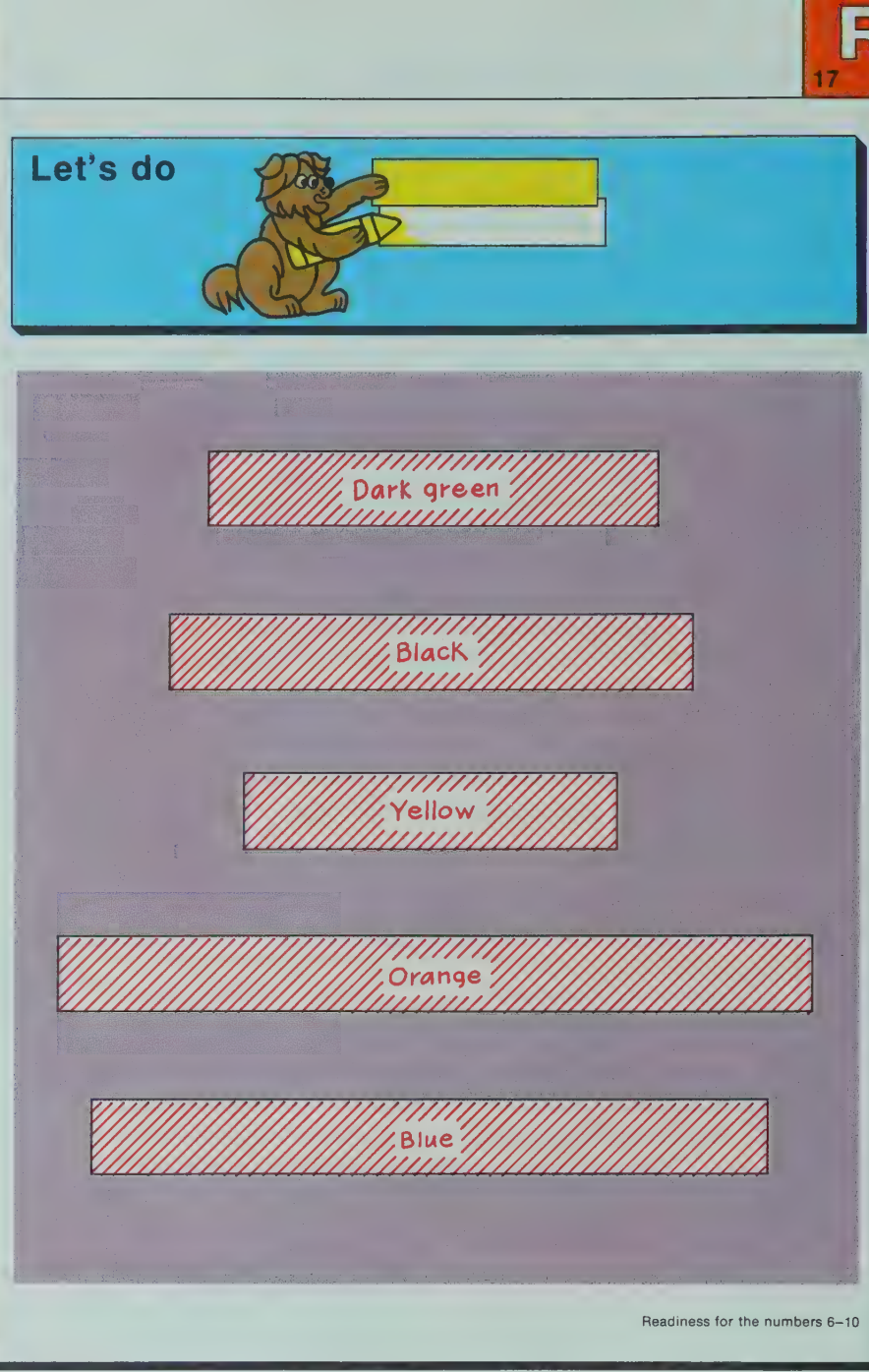
Does your family have any pets?	
Yes	No

Distribute the squares so that each child has one. Ask the question of all the children. Point out the Yes and No columns. Explain that they should come up to the chart and place their square of paper in the column which shows their answer to the question. Note that the graph has only two alternatives and there is a one-to-one correspondence between children and squares on the chart. The completed chart should be discussed. The following kinds of questions are appropriate: "What does this square in the Yes column tell us?" "Is there one square for each child?" "Are there more families that do have pets than don't have pets?" "How do you know?"

Other suggestions for two alternatives are charts showing the number of boys and girls in the class; how many walk to school and how many ride; how many eat lunch at school and how many eat lunch at home. For further reference, see *Pictorial Representation*, 1, John Wiley & Sons, Inc.

Investigation
Page r-17

When you have distributed the printed page, introduce the activity by pointing out the demonstration art. Distribute envelopes containing the strips made for R-1 plus the 5 new dark green, black, brown, blue, and orange strips made for this unit. Explain to the children that they will need only five of their strips but that they must decide which five strips to use. They should do this by finding which strip will exactly cover each blank strip on the page. When they are sure they have found the matching strip, they should color that space on the page the same color as the matching strip. The children who complete this activity quickly might continue the type of free-play activity with the strips that was suggested in the preparation.



Purpose

To provide a general introduction to the numbers 6 through 10

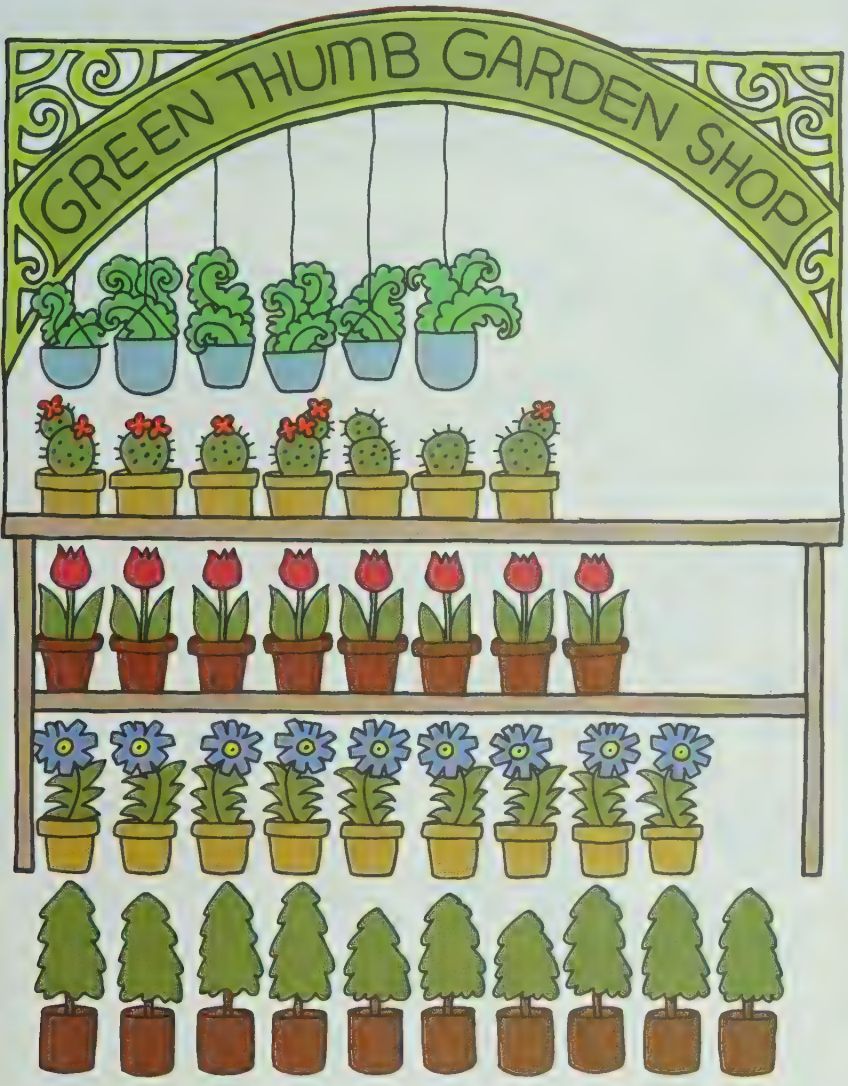
Preparation

Materials

set of strips: dark green (1.5 cm by 9 cm), black (1.5 cm by 10.5 cm), brown (1.5 cm by 12 cm), blue (1.5 cm by 13.5 cm), and orange (1.5 cm by 15 cm)

Distribute all of the strips and encourage the children to make shapes with them such as a house, a rocket, or a tree. Also encourage them to arrange the strips in stair-step fashion from shortest to longest. As they work with the strips, have them use the white strip (the basic unit) to mark off the back side in sections. They will use these sections in the discussion part of the lesson.

Let's talk See Discussion comments.



Discussion
Page r-18

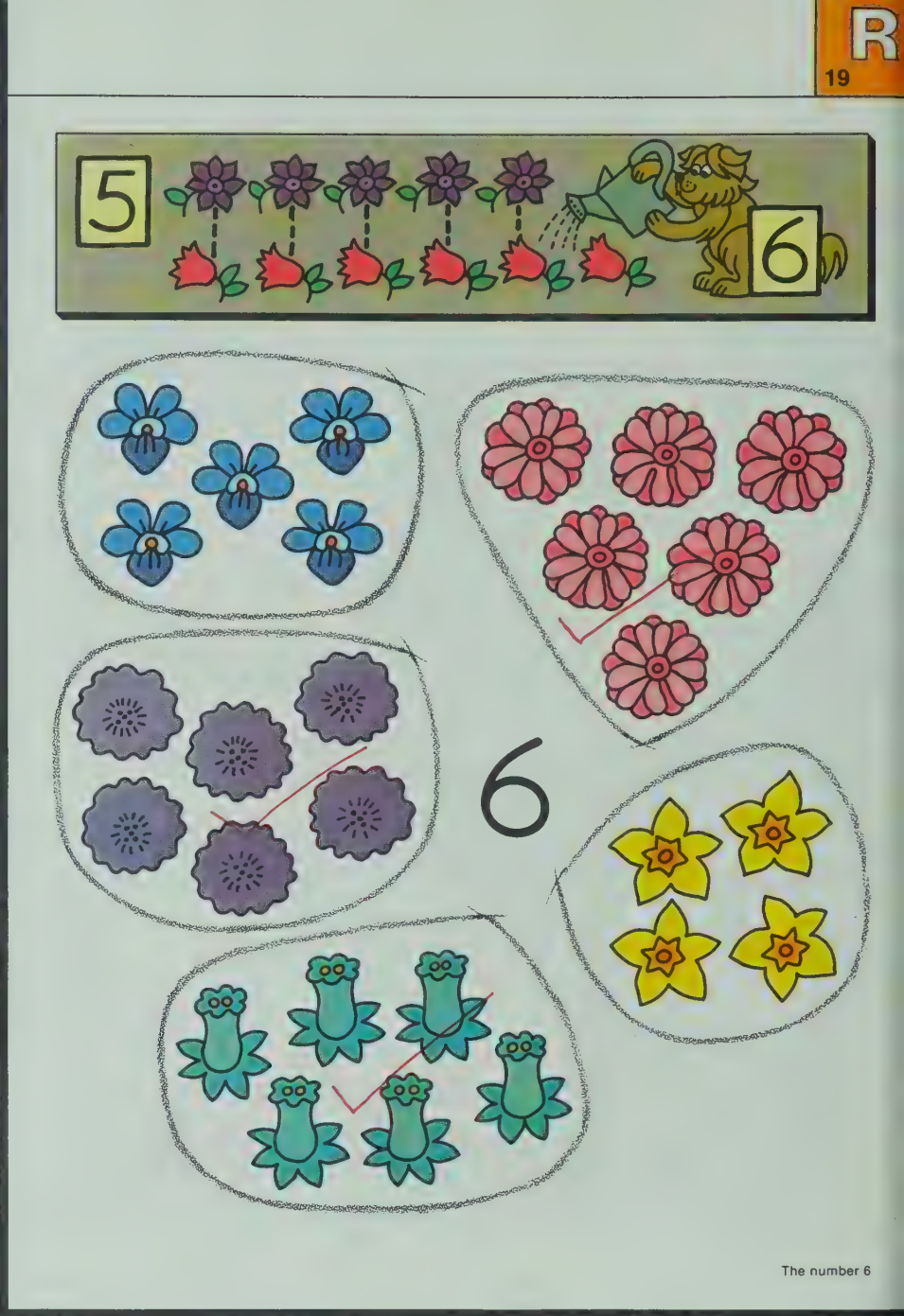
Children will again need their strips for this page. Help them identify the various rows of plants by color. Ask them to use the same strips which they used on page r-17. Direct them to look at the back side; point out the sections that they have marked. Ask them to line up their strips next to the rows and try to match the sections on the strip with the flower pots in each row on the page. Point out that each row has one more plant in it than the row before it. This *one more than* concept is the main idea that should be stressed; number names will be introduced in subsequent lessons.

Follow-up

Use of a pegboard and colored beads may help some children count. Put a strip of tape along the top of the board and label numerals 1 through 6 to correspond with the first six holes on the exposed row. Ask the children in a small group to count as they place the pegs. Beads can then be placed over each peg, and the children can count from 1 through 6 again.

Teaching
Page r-19

Count with the children the set of 5 purple flowers in the art at the top of the page. Then discuss the one-to-one matching with the set of red flowers. Help children observe that there are 5 red matched with the five purple flowers and one more, making a total of 6 red flowers altogether. Finally explain to the children that they should mark with a check all the sets on this page that contain exactly six members.



Objective

Given a set having six objects, the child will be able to identify the number of the set by counting and to associate the numeral 6 with the set.

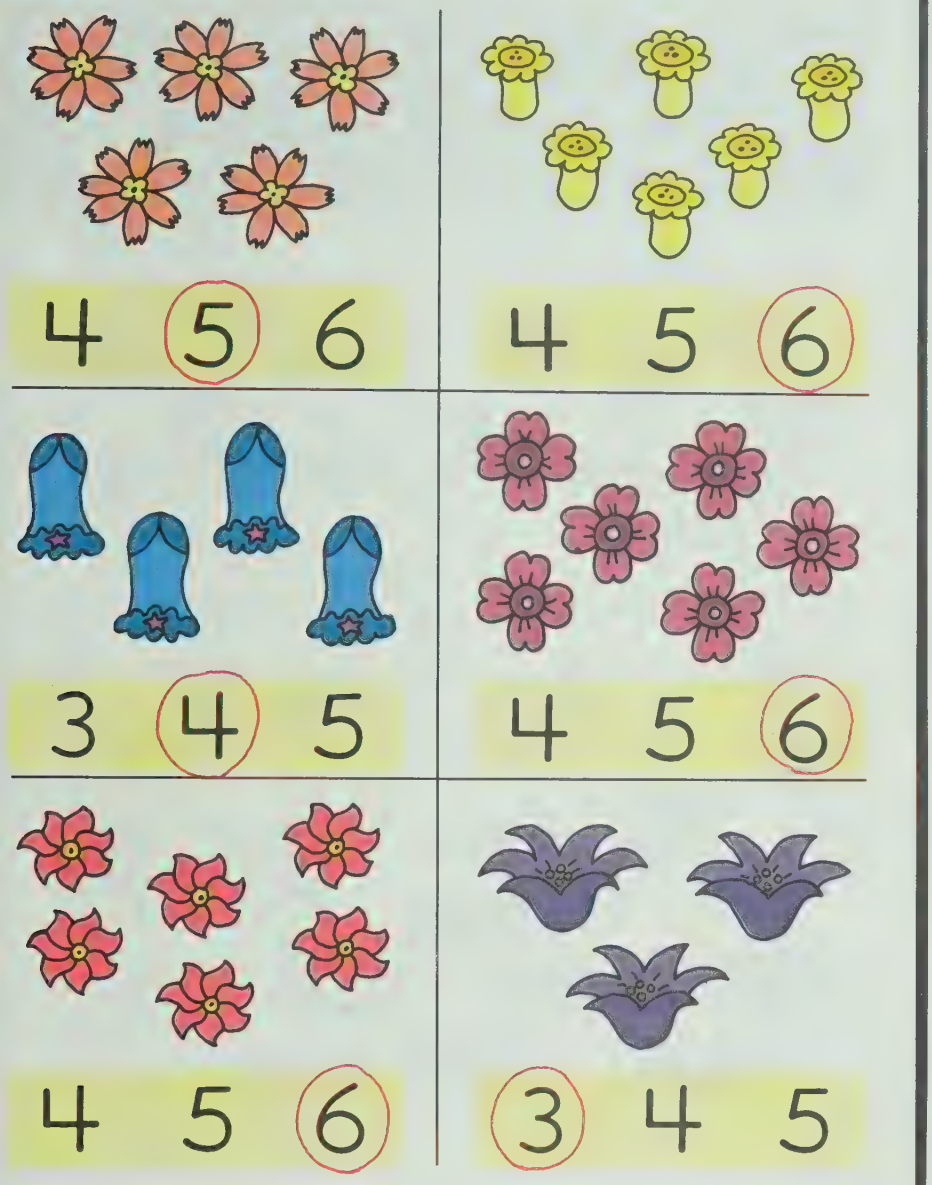
This lesson continues the development of the cardinal numbers from 0 to 10. Recall that counting activities become more appropriate as the numbers children study become larger.

Pre-Book Activity

Materials

objects for set demonstrations

Ask volunteers to identify the number of several sets containing five or fewer objects, and then exhibit a set of five. When the children have discovered the number of this set, put one more object with it and count the objects in the new set. Have the children count with you, since most of them will know, if only by rote, that six follows five. Emphasize that six is *one more than* five. Display various patterns for a set of six so that the children can become familiar with them.



The numbers 3–6

Teaching
Page r-20

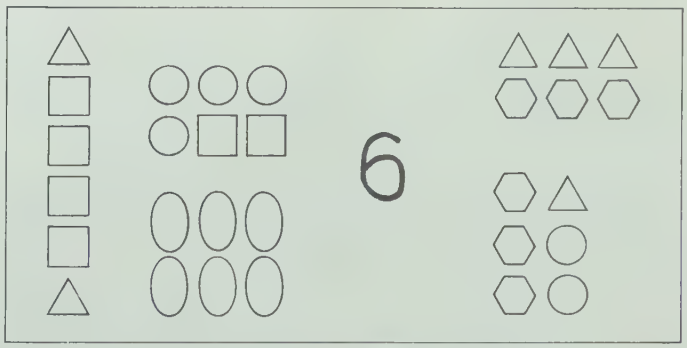
Observe with the children that each frame shows a set of flowers and a choice of numerals below it. Explain that they should find how many are in each set and ring the numeral to show how many.

Resources for Active Learning

Math Activities, Activity 2/1, p. 25, Allyn and Bacon

Follow-up

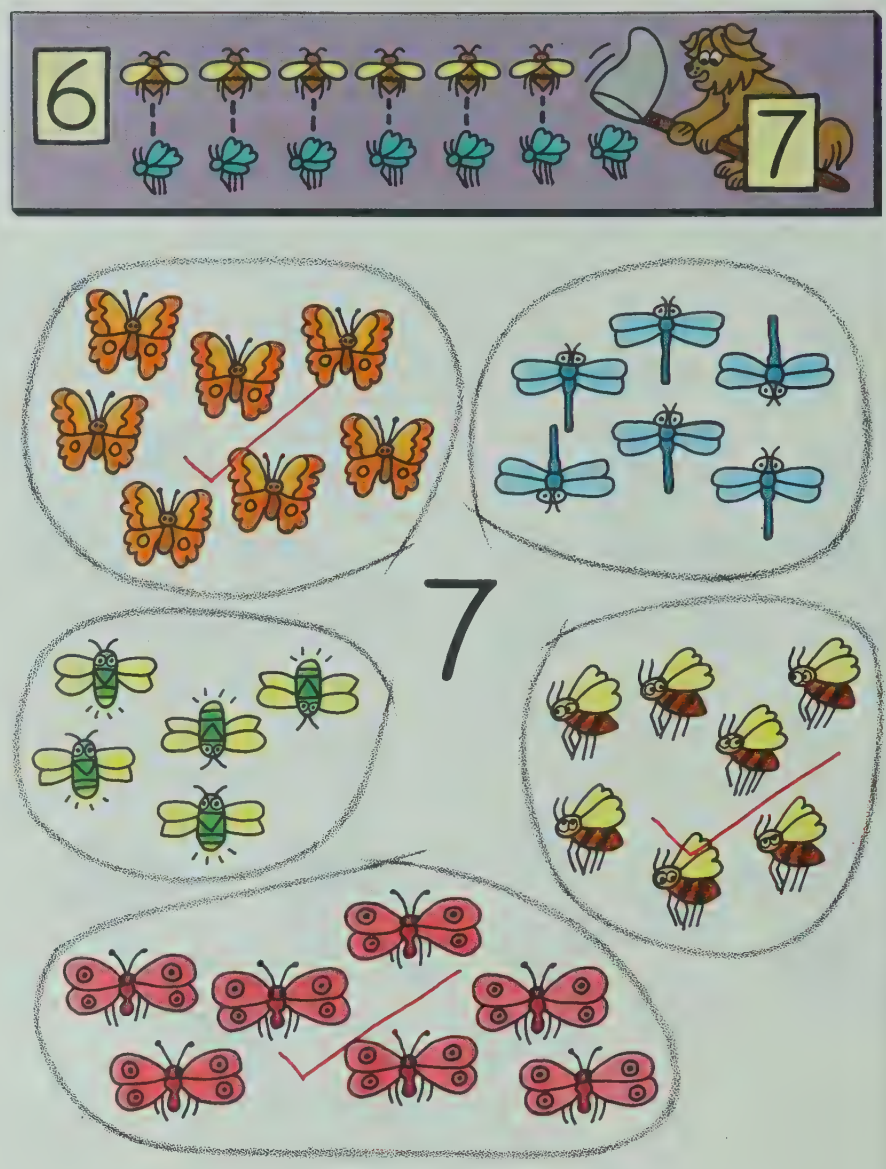
Give each child an envelope containing six sets of six geometric shapes cut from various colors of construction paper. Also, give each one a large sheet of newsprint, paste, and a crayon. Tell the children to put a large numeral 6 in the middle of the sheet of newsprint. Then tell them to arrange the sets of shapes in any combination of six they wish and to paste these sets anywhere on the sheet they labelled for six. Ask the children to continue making sets of six and pasting them on the large sheet until all their shapes are gone. Some children may wish to put all those pieces that are the same shape in each set of six, but urge them to use combinations of shapes and colors. See illustration at the right.



Child's Paper

Teaching
Page r-21

Use the pictures at the top of the page as a basis for discussing the fact that seven is one more than six. Then ask the children to mark with a check all the sets which contain exactly seven members.



Objective

Given a set having seven objects, the child will be able to identify the number of the set by counting and to associate the numeral 7 with the set.

Notice that the set demonstrations should include some counting. Work on developing the children's ability to count according to their need.

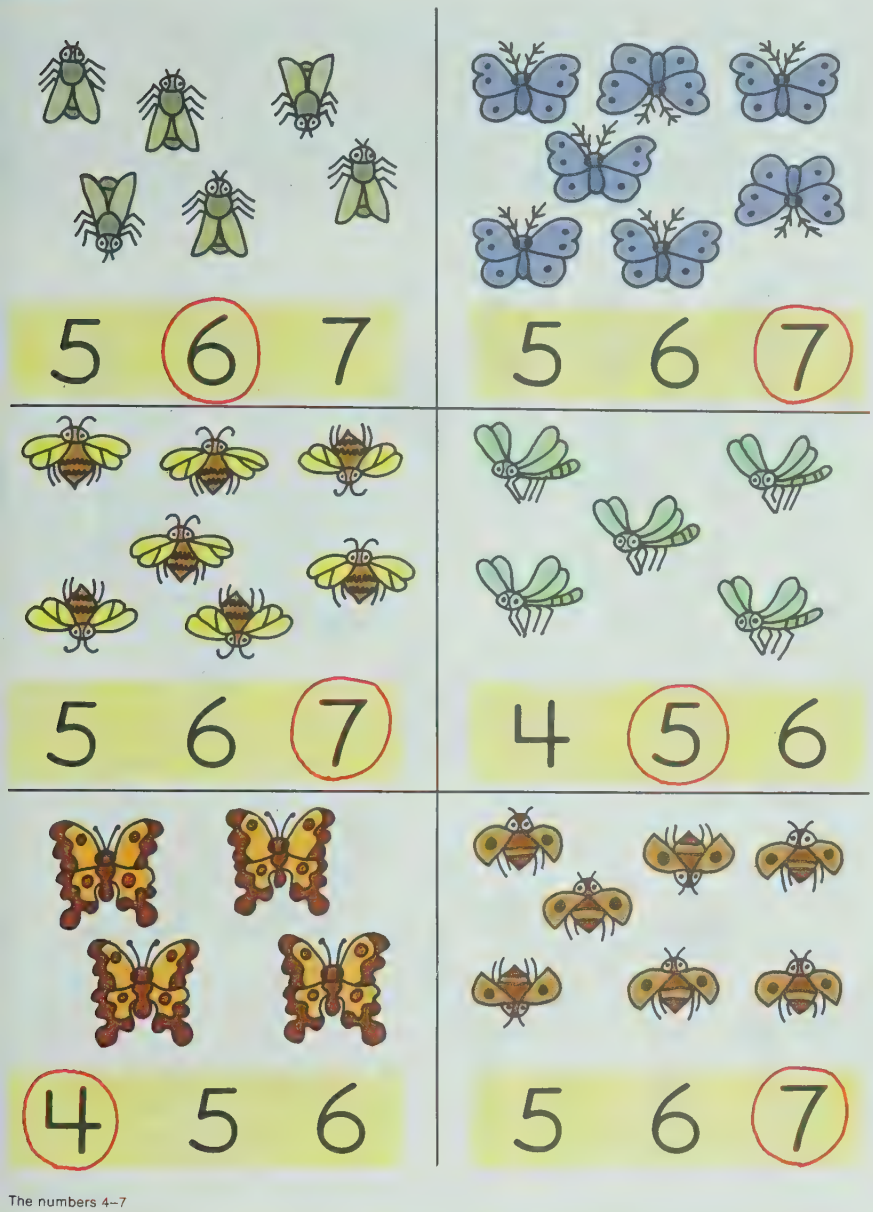
Pre-Book Activity

Materials

felt objects
flannelboard

Show a set of five felt objects on the flannelboard or overhead projector and have the children count them. Place one more object with this set and count again. Next, place another object with the set and have the children count together to seven. Most of the children will know that the number seven follows the number six. However, you should point out when the counting is completed that the number seven is *one more than* six.

Following this, give the children opportunities to identify the numbers of various sets containing seven or fewer objects. Some of the children will need to count very carefully to arrive at the correct number for a given set of seven, while others may recognize the number of the given set without the routine counting activity, as they see the combinations of four and three, five and two, or three and three and one. Each child should have an opportunity to recognize the number of a set.



The numbers 4–7

Teaching
Page r-22

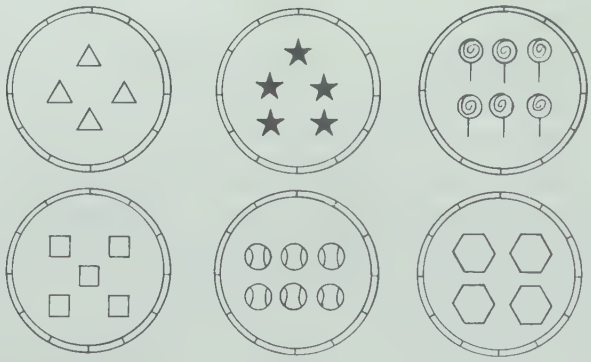
Direct the children's attention to the first frame. Ask how many insects are shown. When a child responds six, explain that they should circle the numeral 6. Encourage the children to complete the remaining frames independently.

Resources for Active Learning

Math Activities, "Circus," p. 32, Allyn and Bacon

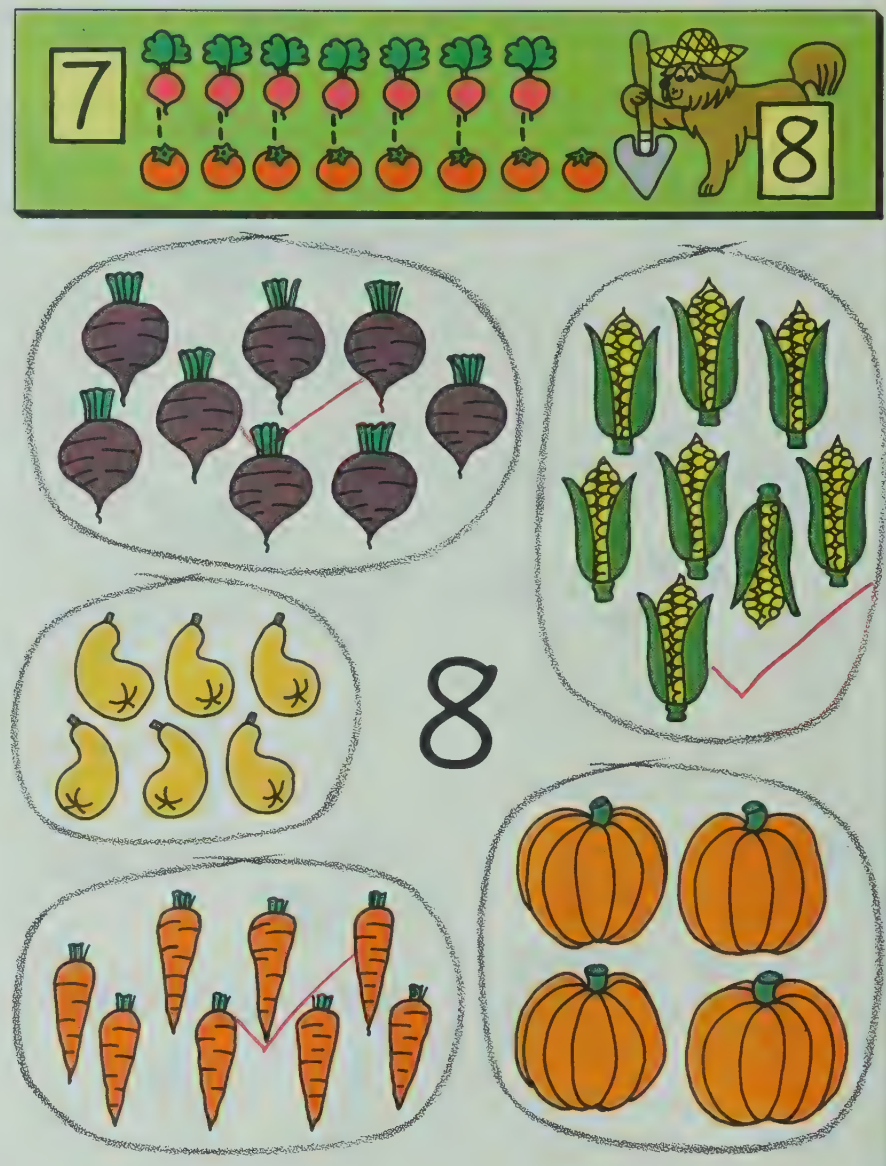
Follow-up

Give each child old catalogues or magazines, scissors, paste, and three or four paper plates or oval sections of tag board. Ask the children to make sets of four, five, six, and seven by cutting out objects or shapes from the colored portions of the magazines. Suggest that they decide on a set and paste it on a plate or tagboard, without labelling it. (See samples illustrated at the right.) Collect all the completed plates and use them for a later activity or warm-up.



Teaching
Page r-23

Use the illustration at the top of the page to further develop the idea that eight is one more than seven. Then ask the children to mark with a check all those sets which have exactly eight objects. Remind the children that they should count the sets to find out which have eight members.



The number 8

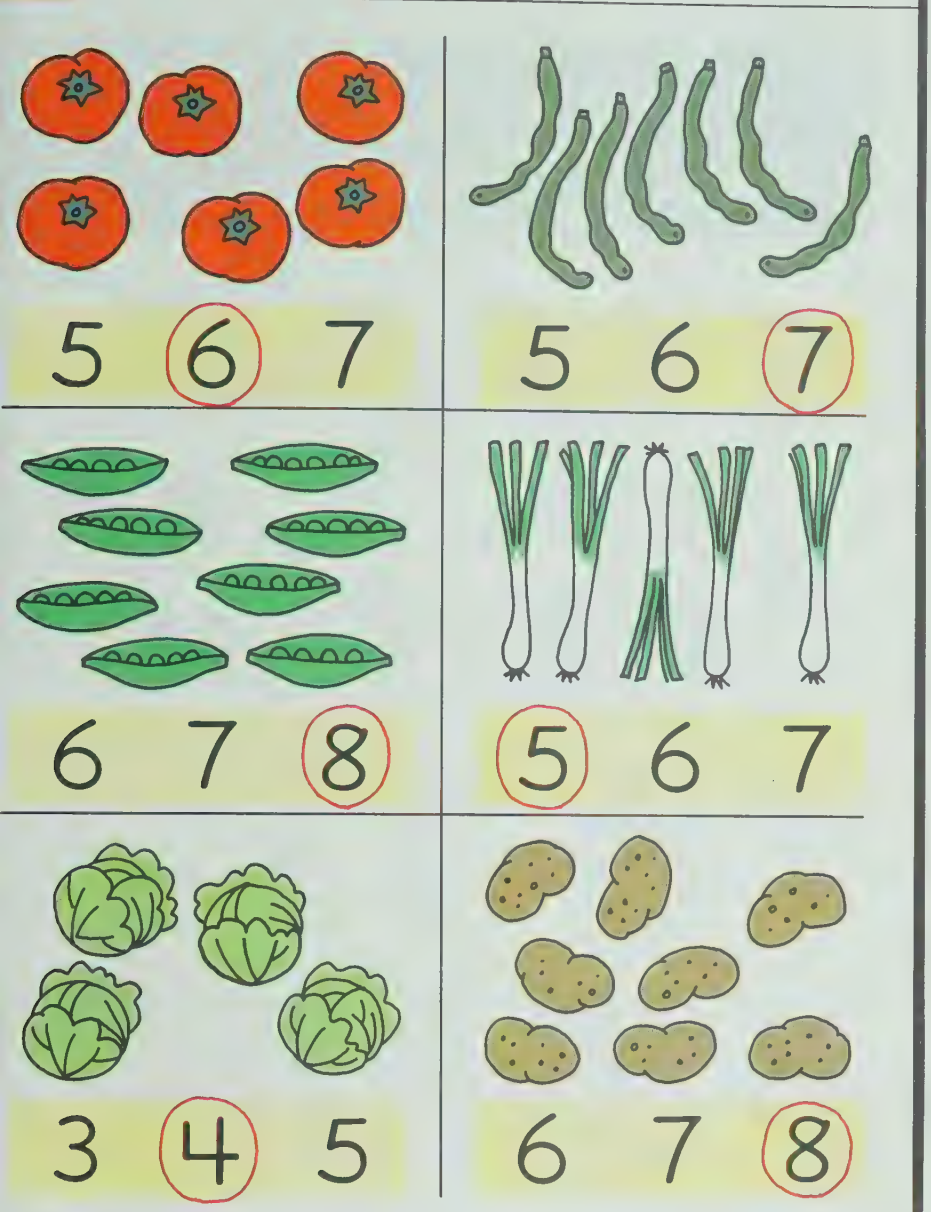
Objective

Given a set having eight objects, the child will be able to identify the number of the set by counting and to associate the numeral 8 with the set.

As children study greater numbers, set demonstrations, together with counting activities, become increasingly important. Develop as many counting activities suitable for individual use as possible.

Pre-Book Activity

If children made the paper plates suggested in the follow-up for page r-22, you might use these plates in a matching game. Prepare numeral cards for numbers 1 through 8. Set up the plates (or other set materials you prepare) along the chalk tray. Pass out the numeral cards for 1 through 7 to children and ask them to place their numeral card beside the plate which shows a set. Children should notice that one plate has not been matched. Count the matched plates and cards. As you come to the eighth and unmatched plate, count the objects shown on it and introduce the number 8 as one more than seven. Finally, display the numeral 8 and join the numeral card for 8 with the other cards for 1 to 7. Then rearrange the plates and redistribute the numeral cards to other children; they in turn should match their card to a plate. Encourage others to check that the matchings are correct.



The numbers 3–8

Teaching
Page r-24

Call the children's attention to the first frame. Count the tomatoes with them. Then ask the children to circle the numeral which shows how many tomatoes are in the set. Explain that you would like them to complete the page on their own. Ask them to circle the numeral which tells the number of the set in each frame.

Follow-up

Various counting activities should be encouraged. Any of the following would be suitable.

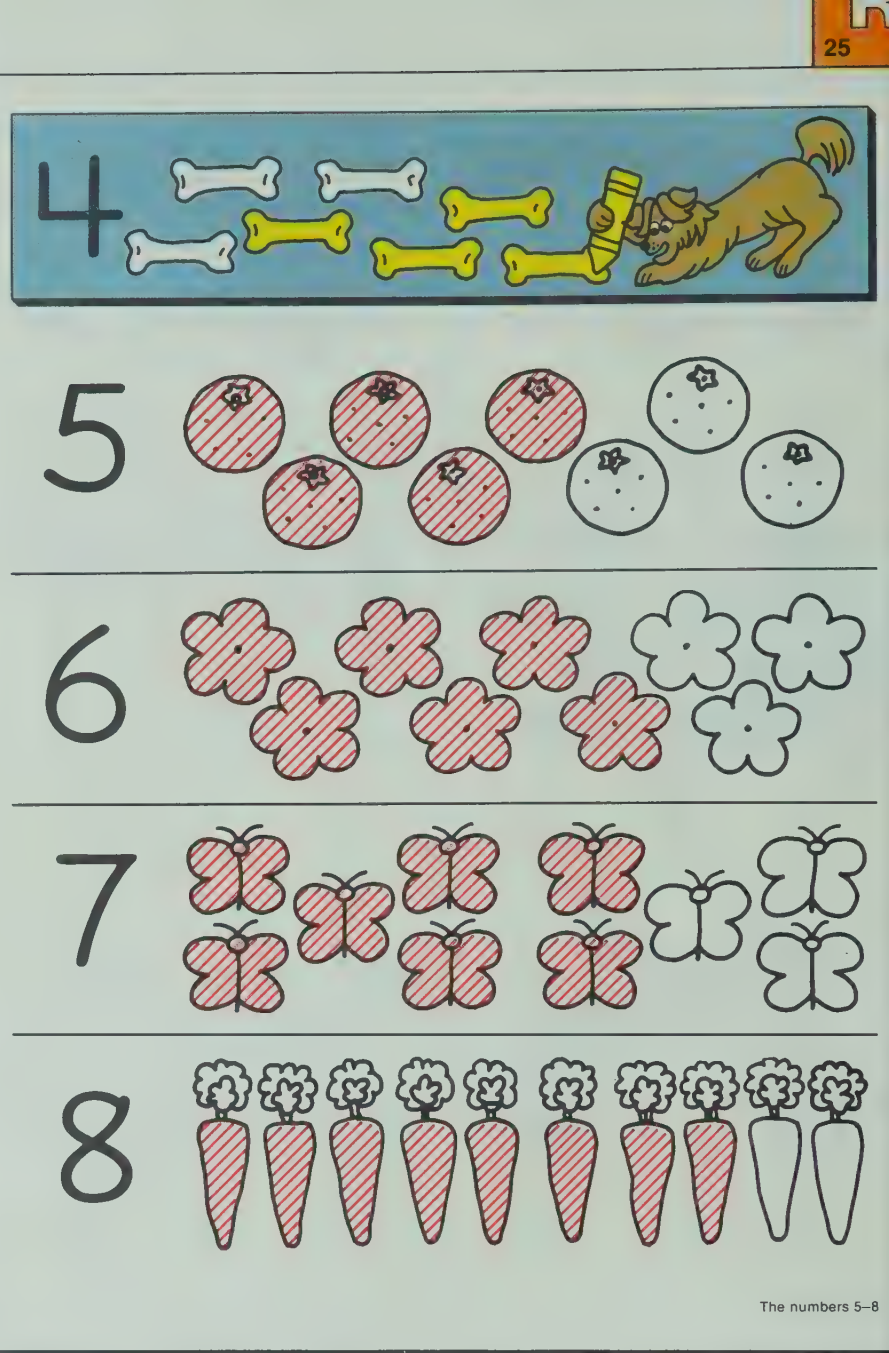
Have a set of numeral cards for 0 through 8. (It is helpful to represent the number by dots as well as by the numeral.) Ask the children to put as many paper clips on the card as the numeral suggests.

With similar cards use clothespins and coat hangers. Staple the cards to the coat hangers. Ask the children to attach as many clothespins to the hanger as is indicated by the numeral on the card.

Add cylinders or jars to the set of "listening cylinders" suggested on page r-8. Remind children to listen and guess first how many are in the cylinder and then check their guess by counting.

Teaching
Page r-25

Use the pictures at the top of the page as an introduction to this activity. Explain to the children that the numeral on the left tells them how many items they should color in each frame. Ask a child what number is indicated in the first frame, and then explain that since the numeral indicates five, they should color five of the oranges.



The numbers 5–8

Objective

Given the numeral 5, 6, 7, or 8, the child will be able to draw or match a set of that number.

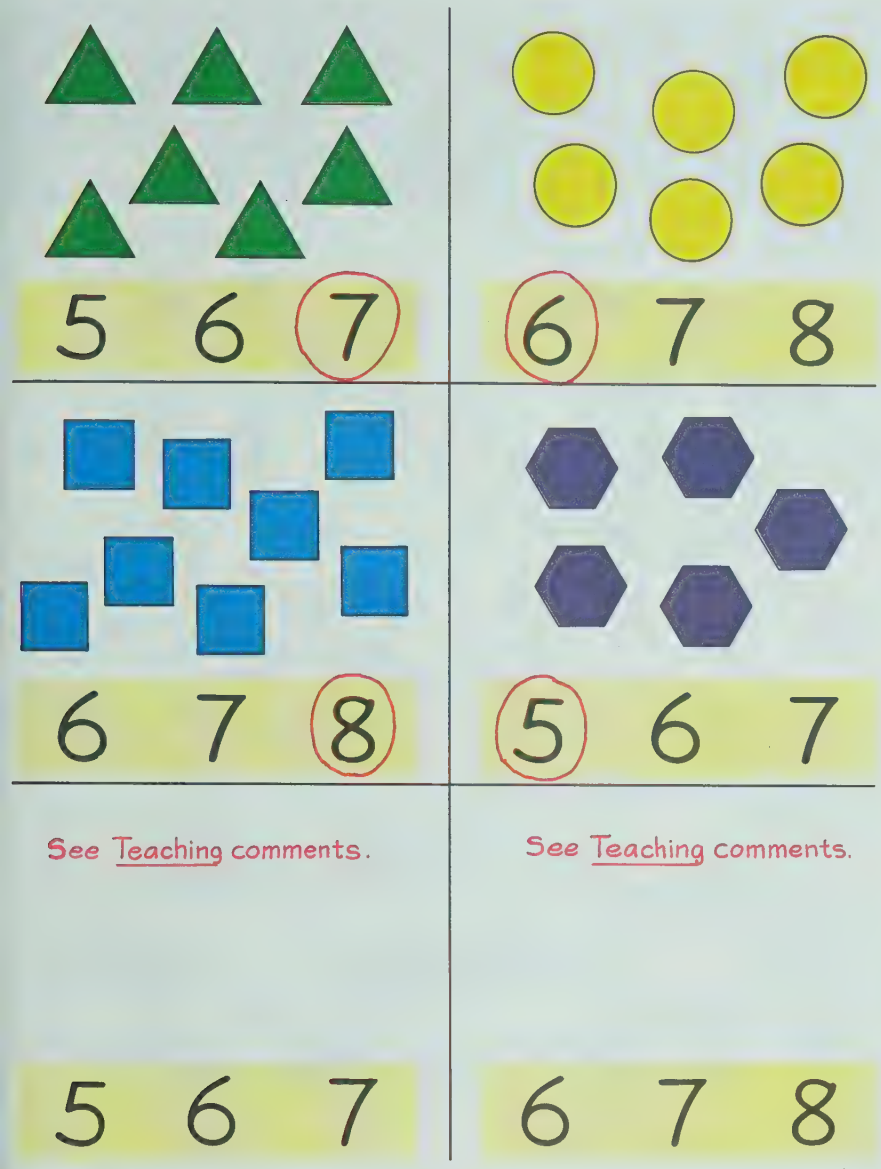
This lesson reviews the numbers studied so far in this module. Note that counting is essential with these relatively larger numbers.

Distribute the table tennis balls (or beans) among the cans to make sets ranging from 0 through 8. Shake a can and ask the children to guess how many objects are in the can. Then have a child check the guess by counting. Encourage all the children to count with him as he removes the objects, one at a time, from the can. Continue this activity with the other cans, repeating it with some of them so that many children can participate.

Pre-Book Activity

Materials

coffee cans, 9
table tennis balls, 36 (Large beans and smaller cans could be used as a substitute.)



The numbers 5–8

Teaching
Page r-26

For each of the top four frames, explain to the children that they should find how many objects are pictured and circle the matching numeral. Work through the first frame with them. Since there are seven triangles, the numeral 7 should be ringed. Also explain that for the bottom two frames they should make sets of their own and tell how many are in each set by ringing the appropriate numeral. Allow the children freedom in their drawing. It would be helpful to use children's sets as examples for further discussion by making sets of X's on the chalkboard for the number of items children have drawn.

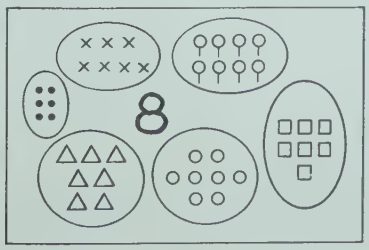
Resources for Active Learning

Developmental Math Cards, "Roll a Number," A-10; "Birthday Months," A-20, Addison-Wesley

Math Activities, "Taps," p. 30, Allyn and Bacon

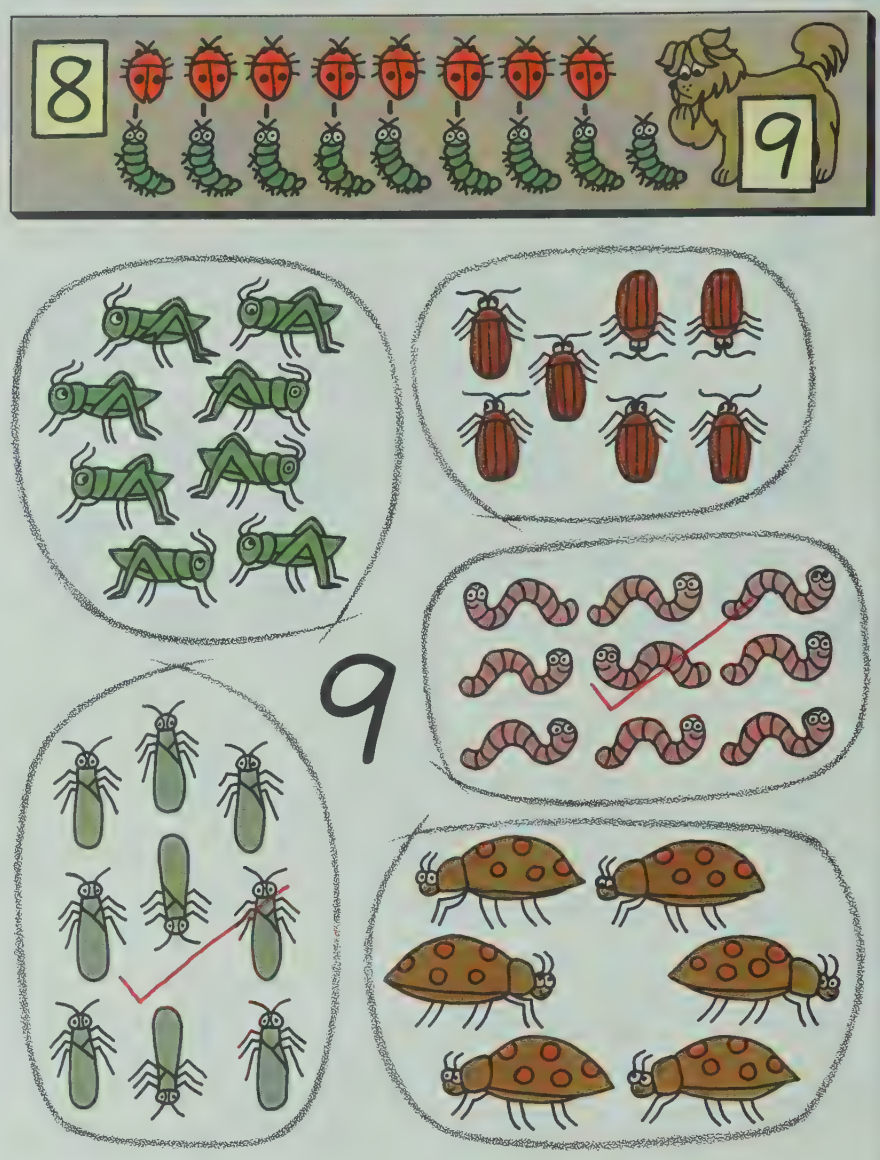
Follow-up

Duplicator masters such as the one below would give children further practice in identifying the numbers of sets. Prepare a worksheet with a large numeral in the centre surrounded by sets, only some of which match the numeral. Ask children to mark with a check all the sets which *do not* match the numeral.



Teaching
Page r-27

Relate the demonstration illustration to your pre-book introduction to 9. Then ask the children to mark with a check the sets having exactly nine objects. Point out the need for counting the sets in order to determine how many objects they contain.



The number 9

Objective

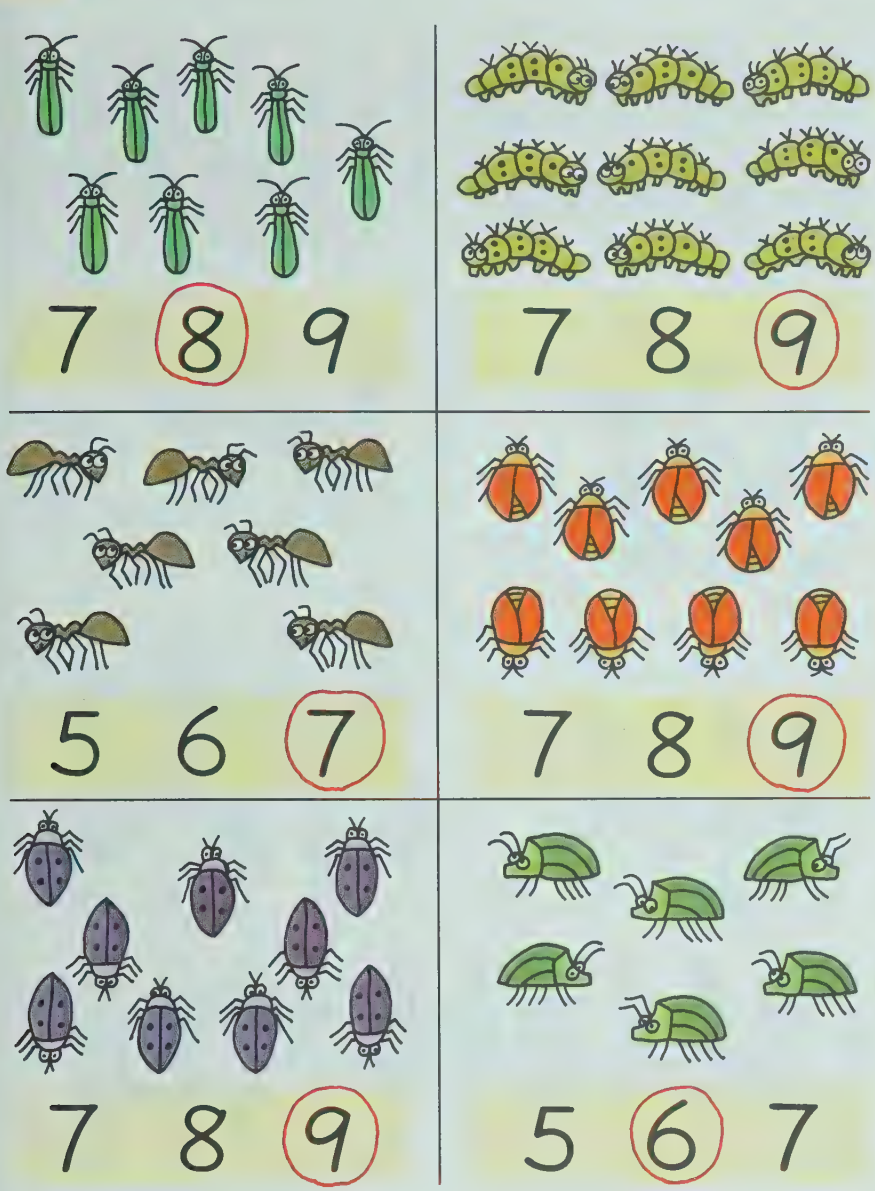
Given a set having nine objects, the child will be able to identify the number of the set by counting and then associate the numeral 9 with the set.

Pre-Book Activity

Materials

counters, at least nine per child
flannelboard and felt objects

Distribute the counters so that children have at least 9 apiece. Prominently display a set of six objects on the flannelboard or draw a set on the chalkboard. Ask children to use their counters to make a set having *one more than* the set you display. Ask them how many are in your set and how many are in their set. Then add one more object to your set. Again, ask children to make a set with one more than yours. Have them identify the number of your set and the number of their set. Repeat this procedure so that children next make a set of nine. Ask someone to give the number of this set which has one more than your set of 8. Display the numeral 9. If children are anxious to continue by building a set of 10 and have enough counters to do so, allow them such freedom but do not stress this activity. The number ten will be developed in the next lesson. It would be suitable to show other examples of sets of nine, using plates with pictured sets, displays of geometric figures, or sets of clothespins or blocks. Encourage the children to count the objects in each set as you discuss them. Continue to point out that the numeral 9 represents the number of each set.



The numbers 5–9

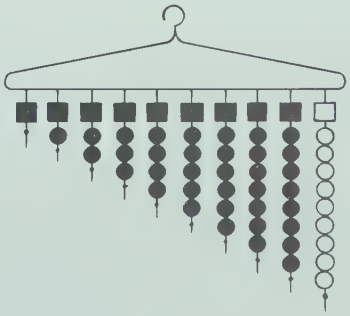
Teaching
Page r-28

Although the format of this page is not new to the children, it would be helpful to work through the first frame with them. Ask them to find how many green bugs there are. Again stress the need for counting. Then ask them to circle the numeral which shows how many are in the set. Encourage children to continue the page independently. Observe their work and ask any child who is making many errors to count the sets with you.

Follow-up

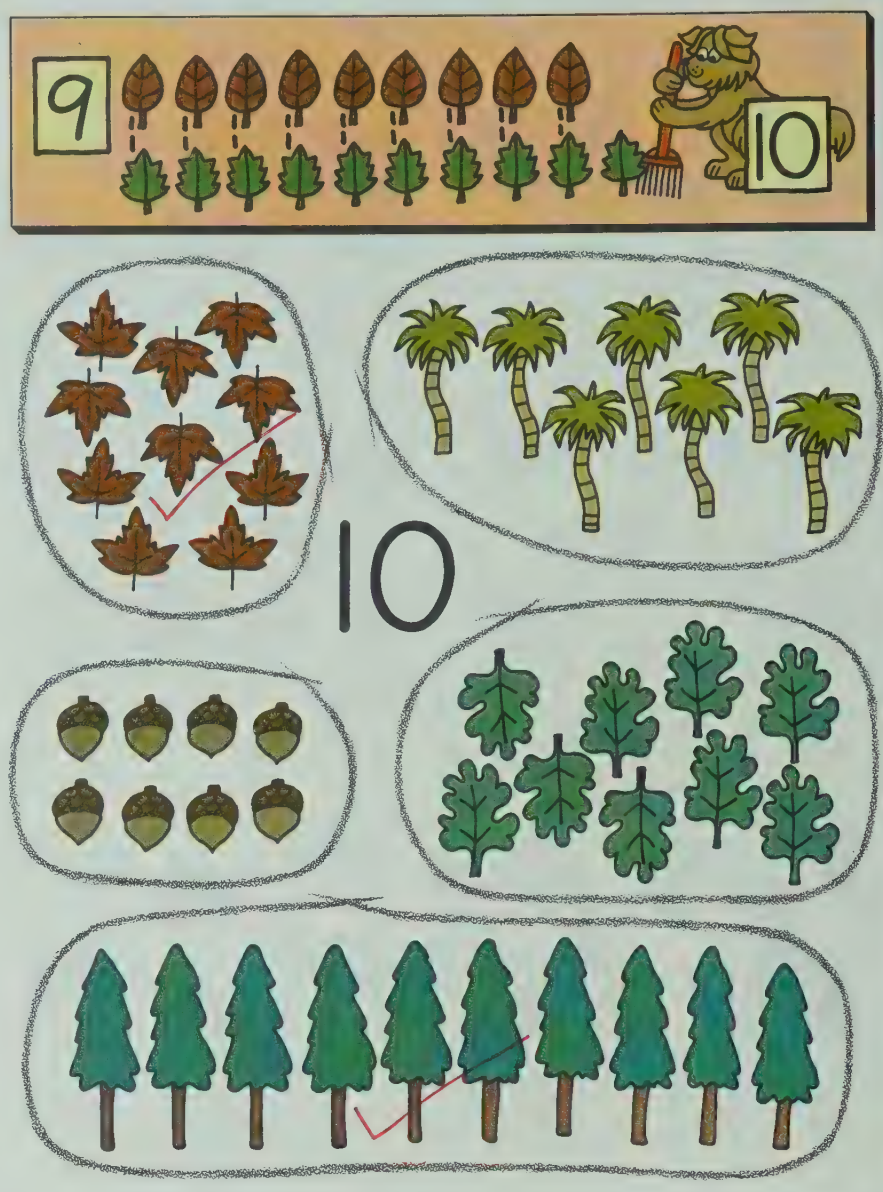
Continue to supply materials for individual counting activities.

To show the "one more" idea for the numbers one through nine, you might use a wire coat hanger, nine shoestrings, and 45 beads. It is preferable to have nine beads of one size and color, and 36 of another shape or color. Mixed colors and shapes can be effective if you choose the beads judiciously so as not the confuse the children by making the pattern more complicated. See illustration at the right.



Teaching
Page r-29

Use the demonstration illustration to stress the idea that ten is one more than nine. Then point out the numeral 10 in the centre of the page and ask the children to mark with a check those sets having exactly 10 members. Stress the need for counting.



The number 10

Objective

Given a set having ten objects, the child will be able to identify the number of the set and associate the numeral 10 with the set.

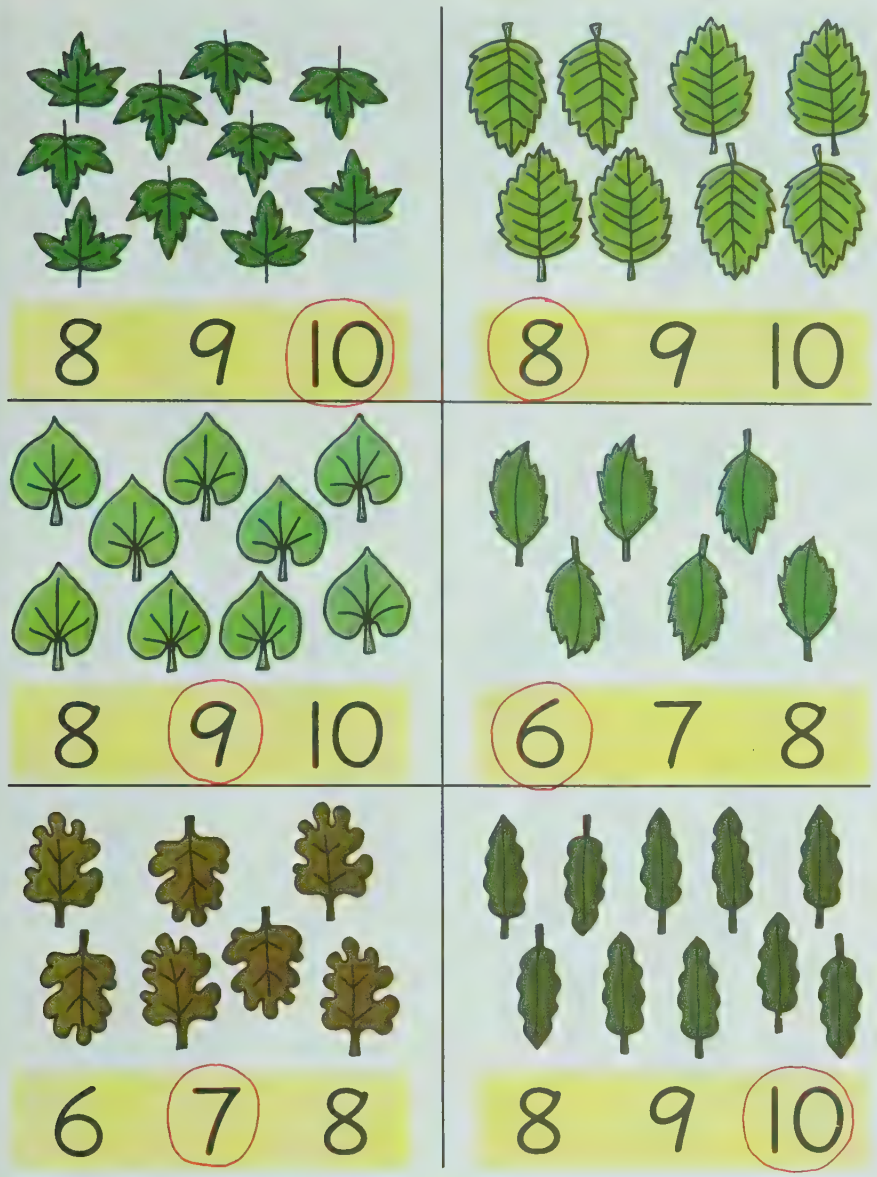
Pre-Book Activity

Materials

paper cups

Display a set of nine paper cups and have the children count the cups in this set. When they have finished, put one more cup with the set and have the children count to ten. Afterwards, stress the fact that ten is one more than nine.

Display another set of ten objects and the numeral for ten. Then give the children an opportunity to count several sets containing ten or fewer objects that have been drawn on a large chart. Place the chart so that the children can point to each object as they count. You might also use counters as suggested in the pre-book activity for the preceding lesson.



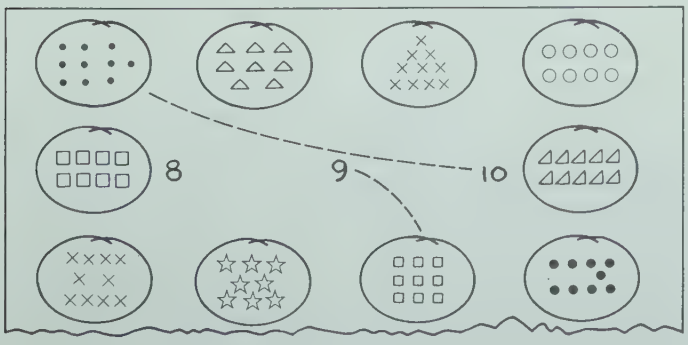
The numbers 6–10

Teaching
Page r-30

Count the leaves in the first frame with the children. Ask them to explain what they should do with the choice of numerals below the set. When they understand that they should circle the numeral which tells how many leaves are in the set, encourage them to complete the remaining frames independently.

Follow-up—Matching Numerals with Sets

Give the children a worksheet similar to the one below to provide more practice in recognizing sets of eight, nine, or ten. Tell the children to draw a line from each set to the numeral showing how many are in the set.



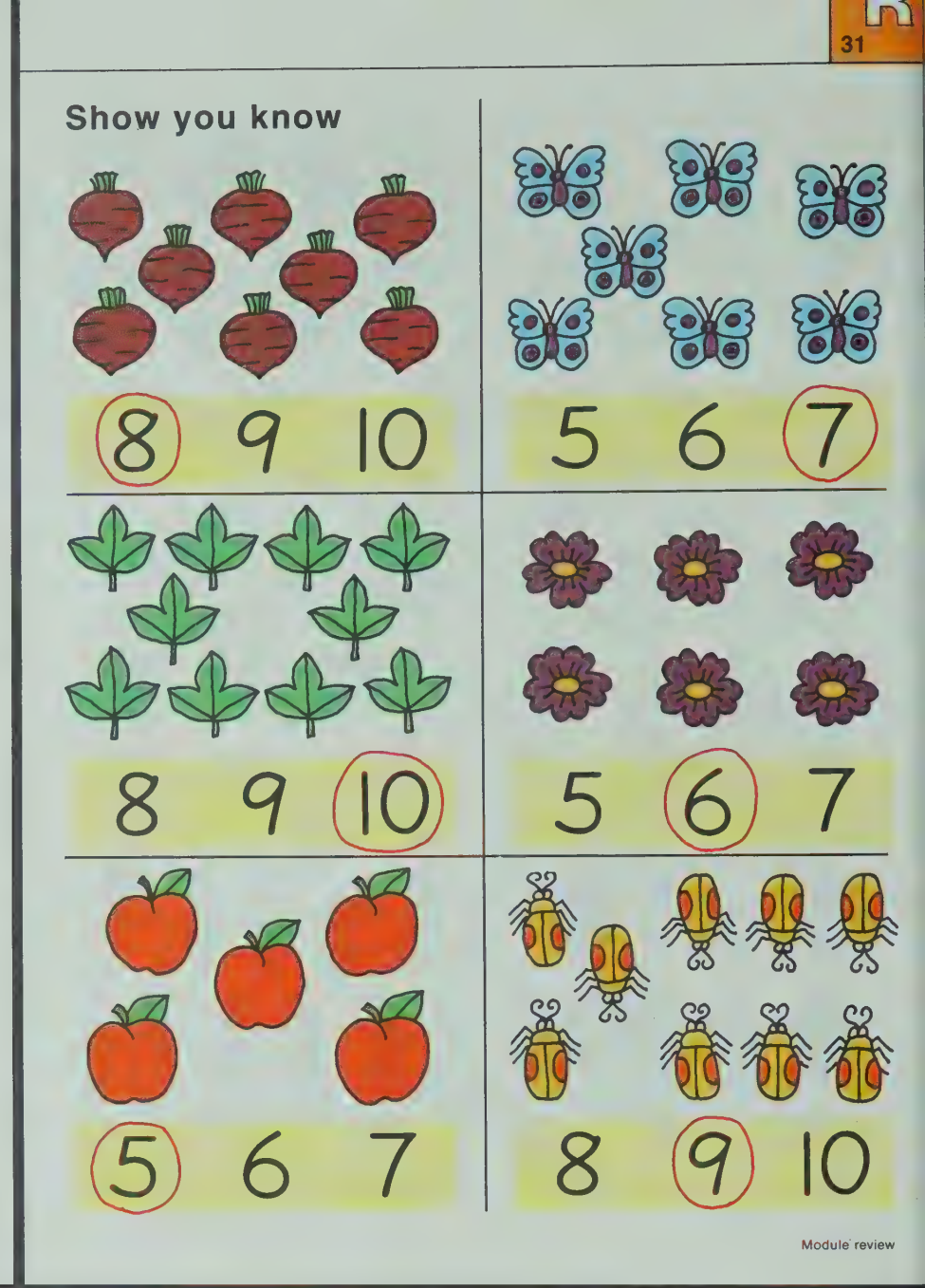
Mathematics

Introduction of the number ten presents no special mathematical problem in itself. It is the numeral for ten which presents the problem. In our numeration system, ten different symbols are used for the numbers zero through nine, and then a place-value scheme is employed in constructing numerals for all other whole numbers. Rather than create a new symbol for the number ten, we combine two of the old symbols.

Clearly, at this stage in learning, many children are not prepared to understand the meaning of place value. For this reason, treat the numeral for ten as if it were an eleventh symbol. *Do not attempt to explain the place-value meaning of this symbol.* The exact meaning of place value will be carefully developed for the children in subsequent books of this program.

Teaching
Page r-31

Since children should be familiar with the format of this page, ask a child to explain what is expected of them. Be sure they realize that they should first count the objects in the set and then ring the numeral which shows how many are in the set. Encourage them to complete the page independently.



Objective

The child will demonstrate his ability to work with the concepts presented in this module.

Page r-31 may serve as an evaluation page or simply as review. Page r-32 is intended as a change-of-pace activity which utilizes the strips that you provided earlier.

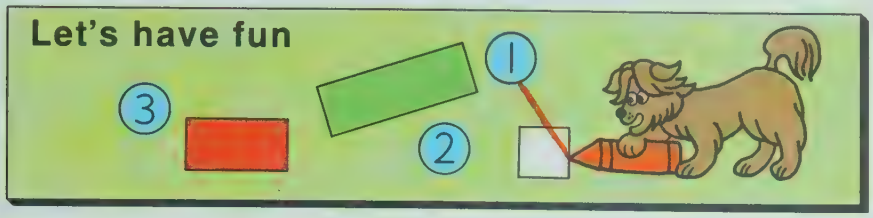
Pre-Book Activity

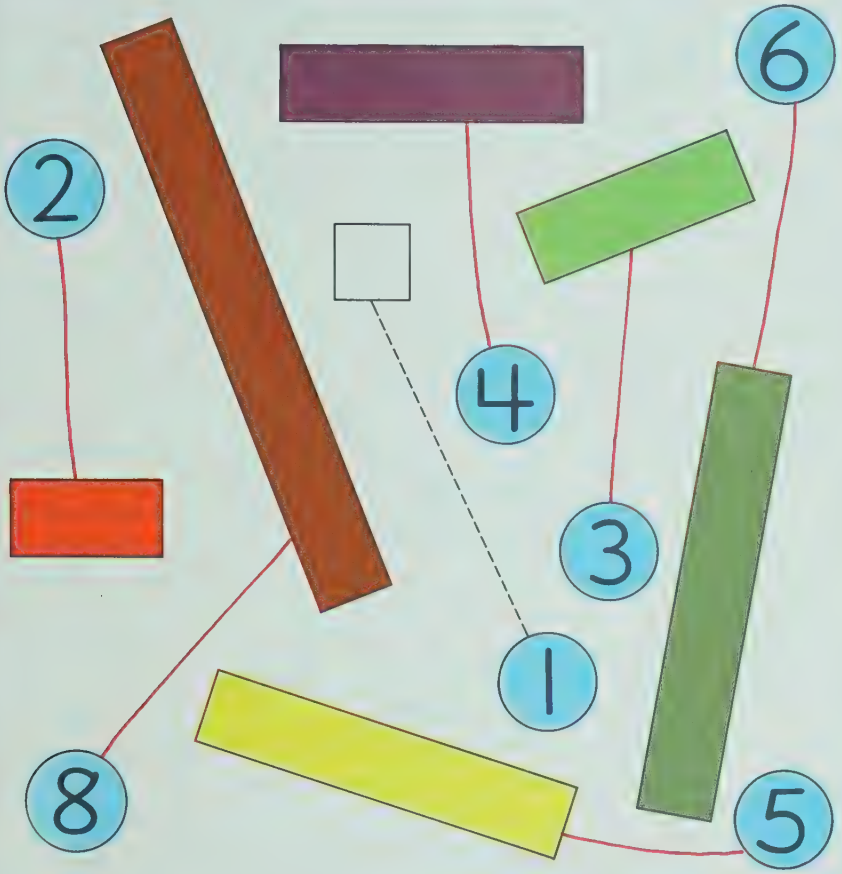
Materials

display numerals
objects for set demonstrations
strips

Provide counting activities for sets containing ten or fewer objects. For example, while you bounce a playground ball, have the children count silently. When you stop, they should give you the number of the last bounce. You may have them choose the proper numeral from cards propped on the chalk tray, clap their hands the same number of times, or use some other device to show that they listened.

Alternatively, you might prepare a quantity of perception cards having sets zero through nine on them. Prop the cards on the chalk tray and seat the children in a semicircle on the floor around the sets. Start a variation of "I Spy" by saying, "I spy a set of one less than ten." Call on a child to respond by choosing the proper set picture and saying, "Is it the set of nine?" Continue the game, changing the set pictures frequently and varying the questions as long as the children are interested.





Change of pace

Teaching
Page r-32

Children will need the colored strips for this page. When you have distributed the strips and page r-32, explain to the children that they should use their strips to help them find the numeral which matches each strip. Remind them to look at the sections marked on the backs of the strips. Help them realize that by counting these spaces they can find the number which corresponds to the strip. You might suggest to the children that they again use the white strips as counters to see how many are needed to cover each colored space. This number of strips will be their answer. Help any children who seem to have difficulty.

Resources for Active Learning

Math Activities, "Partners," p. 48, Allyn and Bacon

Math Workshop: Games and Enrichment Activities, "Collections and Labels," p. 13, Encyclopaedia Britannica Educational Corp.

Follow-up

Give children a worksheet similar to the following. Direct them to complete each row by drawing the number of objects shown by the numeral left of the row.

5		7	
2		1	
9		4	
6		10	
3		8	

Addition and Subtraction Concepts

Pages r-33 to r-48

General Objectives

To introduce the numbers through 10 as combinations
To introduce the concept of addition
To introduce the concept of subtraction

In this module children will be studying the combinations of 10 or less. The strips used in the previous two modules are used here in a few lessons as well as in the investigation lesson. Children are expected to use counting to determine how many are in a given set. Such counting is used in an informal introduction to addition. Children should gain a thorough understanding of the concepts underlying addition without being required to define the operation or master the combinations. The notation for the addition operation should not be stressed. The primary intention is merely to expose the children to the idea of addition in a precise, mathematical way.

Mathematics

Addition of cardinal numbers is defined as the union of disjoint sets.

Two sets are said to be *disjoint* with respect to each other if they contain no common elements. An example of two sets which are disjoint is the set of all teachers in your school and the set of all children in the school. Two sets which are not disjoint are the set of all letters A through P and the set of all letters G through Z, since the letters G through P are common to both sets. The *union* of two sets is defined as the set consisting of all elements in one, in the other, or in both sets. The *sum of two cardinal numbers* a and b is defined as follows. Consider two sets A and B from the cardinal numbers a and b, respectively, such that A and B are disjoint sets. The cardinal number of the union of A and B is the sum of cardinal numbers a and b.

Notice that the definition of the sum of two cardinal numbers is little more than a formal statement of the way we think about addition. That is, we want to find the sum of the numbers 3 and 4; we select a set of three objects and a set of four objects (disjoint sets); we push the two sets together (for a union) and then consider the cardinal number of the resulting set as the sum of 3 and 4.

Consider the first exercise on page r-39 for an example of the way in which these ideas are used to present addition to children. Pupils are asked to identify the number of bugs on one leaf, the number of bugs on another leaf, and then the number of bugs altogether. In this way, the idea of the union of two sets is presented without having the children actually move physical objects.

Teaching Red Module, Unit R

Materials

counters, at least 10 per child
demonstration numeral cards that show a numeral and a matching set
felt objects of different colors
flannelboard
newsprint
numerals for the flannelboard
objects to be used in sets
rhythm instruments
strips

Vocabulary

combination	number
count	numeral
less	set
more	

The suggested materials include the 10 strips used in the previous two modules. Emphasis is placed on the spaces the children marked on the back of the strips; it is not intended that the child be able to associate a number with a strip by looking only at its color. It is expected that he will be able to associate the strip with a particular number because he can count the spaces he has marked on the back. When the white strip is considered the unit or one strip, the following associations may be made:

red—two
light green—three
purple—four
yellow—five
dark green—six
black—seven
brown—eight
blue—nine
orange—ten

You will notice that the technical words such as *addition, subtraction, plus, minus,* and *equals* are not included in the vocabulary list. Since the intent is simply

to develop understanding of basic concepts of addition and subtraction, it is not necessary, at this level, to introduce the notation or the formal vocabulary associated with these concepts. It is important to bring the ideas out clearly through the use of familiar language, without involved notation. You can avoid use of technical language by saying, for example, "Three *and* four *are* seven," rather than "Three plus four equals seven."

Lesson Schedule

Work through these pages at a pace consistent with the children's understanding. Give all the children many opportunities to handle and count sets, to group and regroup a specific number of items, and to work out the resulting combinations.

Evaluation of Progress

This module focusses on the union of sets and the associated concept of addition; the basic ideas underlying subtraction are also brought out, however, by calling attention first to the total number of objects in a set, then to the number of objects in certain subsets of the given set. Subtraction is brought into sharper focus at the end of the unit. For example, the subtraction concept is involved in such problems as those on page r-46, where the children are asked to color objects to make two sets. If you choose, this concept can be emphasized earlier in the unit. For example, with pages such as r-39, instead of first focussing attention on each subset shown and then the total number, you could begin by having children draw a ring around the total number and then the number of the two subsets. Since the purpose is simply to expose the children to the basic ideas, you should not yet introduce the formal terminology of subtraction.

Although this module introduces concepts of addition and of subtraction (pages r-45 and r-46), the child should by no means be expected to master the combinations. As in the preceding modules, the evaluation should be based on the child's ability to recognize the number and the corresponding numeral of a set containing ten or fewer objects.

Resources for Active Learning

General Activities

Nuffield Project: *Pictorial Representation* 1, Wiley
Primary Balancing unit, Webster, McGraw-Hill

(See the introductory notes for the yellow module, Unit R, page 71, for suggested manipulative devices and commercial games.)

Activities for Continued Use

As mentioned in the introductory notes for the previous module, graphing can begin as early as the Primer level. The following activity expands graphing to include questions that have multiple alternatives.

Materials: large chart paper; black crayon or pencil; glue or paste; small squares of colored paper (one per child)

Procedure: Choose a question that has multiple alternatives, such as "What is the month of your birth?" Prepare your chart to show all 12 months.

What is your birth month?							
Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.

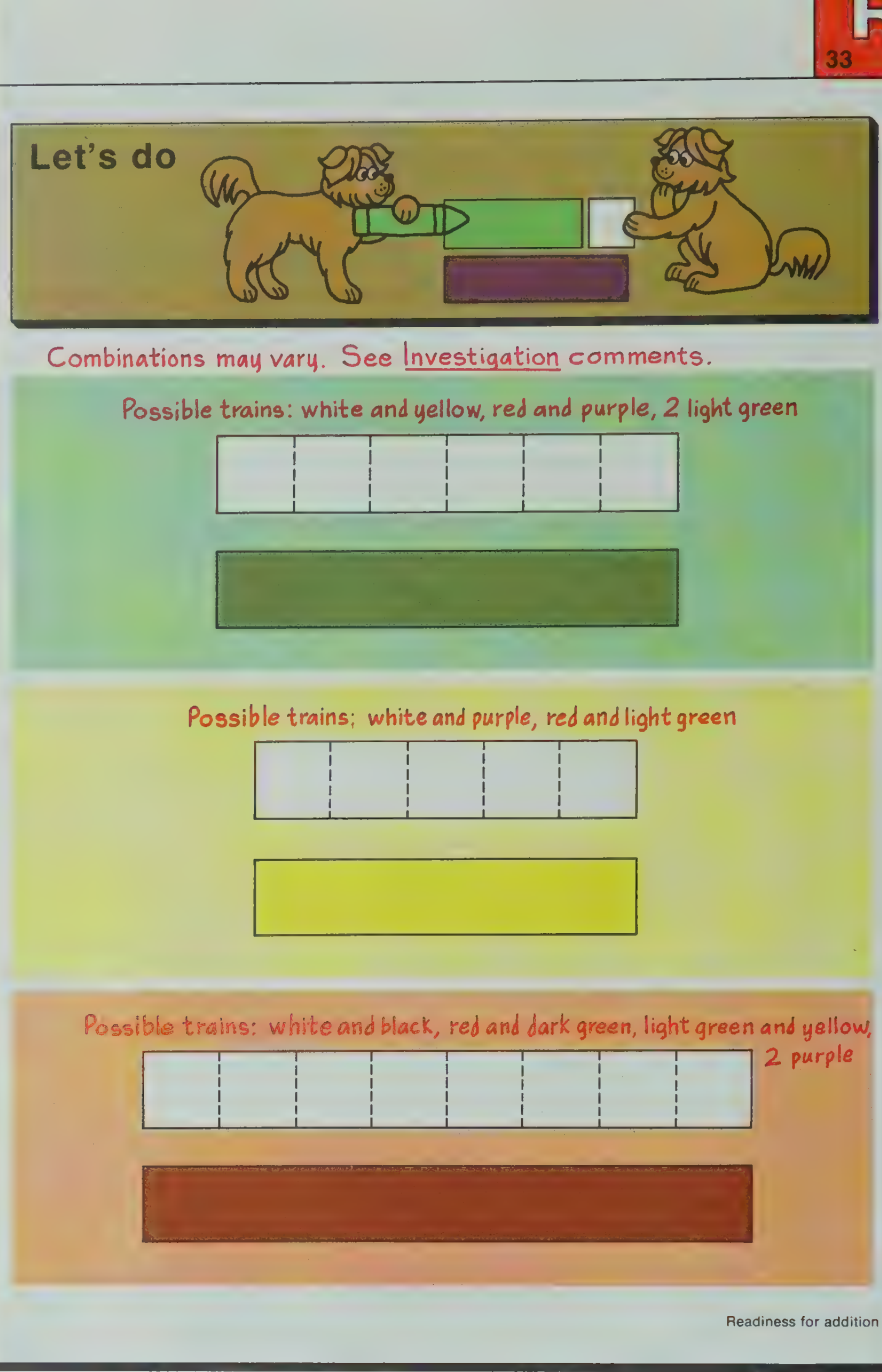
Ask the children to glue their square on the column of the month of their birth. Point out specific columns as needed for individual children. When they have finished, use the chart as a basis of discussion. Ask questions like these: "What month has the most birthdays? . . . the least?" "Does June have more birthdays than October?" "Does November have fewer birthdays than January?" "Is there any month that has no birthday?" "Do January and February together have more (or fewer) birthdays than September?"

Other suggestions for topics for graphs with multiple alternatives are favorite color, favorite ice cream, ways of coming to school, color of children's hair, color of children's eyes.

Investigation
Page r-33

Use the demonstration illustration to introduce the investigation activity. Explain to the children that they are to find two strips and make a train to match the length of the pictured strip. When they have chosen two strips, they should color the sections to show which two strips they used. If you prefer, you can ask the children simply to show you their choice rather than color the picture. Note that various answers are possible. For example, above the yellow strip they might use a train of the purple strip and a white strip, or a light green strip and a red strip.

Since this activity is provided simply to develop *readiness* for a study of addition concepts, the addition combinations as such need not be discussed. Work with specific combinations will be treated in subsequent lessons.



Purpose

To introduce the combinations for numbers ten or less

This investigation and discussion introduces the concept of thinking of a number in subsets, such as thinking of five in terms of three and two, or four and one.

Preparation

Materials

strips

Encourage the children to use the strips to make designs or pictures. As they do, point out the section marks on the back sides. Then suggest that they try to find two strips which together match a third strip. For example, suggest that they place two red strips end to end to form a "train." Then ask them to find a strip which matches the train of two red strips (the purple strip).

Suggest that they find other pairs of strips which they can make into trains that match a single strip.

Let's talk
See Discussion Comments.

Readiness for addition

Discussion
Page r-34

The illustrations on this discussion page stress some of the combinations to be studied in this module. Use them as a basis of discussion to develop the addition concept. For example, elicit from the children the observation that 3 birds are in the tree and 1 bird is flying, so there are 4 birds altogether. Similarly, there are 2 monkeys swinging from vines and 3 monkeys sitting in the tree, so there are 5 monkeys altogether. Use the words joining, combining, total, and altogether, as appropriate.

Resources for Active Learning

Math Activities, "Rod Numbers," p. 34, Allyn and Bacon

Follow-up

As an extension of the investigation, challenge the children to choose one of the dark green, black, brown, blue, or orange strips and to find as many trains as possible to match their chosen strip. If you want children to record their trains, you might duplicate a sheet showing a grid of parallel lines 1½ cm apart which can be used as guidelines. Children should color the squares on this sheet as they did the sections on the investigation page.

Teaching
Page r-35

Call attention to the demonstration art at the top of the page. Elicit from the children the observation that the puppy is circling the numeral 2 because there are two fish in the bowl. Explain that for each frame they should determine how many items are pictured and circle the corresponding numeral. When they have finished, use the page to review the numbers 1 through 5.

Review of the numbers 0–5

Objective

Given sets of 5 or less, each shown in two subsets, the child will be able to identify the number of the set.

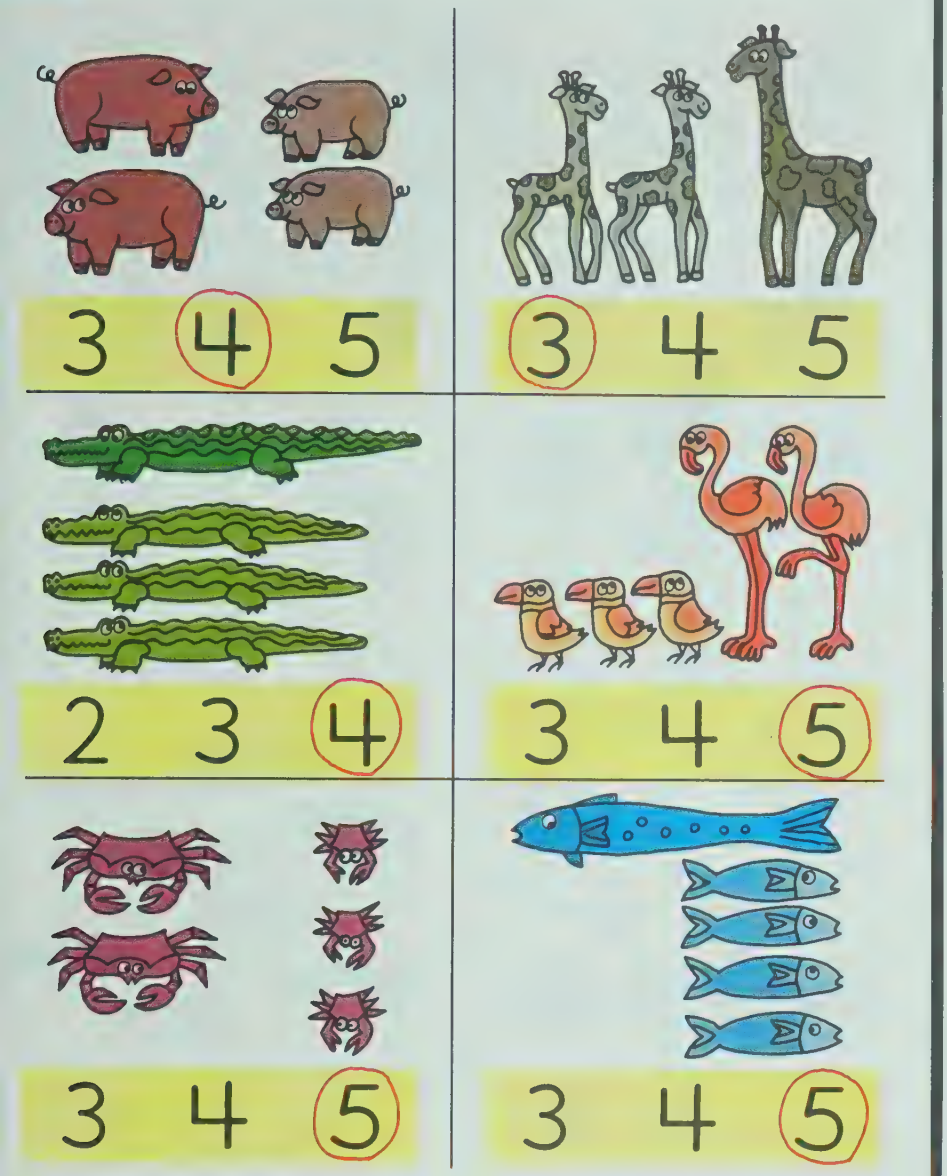
Pre-Book Activity

Materials

rhythm instruments

To review the numbers 1 to 10, you might guide children through the following activity.

Give each of the children a rhythm instrument. Choose a leader to play the tom-tom. The leader should decide on a number from 1 to 10 and pound out that many beats on the tom-tom. The children are to respond by mimicking the same number of beats according to the rhythm set by the leader, who then should direct with his drumstick. Let the leader choose a successor after each number. (Substitute an oatmeal box for the drum and let the rest of the children lightly smack their thighs in rhythm, if instruments are not available for all of the children.)



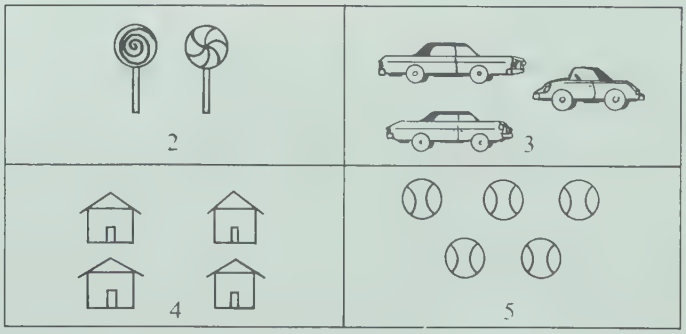
Readiness for sums less than 6

Teaching
Page r-36

Here sets of 5 or fewer are presented in combination form. First ask the children to identify how many are in each frame and to circle the correct numeral. Then use each frame to discuss a number in combination form. For example, in the first frame, there are 4 pigs—2 big pigs and 2 little pigs—so 4 is shown as a combination of 2 and 2. Similarly, the 3 giraffes shown may be thought of as a set of 2 and 1. The alligators may be thought of as a set of 1 and 3, and the birds are a set of 5 as a combination of 3 and 2. Finally, the crabs show 5 as a combination of 2 and 3, and the fish show 5 as 4 and 1. Observe with the children that each of these last 3 frames shows 5 objects, but in each case they may be thought of as a different combination.

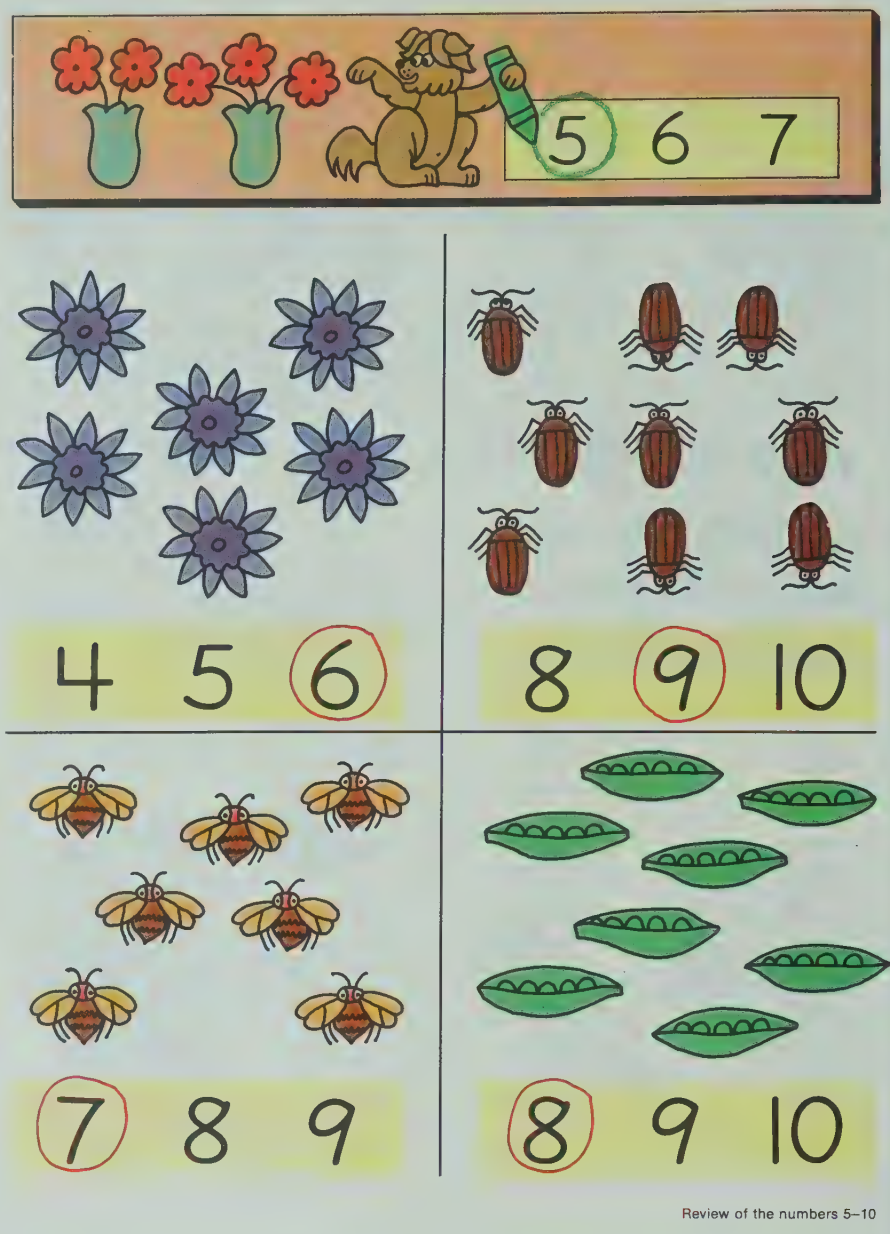
Follow-up

Give the children sheets of newsprint and guide them in folding them in half twice so that they have four sections. Ask them to choose two different colored crayons. At the bottom of each section of paper, they should write the numerals 2, 3, 4, or 5. Then ask them to draw a set according to these numerals and color some objects in a set one color and the others another color. When they have finished, encourage them to describe how they colored each set.



Teaching
Page r-37

Call attention to the demonstration picture, pointing out that 5 is being circled because there is a total of 5 flowers. Explain to the children that they should again identify the number of objects in the set and circle the corresponding numeral. Remind the children that they should count the items in each frame in order to find how many there are.



Review of the numbers 5–10

Objective

Given sets of 10 or less, each shown in two subsets, the child will be able to identify the number of the set.

This lesson reviews numbers 0 through 10 and shows the sets as combinations.

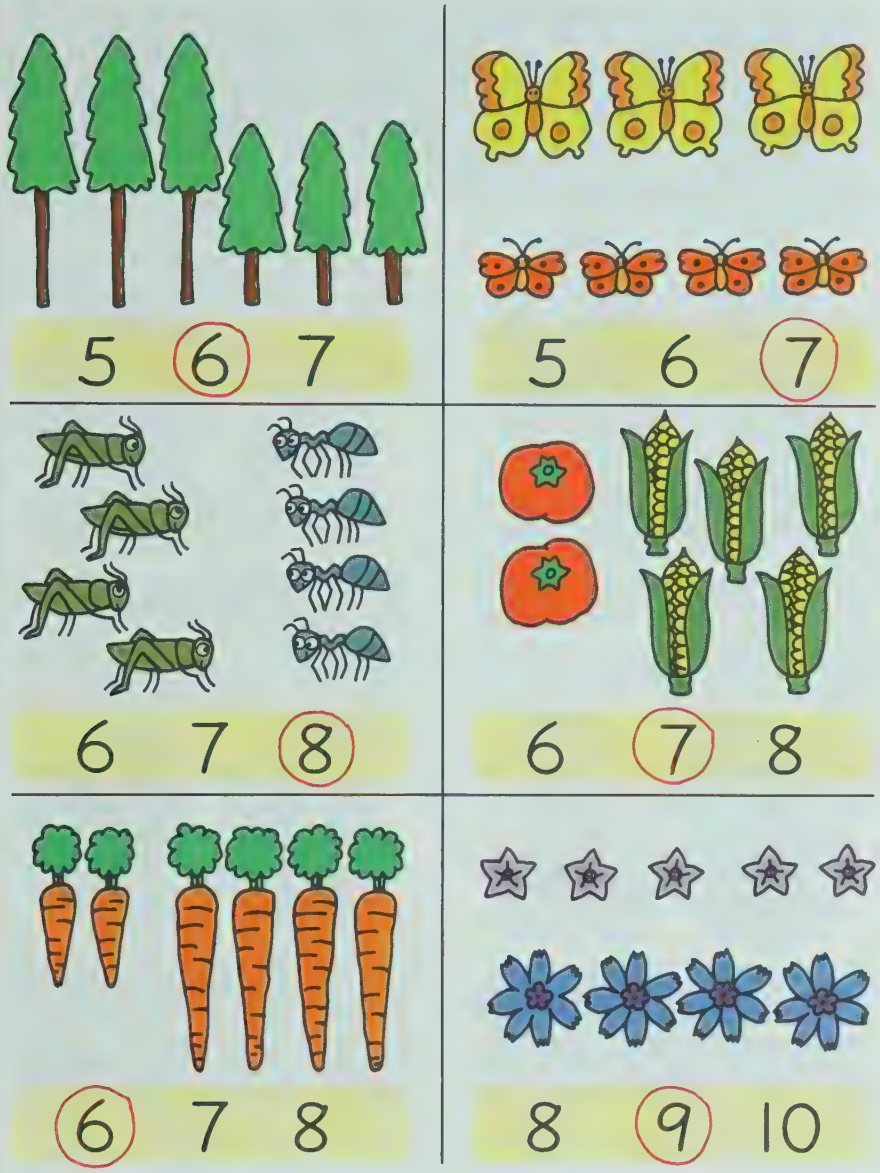
Pre-Book Activity

Materials

strips

A counting activity would be suitable to review the numbers 0 through 10. Prepare a set of numeral cards for 0 through 10 which show both the numeral and a set for that number. Distribute these cards at random among the children. Then ask all the children who do not have cards to put their heads down and those who have cards to position themselves in order from 0 through 10 in front of the room. When they have done so, call "Heads up" and have the children count from 0 to 10. Then again ask children to put their heads down. Each of those with cards should choose a classmate to take his card. Then this new group of children should position themselves in numerical order. Again, call "Heads up" and count with the children from 0 through 10. Continue this activity until all have had a turn with a numeral card.

You might also use the strips to review the arrangement from 1 through 10. Encourage children to build a stair-step arrangement from 1 through 10 and to associate the counting numbers to each of the strips respectively. Again point out the marked side and have them count the sections on the back of various strips.



Readiness for sums 5–10

Teaching
Page r-38

Call attention to the first frame. Ask the children how many trees are shown. When they respond "six," ask them to circle the numeral 6. Then ask them how many tall trees and how many short trees. Help them see that 6 may be thought of as 3 and 3. When you are sure children understand the directions ask them to tell how many objects are in each frame by circling the correct numeral. Then work through each frame with them and talk about the combinations, such as 7 shown as 3 and 4 in the second frame.

Follow-up

Prepare 55 beans by painting one side of each bean (spray paint is appropriate). Place the beans in ten transparent covered containers so that the beans can be shaken and viewed. Place one in the first container, two in the second, and so on until you have a set for 1 through 10. A child or group of children should choose one container to study. Instruct them to shake a container such as the 6 container, and observe the combination formed by the different colors of the top side of the beans. Encourage them to count how many beans come up painted and how many come up unpainted for each shake. You might also supply numeral cards so that children can show their observations by choosing the two appropriate cards.

Teaching
Page r-39

Children will need guidance in working with the format of this page. Use the demonstration illustration as an introduction. Point out that in the first row the numeral 1 has been circled because there is one bug on the first leaf. Then ask them to tell you why the numeral 2 has been circled in the second row (to match the 2 bugs on the second leaf). Finally, discuss why the numeral 3 has been ringed.

Then call attention to the first frame. Work through it with the children. Ask them to circle the numeral in the first row which tells how many bugs are on the first leaf. Then ask them to circle the numeral in the second row which tells how many bugs are on the second leaf. Finally ask them to circle the numeral in the last row to show how many bugs there are altogether in this frame. Continue with this type of direction for the rest of the exercises. Encourage the children to point out the combinations that they have found: 5 is 3 and 2; 4 is 1 and 3; 4 is 2 and 2; 6 is 2 and 4.

Concept of addition

Objective

Given two subsets for a number 10 or less, the child will be able to identify the combination for the number.

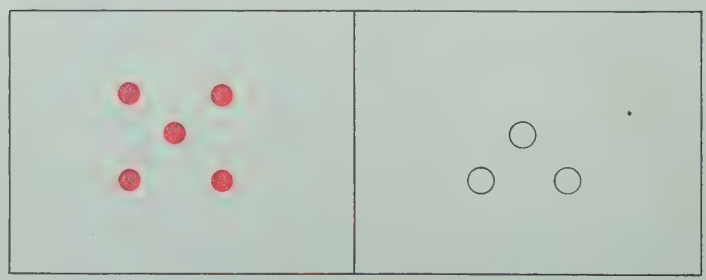
This lesson should serve to strengthen skills in identifying numerals as well as develop the idea of combinations.

Pre-Book Activity

Materials

newsprint
counters, 10 per child

Present a demonstration activity using large colored beads or blocks or the flannelboard. Give each child a sheet of newsprint to be folded in half. Show a set of 10 or less, using two different colored subsets, such as 5 red beads and 3 blue beads. Ask the children to use their counters to show this combination on the sections of their newsprint.



Ask questions such as "How many do I have altogether?" "What combination do my different colored blocks show?" "Can anyone name another combination for this number?" Work through several numbers and several combinations.

Concept of addition

Teaching
Page r-40

Children will again need guidance to work through this page. Call their attention to the first frame. Ask them to circle the numeral in the first row which tells how many dragonflies are on the first leaf. Then ask them to circle the numeral in the second row which tells how many dragonflies are on the second leaf. Finally ask them to circle the numeral in the third row which tells how many dragonflies there are altogether. Stress that 6 may be thought of as 3 and 3. Work through the remaining exercises similarly.

Resources for Active Learning

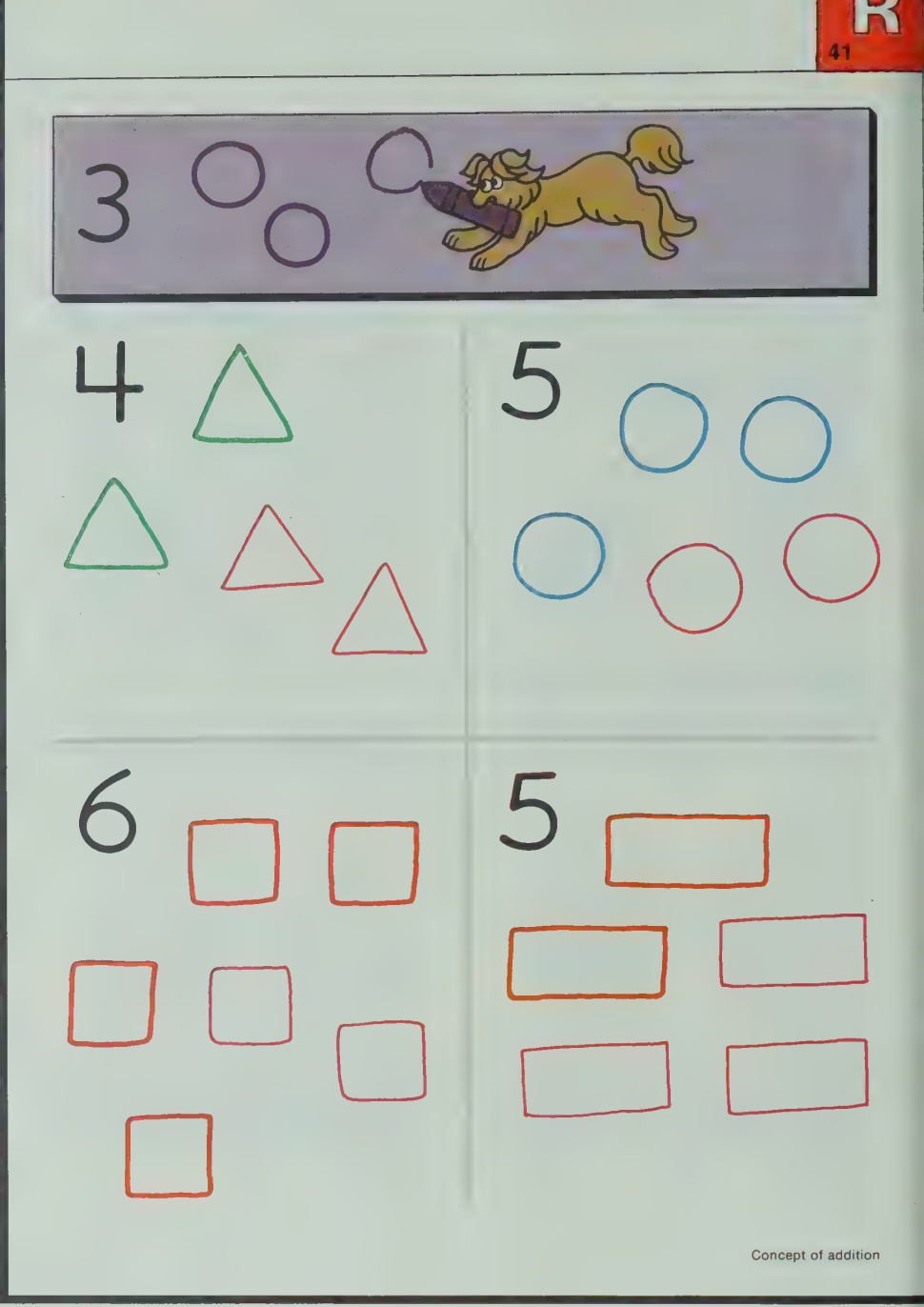
Mathex: Numeration No. 2, Activity 7, p. 8, Encyclopaedia Britannica Educational Corp.

Follow-up

Give each child a brown crayon, two colors of tissue paper, and a large piece of construction paper. Direct them to fold their paper in halves (or fourths if size permits). Guide them in drawing a tree trunk in each section. Then explain that you want them to choose one of the numbers 7, 8, 9, or 10. Then they should think of a combination for this number and use that combination to cut and paste that many tissue paper leaves on the tree. They should show their combination by their choice of colors. For example, a child might show 9 by using 6 leaves of dark green and 3 leaves of light green.

Teaching
Page r-41

Use the demonstration illustration at the top of the page to introduce the activity. Explain that the puppy is drawing a circle so that there will be enough circles to match the numeral 3. Then call attention to the first frame. Explain to the children that they should draw as many more objects as are needed to make a set that matches the numeral 4. Note that, although many will complete the set by drawing 2 triangles, they might correctly complete the set by drawing 2 other figures. With any who draw different figures point out the combination that their drawing illustrates, such as that of 4 as 2 and 2. As children work, stress the importance of counting to see how many they have drawn and how many altogether.



Objective

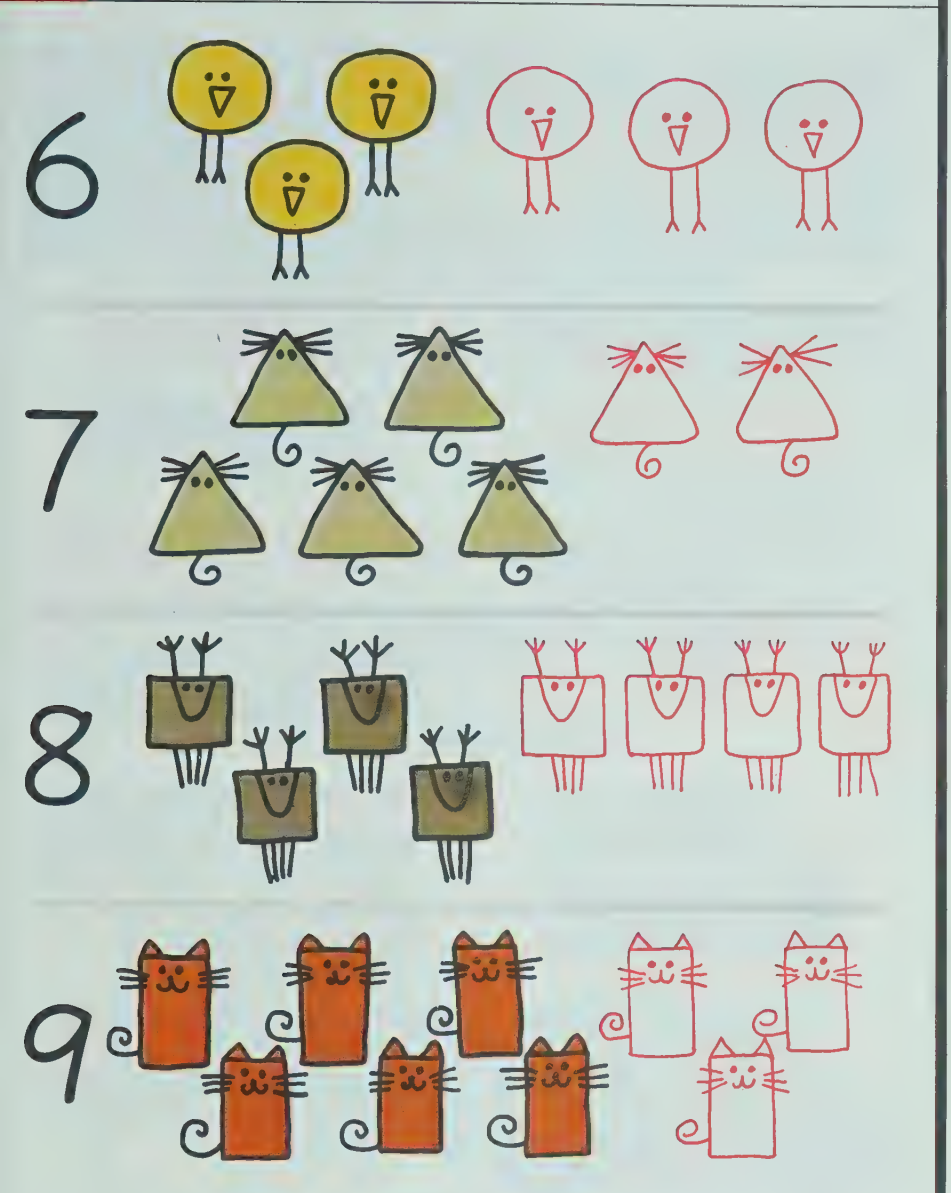
Given a numeral and an incomplete set, the child will be able to complete the set to match the numeral.

Pre-Book Activity

Materials

flannelboard
felt objects and numerals

Show on the flannelboard the numeral 7 and a set of 4 objects. Explain to the children that you want to complete this set so that it will have 7 objects. Encourage a child to add one more object to the set. Then ask the children if there are 7 objects. Continue by having a child add felt objects, one at a time, until a set of 7 is reached. Work through a similar demonstration for other sets.



Concept of addition

Teaching
Page r-42

The activity on this page is similar to that of the preceding page. Direct the children to draw enough objects so that each set has as many in it as are indicated by the numeral. According to the needs of the children, you might want to work through the frames with them. For example, ask them if there are 6 figures in the first frame. When they respond "no," direct them to draw one more. Then again ask if there are 6 objects. Continue one by one, until children have completed the set of 6. Work through similar exercises as needed.

Resources for Active Learning

Math Activities, "Loading the Steps," pp. 32–33; "Airplane Ride," p. 67, Allyn and Bacon

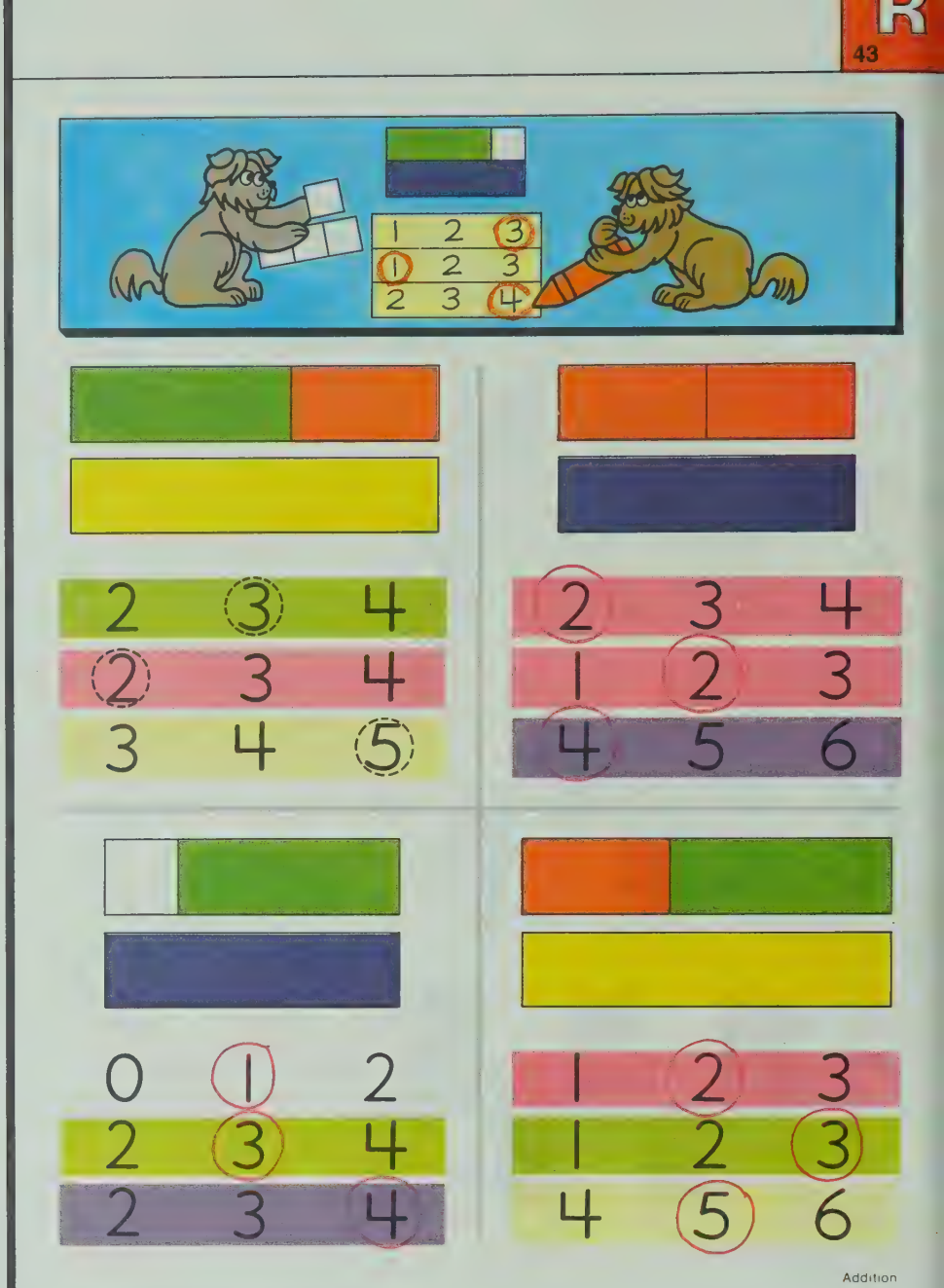
Follow-up

This might be a good time for more concentration on ordinal numbers. One possible approach could be giving balloons of different colors to each of six children. Ask each child to try to blow up his balloon to about the size of a grapefruit. Show the size with your hands. As soon as their balloons reach this size, the children should quickly form a line in front of the class to show which is first, second, third, fourth. Or, ask the children to tell the color of the fourth balloon, the second balloon, the sixth balloon, and so on. The balloons can also be used to compare sizes, such as the largest, the smallest red, or the largest green.

Teaching
Page r-43

Use the illustration at the top of the page to introduce this activity. Suggest that the children make a train with the light green strip and the white strip and match it with the purple strip. Then ask them to count the number of spaces on the back of the light green strip. Help them see that since there are 3 spaces on the back of the strip, 3 has been circled in the first row of numerals below the strips. Similarly, the 1 has been circled in the second row to indicate one space for the white strip, and, finally, the 4 has been circled to indicate the four spaces on the back of the purple strip.

Call attention to the first frame. Encourage the children to set up their strips as shown. Remind them to count the number of spaces on the back of each strip in order to know what number to circle. Work through the frames with the children as necessary. If they are capable, encourage independent work.



Objective

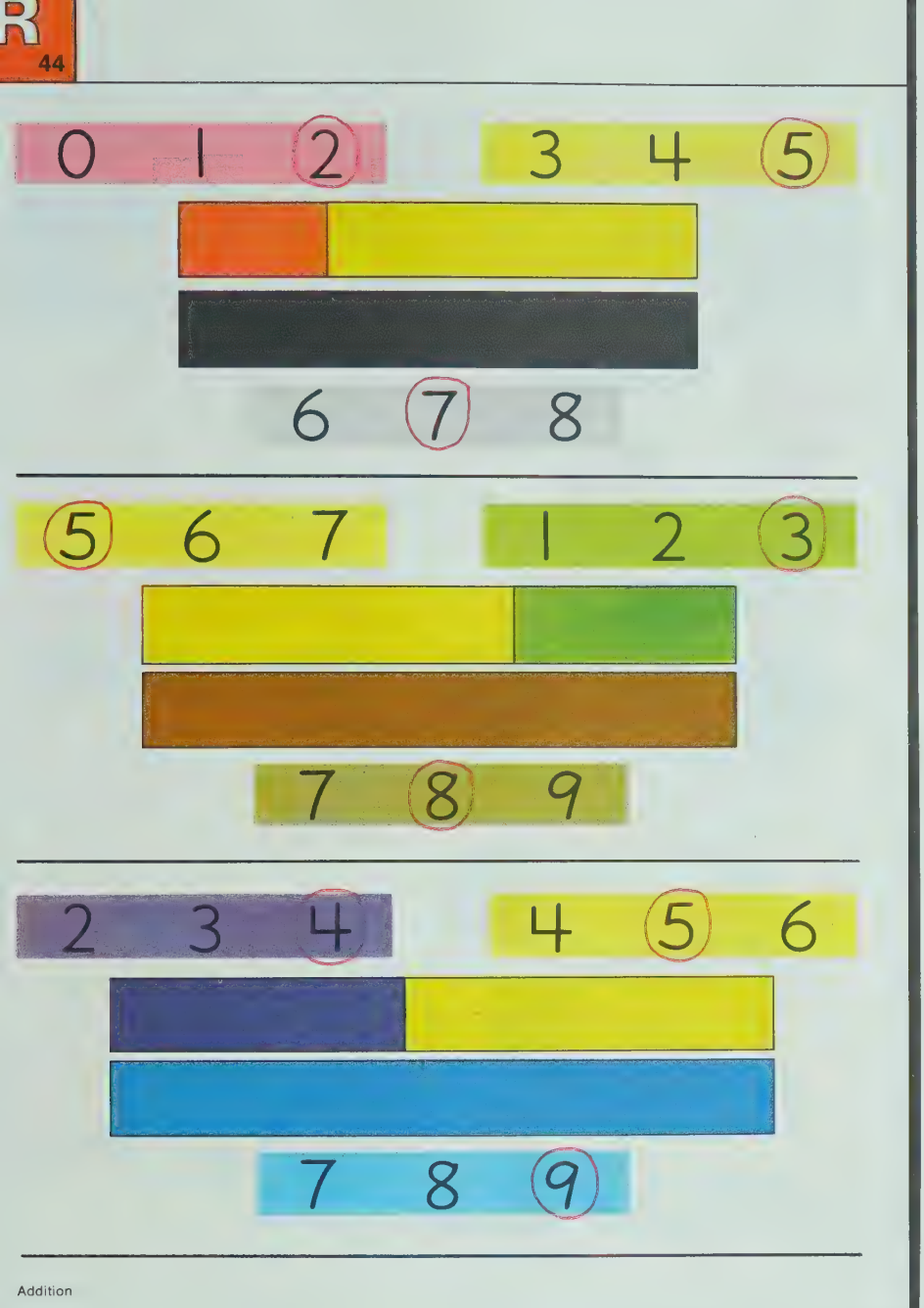
Given a train of strips matched to a single strip, the child will be able to circle numerals to show the combination represented by the strips.

Pre-Book Activity

Materials

strips

After the strips have been distributed, give children an opportunity to play freely with them, creating various designs or patterns. You might suggest that they try to show a butterfly or a flower or an airplane with the strips. Then ask them to select a particular strip, such as the brown strip, and try to find 2 strips that could be put into a train that would be as long as the brown strip. Help them see that various pairs are suitable, such as 2 purple strips, or the light green strip and the yellow strip. Then ask them to count the number of spaces on the back of their strips. Are there as many spaces on the backs of the two strips used in the train as are on the back of the brown strip? When children have realized that the spaces on the backs of the strips can be counted and matched, direct them to page r-43.



Teaching
Page r-44

Call attention to the first frame. Point out the use of the red and the yellow strips and the black strip. Ask children to count the number of spaces on the back of the red and the yellow strips and to circle the numerals above the strips accordingly. Then ask them to mark the numeral in the row below the black strip which indicates the number of spaces on the back of it. Work through other frames similarly or, if children are capable, encourage them to work independently.

Resources for Active Learning

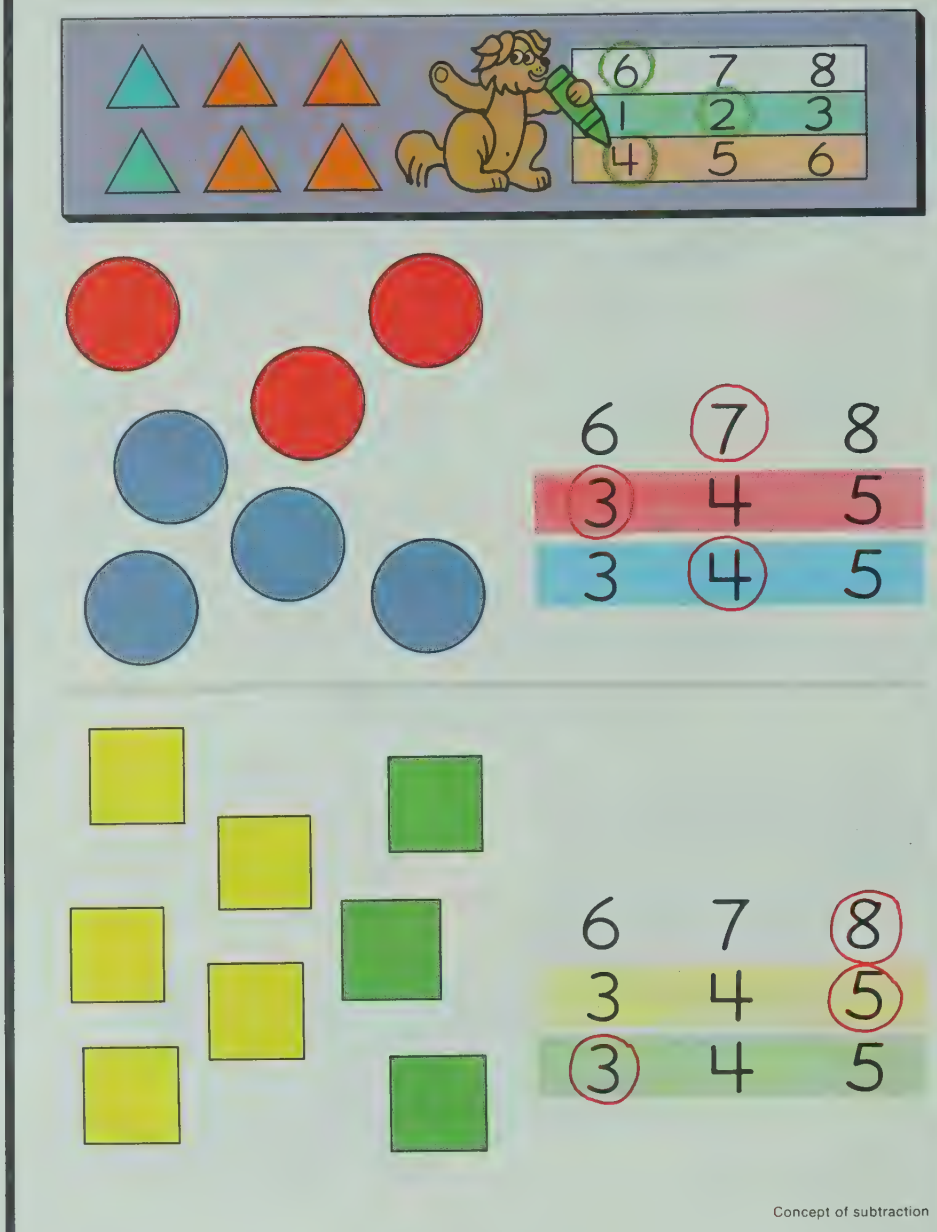
Math Activities, "Match It," p. 33, Allyn and Bacon

Follow-up

Give the children a duplicated sheet having 10 rows of 10 objects and a large-sized numeral (6, 7, 8, 9, or 10) at the beginning of each row. Tell them to choose two crayons and to color sets in each row according to the numeral indicated. They may use a different pair of crayons for each row if they wish. See the sample worksheet at the right.

Teaching
Page r-45

Use the demonstration illustration as an introduction to this activity. Ask the children how many triangles are shown. Point out the circled 6. Then ask how many triangles there are of each color. Help children associate the numbers of these subsets with the circled numerals 2 and 4. Stress that 6 may be thought of as 2 and 4. Work through the other examples with the children similarly. Stress that 7 may be thought of as 3 and 4 and that 8 may be thought of as 5 and 3.



Concept of subtraction

Objective

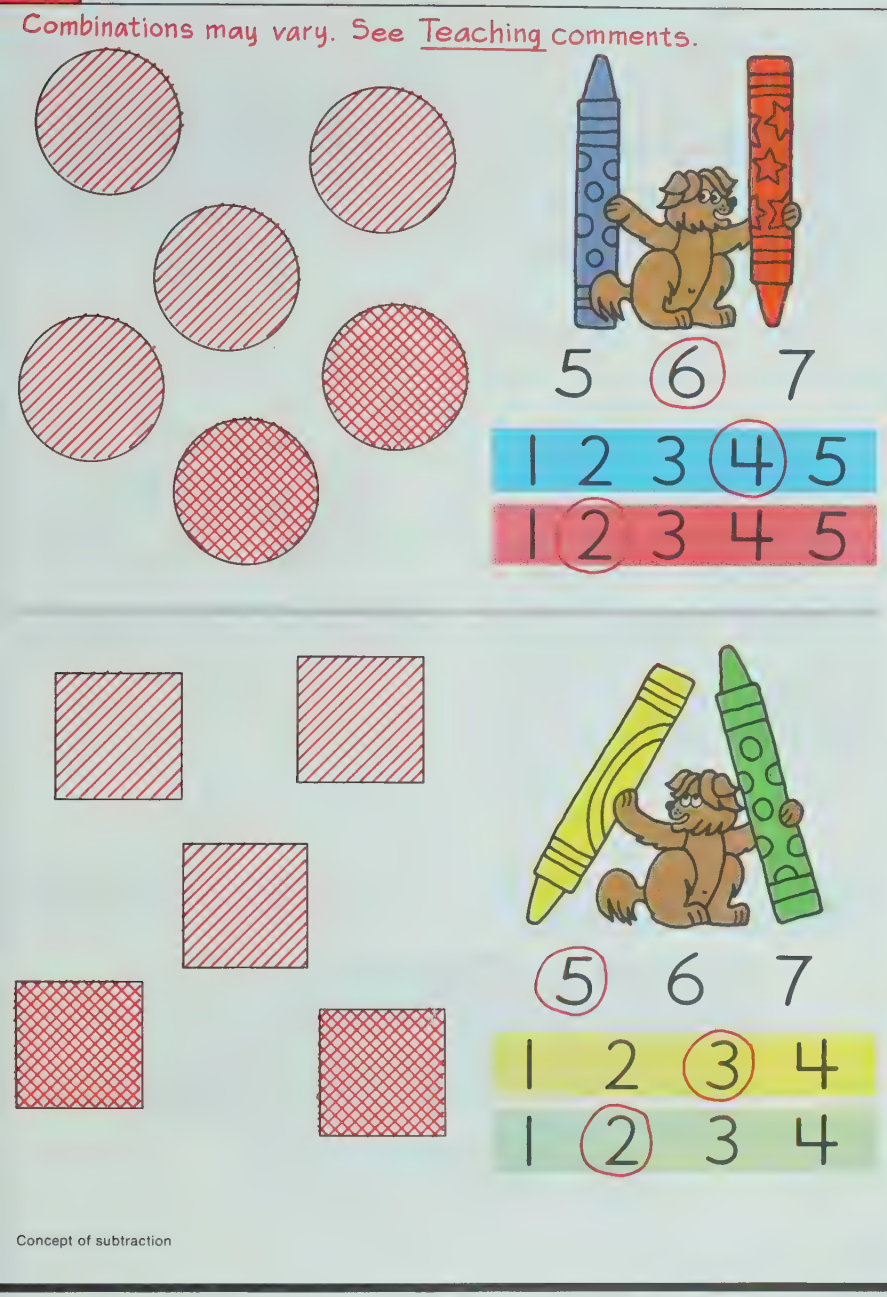
Given a set shown as two subsets, the child will be able to identify the number of the set and the numbers of the two subsets.

Pre-Book Activity

Materials

flannelboard
felt objects of different colors

Place a set of about seven felt objects on the flannelboard arranged in two subsets of different colors. Ask the children first to identify how many objects are in the whole set. Then ask them to tell how many objects are in each of the subsets. For example, place 3 red and 5 green felt objects on the flannelboard. Elicit from the children that there are 8 objects shown, 3 red and 5 green. Also stress that we can think of 8 as 5 and 3. Work through other examples. If possible, have the children manipulate counters of two different colors to match the sets which you show on the flannelboard.



Concept of subtraction

Teaching
Page r-46

Although this page is similar to the preceding one, here the children choose the subset arrangement by coloring objects of their choice. Call attention to the first frame. First ask them how many circles are shown and direct them to ring the numeral 6. Then explain that they should color some of the objects red and some blue. After they have colored them they should indicate how many of each color they have shown by ringing the matching numeral in the red and blue rows. Work through the second frame similarly. Stress that 5 may be thought of as 3 and 2, or 1 and 4, according to the combinations children choose.

Follow-up

Give each child an assortment of geometric shapes cut from different colored construction paper. Also give them paste and a large sheet of construction paper. Guide them in folding the paper into four sections. Instruct them to paste sets into each section according to labels for each section that you show on the chalkboard.

8	7
6	9

Explain that they should use two colors for each set. When they have finished, encourage them to explain the combination they have chosen for each number.

Teaching
Page r-47

You might want the children to work on this page independently or you might work through it with them as a review. Explain that in the first two frames they should circle the numeral which tells how many are in each frame. For the second two frames, they should circle the numerals which tell how many are in each subset and how many altogether. Finally, in the last frame, they should use their strips to match the strips that are illustrated and then, by counting the spaces on the back of the strips, they should circle the numeral that indicates which combination is shown.

Objective

The child will demonstrate his ability to work with the concepts presented in this module.

Pre-Book Activity

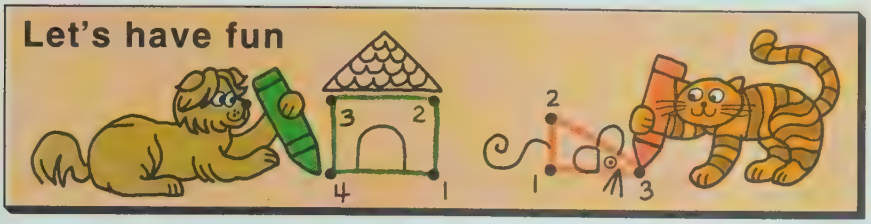
Materials

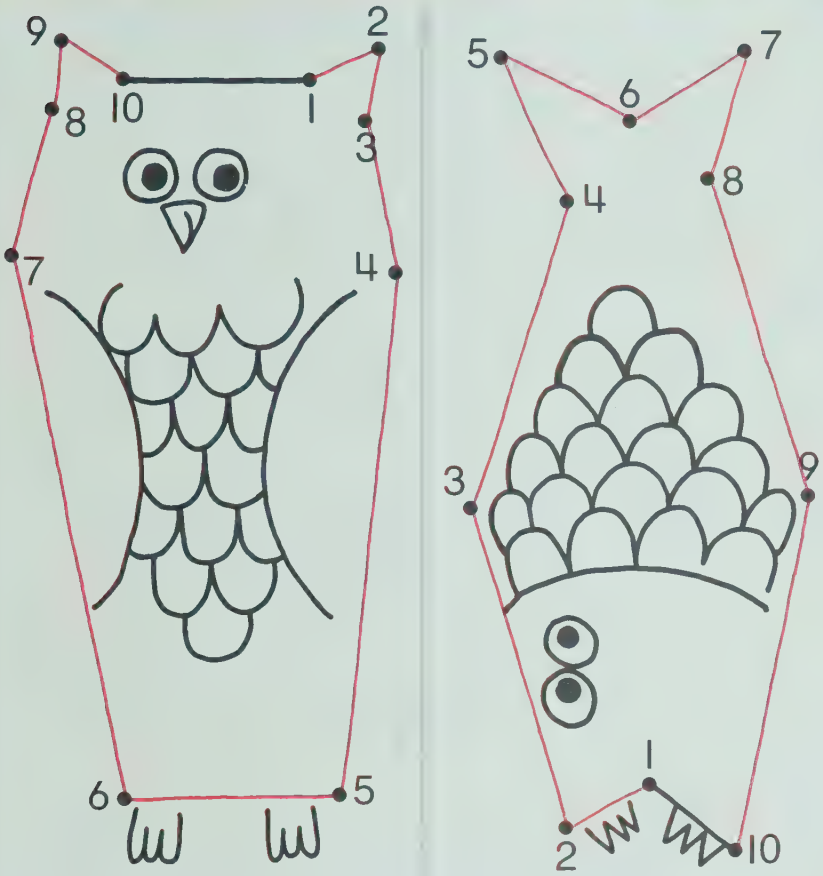
counters
strips

Distribute enough counters so that every child has at least 10. Explain to the children that they might want to use these counters to help them find the answer for problems you will give them. Make up some simple problems which use combinations under ten. Sample problems might be something like this:

1. Suzy and Pete like caramel suckers. They could get two suckers for five pennies, but they have only four pennies. How many more pennies do they need so that each of them can have a sucker?
2. The teacher wants a paintbrush for each jar of paint. She found eight brushes, but there are nine jars of paint. Does she have enough brushes? If not, how many more does she need?
3. Jon and Louis were playing marbles. Jon had six marbles and Louis had three. How many marbles did they have altogether? Did they have more than ten marbles? How do you know?

Keep the discussion appropriate to the level of the children's abilities and interests.





Counting

Teaching
Page r-48

Use the demonstration illustration to explain the procedure of connecting dots. Be sure children realize that they should connect the dots in numerical order. Explain that they should start at the dot beside the numeral 1, draw a line to the dot near 2 and continue through 10. Urge the children to work carefully. Be sure they realize that they should not cross over the black line down the middle. After they finish the page, ask them to identify and color the pictures. You may have to suggest to some that they turn their page sideways before they recognize that the completed drawing on the right is a fish.

Resources for Active Learning

Math Activities, "Race to Uncle Jack's House," pp. 49–50, Allyn and Bacon

Follow-up

For an individualized activity, have children look for objects buried in sand. Have a small box into which you have poured coarse-grained sand and have buried ten red and ten blue objects such as little rubber toys. Also, prepare task cards which instruct the child to look for combinations, such as "Find 7 as 5 and 2" or "Find 9 as 4 and 5." Instruct the children to choose a task card and look for the objects in the sand to find the correct combination. They might place them beside the task card to show what they have found, or they might simply show you the objects they have found.

Glossary

addend Any one of a set of numbers to be added. In the equation $4 + 5 = 9$, the numbers 4 and 5 are addends.

addition An operation that combines a first number and a second number to give exactly one number. The two numbers are called addends, and the one number that is the result of combining the two numbers is called the sum of the addends.

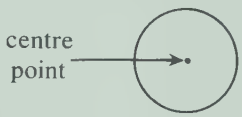
angle Two rays from a single point.

area The area of a closed figure or region is the measure of that region as compared to a given selected region called the unit, usually a square region in the case of area.

bisect To divide in half or to find the midpoint.

centimetre A unit of length. One centimetre is $\frac{1}{100}$ metre.

centre point A given point in the interior of a circle, such that all the points on the circle are the same distance from this given point.



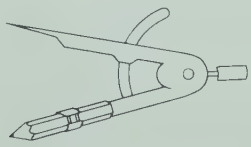
circle A set of points, all of which are a specified distance from a given point called the centre or centre point.

circumference The distance around a circle.

closed figure Intuitively, a figure is closed if you can begin at any point and trace the figure, returning to the initial point.

commutative principle *See* order principle.

compass A device for drawing models of a circle.



congruent angles Two angles are congruent if they are the "same size."

congruent segments Two segments are congruent if they are the "same size."

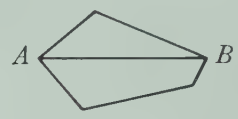
count To name numbers in regular succession.

cube A rectangular prism (box) such that all faces are squares.

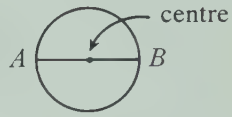
decimetre One tenth of a metre. Ten centimetres.

denominator The number indicated by the numeral below the line in a fraction symbol.

diagonal A segment joining two nonadjacent vertices of a polygon. In the figure, the diagonal is segment AB.



diameter A chord that passes through the centre point of the circle.



difference The number resulting from the subtraction operation.

digits The basic Hindu-Arabic symbols used to write numerals. In the base-ten system, these are the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

disjoint sets Disjoint sets have no common elements. That is, when S and T are two sets such that $S \cap T$ is empty, we say that the sets are disjoint with respect to each other.

distributive principle *See* multiplication-addition principle.

division An operation related to multiplication, as illustrated:

$$3 \times 4 = 12 \begin{cases} \nearrow 12 \div 3 = 4 \\ \searrow 12 \div 4 = 3 \end{cases}$$

edge An edge of a space figure is one of the segments making up any one of the faces of the space figure.

empty set The set that has no objects in it.

equality (equals, or $=$) A mathematical relation of being exactly the same. The statement $4 + 5 = 6 + 3$ claims that the number $4 + 5$ is exactly the same as the number $6 + 3$.

equation A mathematical sentence involving the use of the equality symbol. Examples: $5 + 4 = 9$; $7 + \square = 8$; $n + 3 = 7$.

equivalent sets Two sets that may be placed in a one-to-one correspondence.

even numbers The whole-number multiples of 2 $(0, 2, 4, 6, 8, 10, 12, \cdots)$.

face The face of a given space figure is any one of the plane geometric figures (regions) making up the space figure. For example, in a cube each of the square regions is a face of the cube.

fraction A symbol for a rational number.

Example: $\frac{2}{3}, \frac{5}{8}, \frac{7}{2}$.

graph (1) A set of points associated with a given set of numbers or set of number pairs. (2) A picture used to illustrate a given collection of data. The data might be pictured in the form of a bar graph, a circle graph, a line graph, or a pictograph. (3) To draw the graph of.

greater than ($>$) One of the two basic inequality relations. Examples: $8 > 5$, $28 > 25$, $80 > 50$.

grouping principle (associative principle) When adding (or multiplying) three numbers, you can change the grouping and the sum (or product) is the same.

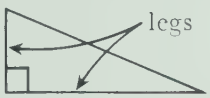
Example: $2 + (8 + 6) = (2 + 8) + 6$
$3 \times (4 \times 5) = (3 \times 4) \times 5$

hexagon A six-sided polygon.

inequality ($>$, $\neq$, $<$) In arithmetic, a relation indicating that the two numbers are not the same, or that one is greater (or less) than the other.

intersection of two sets The set containing those objects and only those objects that are in both of two sets. Example: Set $A = \{c, d, e, f\}$: Set $B = \{e, f, g\}$. The intersection of these two sets is the set $\{e, f\}$. We write: $A \cap B = \{e, f\}$.

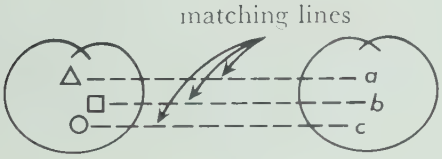
legs of a right triangle The two sides of a right triangle other than the hypotenuse.



length (1) A number indicating the measure of one line segment with respect to another line segment, called the unit. (2) Sometimes used to denote one dimension (usually the greater) of a rectangle.

less than ($<$) One of the two basic inequality relations. Examples: $5 < 8$, $25 < 28$, $50 < 80$.

matching lines Lines used to indicate or denote the correspondence between the objects in two sets.



measure (1) A number indicating the relation between a given object and a suitable unit. (2) The process of finding the number described above.

metre A unit of length in the Metric System that is 100 centimetres.

minus ($-$) Used to indicate the subtraction operation, as in $7 - 3 = 4$ (read, "7 minus 3 equals 4").

mixed numeral A symbol given for a rational number greater than 1 that is a combination of a whole-number symbol and a fraction symbol.

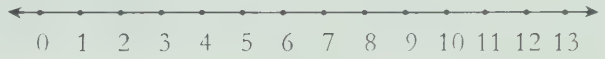
Examples: $2\frac{1}{2}$, $3\frac{2}{3}$, $5\frac{1}{4}$.

multiplication An operation that combines a first number and a second number to give exactly one number. The two numbers are called factors, and the one number which is a result of combining the two numbers is called the product of the two numbers.

multiplication-addition principle (distributive principle) This principle is sometimes described in terms of "breaking apart" a number before multiplying.

Example: $6 \times (20 + 4) = (6 \times 20) + (6 \times 4)$.

number line A line on which specified points are given number labels or names. The following example illustrates the whole-number line.



number pair Any pair of numbers. In this program, usually a pair of whole numbers.

numeral A symbol for a number.

numerator The number indicated by the numeral above the line in a fraction symbol.

odd number Any whole number that is not an even number.

one-to-one correspondence A one-to-one correspondence exists between two sets when the elements of one can be matched with the elements of the other in such a way that each element of the first set is matched with exactly one element of the second set, and each element of the second set is matched with exactly one element of the first set.

order principle (commutative principle) When adding (or multiplying) two numbers, the order of the addends (or factors) does not affect the sum (or product).

Example: $4 + 5 = 5 + 4$, $2 \times 3 = 3 \times 2$.

parallel lines Two lines which lie in the same plane and do not intersect.

parallelogram A quadrilateral with opposite sides parallel.

parentheses (( )) Symbols used to indicate grouping or order of performing operations. Examples:

$(5 \times 4) - 2 = 18$; $5 \times (4 - 2) = 10$.

perimeter The sum of the lengths of the sides of a given polygon.

placeholder In this program, this term is used to indicate the small box in which you write the solutions to equations.

place value A system used for writing numerals for numbers, using only a definite number of symbols or digits. The system permits a given digit to stand for different numbers, depending upon its location or position within a given numeral. The number a given digit stands for in a symbol is determined by its position in the numeral and by the base being used in the particular system. In the ordinary base-ten system, for example, a numeral 2 in the third place from the right would stand for 200. Other examples are as follows: in the numeral 3257, the 5 stands for 5 tens or 50; in the numeral 36 289, the numeral 6 stands for 6000 (or $6 \times 10 \times 10 \times 10$).

plus ($+$) Used to indicate the addition operation, as in $4 + 3 = 7$ (read, "4 plus 3 equals 7").

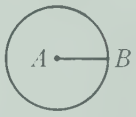
polygon A closed geometric figure made up of line segments.

product The result of the multiplication operation. In $6 \times 7 = 42$, 42 is the product of 6 and 7.

quadrilateral A four-sided polygon.

quotient The number (other than the remainder) that is the result of the division operation. It may be thought of as a factor in a multiplication equation.

radius (1) Any segment from the centre point to a point on the circle (2) The distance from the centre point to any point on the circle.

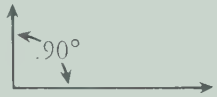


repeated addition Finding the sum of a set of numbers, each of which is the same.

Example: $5 + 5 + 5 + 5$.

repeated subtraction Starting with a number and repeatedly subtracting the same given number from each difference that is obtained.

right angle An angle that has the measure of 90 degrees.



A right angle

right triangle A triangle that has one right angle.

Roman numerals Numerals used by the Romans. Used primarily to record numbers rather than for computing. Examples: IV, IX, XIV.

segment Two points on a line and all the points on that line that are between the two points.

set A group, collection, family, or aggregate of objects. At the heart of the concept of set is man's ability to think of a collection of objects as a single entity.

similar triangles Two triangles are similar to each other if their sides can be matched so that the ratio of the length of each pair of sides is the same.

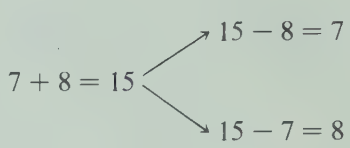
skip count To count by multiples of a given number. Example: Counting by fives—0, 5, 10, 15, 20, · · ·.

solution The number or numbers that result from solving an equation or a given problem.

solve To find the number or numbers that, when substituted for the variable or placeholder, make the given equation true.

square A quadrilateral that has four right angles and four sides that are the same length.

subtraction An operation related to addition as illustrated:



surface area The sum of the area of each face of a figure.

times ($\times$) Used to indicate the multiplication operation, as in $3 \times 4 = 12$ (read, "3 times 4 equals 12").

triangle A three-sided polygon.

union of two sets The set consisting of those objects which are in one or the other or both of two sets. Example: Set $A = \{c, d, e, f\}$; Set $B = \{e, f, g\}$. The union of the two sets is the set $\{c, d, e, f, g\}$. We write: $A \cup B = \{c, d, e, f, g\}$.

unit An amount or quantity adopted as a standard of measurement.

vertex The point that the two rays of an angle have in common.

volume The measure, obtained using an appropriate unit (usually a cube), of the interior region of a space figure.

Index

(All page numbers refer to the pupils' text.)

A Text for Teachers

Investigating Mathematics Learning

Phares G. O'Daffer

Robert E. Eicholz

Charles R. Fleenor

Introducing the Metric System

James Sherrill

J. Norman C. Sharp

ADDISON-WESLEY (CANADA) LTD.
DON MILLS, ONTARIO • READING, MASSACHUSETTS • MENLO PARK, CALIFORNIA • LONDON

INVESTIGATING MATHEMATICS LEARNING

I. Some Thoughts About Learning

Almost everyone has some observations on teaching and learning. A recent quote that has been making the rounds is: "If we tried to teach children to speak, they would never learn." However, in *The Process of Education* (Harvard University Press, 1960), Jerome Bruner observes, "Any subject can be taught effectively in some intellectually honest form to any child at any stage of development." But Linus (of *Peanuts* cartoon fame), considerably less optimistic, laments: "How can I learn 'New Math' with an 'Old Math' mind?"

In a more critical vein, John Holt in *How Children Fail* (Pitman Publishing Co., 1964) asserts: "In our classes, we begin with words, carry on with words, and often fail to get beyond words." He also says, "All too often the mathematics classroom becomes a temple of worship for the right answers, and the way to get ahead is to lay plenty of them on the altar." We know, of course, that many teachers for many years have been doing an excellent job helping elementary school children learn mathematics. Yet, it is worthwhile for us to reevaluate our approaches and, if possible, find even better ways to create situations where children learn more effectively.

The implications of the research of Piaget and others in how children learn mathematics and the observations of countless classroom teachers concerning the directions we should take are well summarized by a familiar Chinese proverb:

I hear and I forget.
I see and I remember.
I do and I understand.

The message of this proverb is that hearing and seeing are not enough: to learn with understanding, the child should experience *active involvement* with mathematical ideas. In order for the child to become actively involved, it has been found that the use of *physical materials* which contain the seeds of the mathematical ideas are valuable and often necessary. Coupled with the idea of active involvement with physical materials is the idea that teachers should encourage *student responsibility* and create conditions in which the student is not always encouraged to rely solely on the teacher but rather to take initiative for figuring out some things for himself.

Z. P. Dienes summarized a multitude of suggestions from researchers and teachers when he said: "It is suggested that we shift the emphasis from teaching to learning, from our experience to the children's, in fact, from our world to their world."

Teachers vary considerably in their views of how best to help children become actively involved with mathematics. While one teacher desires to convert his classroom immediately into a mathematics laboratory, another teacher may prefer a very modest beginning with a limited amount of active student involvement with physical materials inserted into his usual classroom approach. In this text we suggest a number of approaches for modest beginnings and indicate ways in which these approaches might be expanded to provide for a total laboratory approach and a more extensive individualized program.

To introduce one possible approach, let us simulate a teaching strategy by outlining one way to organize a specific lesson. Thus, suppose a teacher wanted to devise a lesson which would help children understand the idea of congruent segments in geometry. First the teacher provides each child with a geoboard and a sheet containing several 5-by-5 arrays of dots. Then he reviews, very briefly, the idea of a segment and the endpoints of a segment. Next, after helping the children see that they can use a rubber-band around two nails to represent a segment on the geoboard, he passes out the investigation suggested in Figure 1.

INVESTIGATION

How many different segments with endpoints just as far apart as these can you find on your geoboard? Record your results on dot paper.

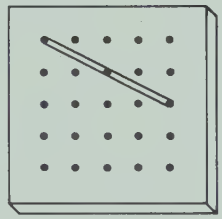


Figure 1

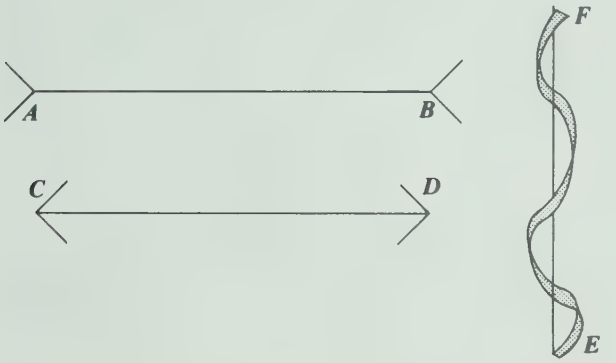
The teacher may choose to have chairs or desks rearranged so that children can communicate with each other as they become involved in the investigation. The teacher will check to be sure that everyone understands the investigation question; then he should encourage the children to find their own way to answer the question and record their findings. (To gain a fuller appreciation of an investigation situation, play the role of the child and complete the investigation yourself.)

Brief discussions among children or between teacher and children may occur during investigations, but the main discussion is most effective after the investigation has been completed. At this time, the teacher might ask such questions as: "How many different

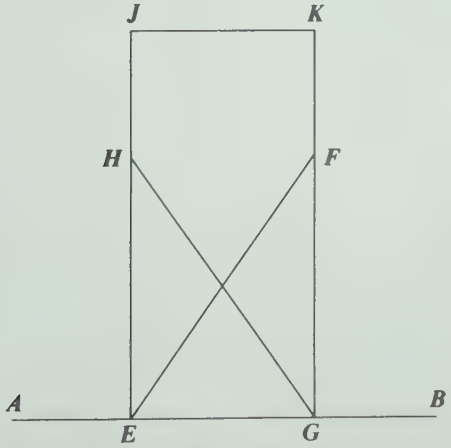
segments did you find?" "How can you be sure that you have found all such segments?" "How could you convince someone that each of your segments has endpoints just as far apart as all the others?" Such questions could then be followed up with a definition of congruent segments: When the endpoints of one segment are just as far apart as the endpoints of another segment, we say the segments are *congruent*. Then ask, "Can you think of some other ways to tell when two segments are congruent?" This question might lead into a discussion of how tracing paper, compasses, or marks on the edge of a piece of paper can be used to determine whether or not two segments are congruent.

After the children have discussed the ideas, the teacher may provide them with some problems which *utilize* these ideas. The child would probably be encouraged to use the ideas for testing congruence of segments that were developed in the discussion. The following are examples of possible exercises.

1. Find 2 segments below that are congruent to each other.



2. Name each pair of congruent segments in this picture.



One way to individualize a lesson is through an *extension* of the exercises. Extending the exercises can provide for remediation, reinforcement, or enrichment. As an extension to individualize this lesson, the teacher might give certain students the follow-up investigation below. (For a fuller appreciation of this lesson, complete the exercises and the investigation yourself.)

INVESTIGATION

Segment AB is not congruent to segment CD.

How many different segments (no two congruent) can you find on your geoboard? Record your results on dot paper.

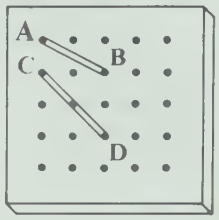


Figure 2

This abbreviated "lesson" provides a preview of one possible technique for encouraging children to become actively involved with physical materials in situations where they take more responsibility in the learning of mathematical ideas.

In the next section of this text, the parts of lessons such as the one described above will be analyzed and discussed. An outline for planning such lessons will be given, and various suggestions for carrying out each part of such a lesson will be proposed.

Since the investigation phase of the lesson provides the encouragement for active involvement by the child and since the kind of investigation used depends upon the type of learning involved, Section III in this text will focus on specific types of learning in elementary school mathematics. For example, the "lesson" described above helped children learn the *concept* of congruent segments; other lessons might be concerned with developing a *skill*, forming a *generalization*, learning a *fact*, or developing an *attitude*. Each type of learning will be analyzed and related to activity-oriented lessons that provide modest beginnings toward an active approach to mathematics learning.

Edith Biggs and James MacLean, in their book *Freedom to Learn* (Addison-Wesley, 1969), state: "A few schools scattered throughout the world are responding with some speed to a message which has been repeated with increasing urgency for some three hundred years. It is a simple message: Schools should be organized, not for teachers to teach, but for children to learn." In the same book, there appears an extensive list of "homemade" materials and devices that can be easily acquired for use in the mathematics classroom. Many materials, from newsprint and drinking straws, to string, popsicle sticks, beans, and homemade geoboards, can be made available to children at minimal cost. Rather than dismiss the possibility of actively involving children with materials in the classroom because no funds are available, a teacher should study this list carefully; he may be amazed by how much can be done with minimum expense.

Teachers sometimes feel that to involve children with physical materials and allow them to communicate with other children in the classroom is to invite

chaos. On the contrary, it has been found that, when children really become involved in using materials to investigate a situation, there may be a bit more low-keyed noise about the room but the usual discipline problems are almost nonexistent. It is helpful if there are tables available in the classroom so that children can work in small groups. If tables are not available, desks could be moved to assist in small-group work. On occasion, an investigation might call for children to leave their desks and to engage in other activity in the room. A simple set of "ground rules" should suffice to make the situation quite manageable.

It is interesting to consider the number of elementary school teachers who prefer to say that they are "helping children learn mathematics" rather than that they are "teaching mathematics." What one says, of course, does not always describe accurately what one does. It does seem important, however, in the light of recent studies and observations about how children learn mathematics, to focus on the child and try to create an environment in which the child has a greater opportunity to make decisions and to become really interested in his study. It is hoped that the following sections of this teachers' text will provide some ideas which may help you improve your ability to "help children learn mathematics."

EXERCISE SET 1

1. What was your reaction to the investigations in this section? **A** Did you become involved in the activity? **B** Were you interested? **C** Did you watch the clock? **D** Did you talk to anyone else while completing the investigation? (If so, was it helpful?) **E** Did the investigation situation help you better understand the idea involved? **F** What other feelings did you have?
2. Which quotation in this section seemed most significant to you? Why?
3. **A** Do you think most teachers teach the same way they were taught as elementary school children? **B** What do they do differently? **C** What are some ways you think our teaching of elementary school mathematics might be improved?
4. Look through the *Investigating School Mathematics* text at your grade level. How do the comments in this first section of the text relate to the approach taken in the child's text?

II. A Plan for a Learning Experience

First consider the practical matter of how the teacher proceeds in the daily task of helping children learn mathematics. A structured outline (inherently flexible) around which daily learning experiences may be planned can be a valuable organizational aid for the teacher and can give him a fresh insight into the role of new approaches to instruction.

Here is the outline that was used in planning the "lesson" in Section I. It has proven to be quite useful, especially for those teachers who have desired to make a beginning toward providing children more opportunities for active involvement with mathematical materials and ideas.

Preparation and Investigation
Discussion
Utilization and Extension

Since this outline offers a variety of possibilities for a teacher to reevaluate his approach to classroom instruction, the following sections provide an examination of its individual elements.

Preparation and Investigation

The investigation phase (often called simply "the investigation") is central to the learning experience. In this phase, the children are encouraged to become actively involved, individually or in groups, in the investigation of a situation that contains the seed for the central idea of the lesson. The investigation should be the "main event" in terms of pupil activity and involvement. The teacher should think of the investigation as a child-centred activity. Completion of the investigation in Figure 3 will help clarify the ideas of investigation.

INVESTIGATION

Can you find an investigation in a text from the *Investigating School Mathematics* series that

(a) uses centimetre strips?
(b) utilizes paper folding?
(c) has a question like "How many can you find?"
(d) involves the geoboard?
(e) encourages children to use graph paper?
(f) asks the children to record their findings?
(g) directs the children to use reference material?

Figure 3

Homemade or commercially produced manipulative materials often provide the stimulus for the situation to be investigated. At other times, even more simple teacher-devised activities provide this stimulus. For example, the suggested investigation in Figure 4 might have been made by a teacher to initiate an investigation in a lesson designed to help children form the generalization, "You can rearrange three addends any way you please, and the sum will always be the same."

Sometimes by asking appropriate questions about a situation of interest to the children the teacher may involve them in an exploration of a central idea to be developed.

Regardless of how an investigation is initiated, a teacher should remember that the investigation situation is specifically designed to encourage children to

INVESTIGATION

Make three slips of paper like these.

Then turn them over and mix them up. Pick any two slips and add the numbers on them. Then add the number on the other slip.

If you do this five times, will you get the same number each time?

Figure 4

take responsibility for the thinking and exploring. Too much "teacher help" can hinder the achievement of these aims.

In an investigation, it is not uncommon to see children deeply involved and assuming full responsibility for completing the task at hand. The teacher, who plays a key role in initiating the investigation, may appear not to be needed as he moves about the room. Occasionally, a brief discussion between teacher and child occurs, but most of the larger-group discussion occurs after the investigation. The investigation itself should embody an attitude toward learning that could be easily stifled by too many words from the teacher. Perhaps, in an investigation, a new adage should replace the old: the teacher, rather than the children, should be "seen but not heard."

The investigation is predicated on the assumption that the best way to minimize the need for words is to substitute an appropriate question for a wordy explanation at a time when a child's interest in a mathematical situation is beginning to ripen.

For example, suppose a certain group of children understand the concept of a triangle and are ready to consider characteristics that distinguish one type of triangle from another. An appropriate question to initiate an investigation might be the one shown in Figure 5. (Try this investigation yourself.)

INVESTIGATION

How many triangles with different shapes can you make that have no nails inside?

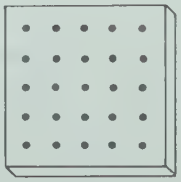


Figure 5

This question is both activity-stimulating and activity-sustaining. It helps involve the child in a search which he will continue with little further motivation. Notice also that the answer is not as important as the experiences the child will have as he responds to the question. Further, the question is sufficiently clear that the child immediately becomes involved with the challenge of the investigation rather than dissipating energy in efforts to understand the question. Another characteristic of this type of question is that it provides for individual differences: when the child is asked "How many can you find?" he can feel successful even if he finds only one. Of course, not all investigations can or should be introduced by this type of question, but it is important for the teacher to recognize that as the children respond to these questions, they will achieve in widely differing ways. In an investigation, the teacher should give recognition for all levels of achievement.

It should be noted that the amounts of time used for the investigations may vary considerably. One investigation may involve a very brief "happening" which sparks a simple idea within the child. Another investigation may utilize a large part of the period of time available for the mathematics lesson and might involve the child in a sustained exploration of a game or a set of manipulative materials.

To set the stage for an investigation of any duration, a preliminary *preparation* phase is sometimes needed. This phase provides for a brief review of key ideas needed for the investigation and for any motivational activity helpful in introducing it. This phase should be kept fairly short and care should be taken to see that this preliminary work does not preempt the central idea or activities involved in the investigation or the work that follows it.

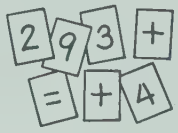
In summary, the investigation phase is the child-involvement phase. It often requires materials, and is usually motivated by a carefully selected question which focusses the student's attention on the central idea of the lesson. Proper consideration of this phase in your lesson planning can be highly rewarding.

EXERCISE SET 2

1. Find some investigations in the *Investigating School Mathematics* text that contain features not mentioned in Figure 4.
2. Choose a lesson from an *Investigating School Mathematics* text and write a description of the role you think the teacher would play in using the investigation phase of the lesson.
3. Choose an idea to be taught and prepare an investigation situation which has the potential of involving the child in working with this idea.
4. Two investigations follow. Give the central idea of a possible lesson based on the use of each one.

A

INVESTIGATION

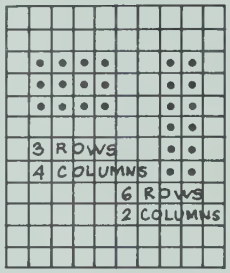


Cut out 7 slips of paper. Put one of these numerals or one of these signs on each one.

How many different equations with 3 addends can you write with your slips of paper? Record each equation you find.

B

INVESTIGATION



The graph paper shows two different ways to arrange 12 counters in a rectangular array.

How many different ways can you arrange 24 counters in a rectangular array? Record your findings by drawing pictures on graph paper.

5. Here is an interesting investigation you may like to try. Through it, you will be introduced to a basic idea of mathematics. Be sure to record your findings and be ready to discuss them further in the next section.

Copy and continue this array of numbers until you reach 52.

Then circle all the prime numbers in the array.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16
17	18	19	20
	. . .		

Notice that the numbers in the right-hand column can be written as $4 \times$ (a whole number). For example: $8 = 4 \times 2$, $12 = 4 \times 3$, and $20 = 4 \times 5$.

Can you make a statement about prime numbers that is suggested by this activity?

Discussion

Following the investigation, a *discussion* phase allows teacher and children to further share ideas in a discussion of what they found in the investigation. The teacher has an excellent opportunity in this phase to ask questions and to supply examples to help children further develop their understanding of the ideas germinated by the investigation.

Another valuable aspect of the discussion phase is that it provides additional opportunities for children to communicate with other children as a means of shaping their ideas. In a good discussion, it is not unusual for children, having reached an impasse in *their* thinking and communication about an idea, to ask the teacher if he can clarify the point. This is when the teacher as a resource person emerges. At other times, when ideas new to the teacher arise, the teacher participates in the discussion, not as a resource person, but as a fellow-learner. Both of these situations can contribute to a comfortable, meaningful discussion, but its potential benefits may never be realized if the teacher monopolizes the discussion to the extent that the children are denied the opportunity to draw their own inferences and make their own decisions. Since it is the child who is involved in the investigation, the child's ideas about the findings should be of primary importance, and the child should supply as many details leading to the understanding of the idea as possible.

By listening to the child and asking appropriate questions, the teacher can build on the child's initial ideas and help him develop a deeper understanding in preparation for further work. This understanding cannot be developed, however, by always asking questions which require simply that a child remember a fact correctly or perform a practical skill. Nor is it sufficient to ask questions to which a child can respond with a guess of "Yes" or "No." Rather, the questions that should be asked often are those that require a deeper thinking on the part of the child.

For examples of the more effective type of question, consider again the investigation described in Figure 7. This investigation, designed to set the stage for the development of the concepts of isosceles triangle, right triangle, scalene triangle, and equilateral triangle, might be followed by a discussion in which the teacher would ask questions such as the following:

1. Can you choose a pair of triangles you found and describe ways in which one is different from another?
2. In what ways are some triangles you found alike? (Note: Children may respond, "Some have a square corner," "Some have two sides the same," "Some have no sides the same," "Some are large," and so on.)
3. How would you describe a triangle that is different from any of the triangles you formed on the geoboard?

As the teacher asks thought-provoking questions and listens to the children's responses, he will be able to find ways to clarify the basic idea of the lesson and to prepare the children for the independent work which is to follow. It is in the latter stages of the discussion that the teacher may want to explain more carefully, show additional examples, and, in general, lead the child to a deeper mastery of the ideas involved.

EXERCISE SET 3

1. Can you find a question in the "Discussing the Ideas" section of an *Investigating School Mathematics* text which **A** asks the children to recall something previously learned? **B** asks the children to restate or explain an idea in their own words? **C** asks the children to interpret a diagram, picture, or explanation? **D** asks the children to analyze a given situation? **E** asks the children to evaluate a given situation?
2. What do you think about the effectiveness of the investigation described in Figure 5 as a means of meeting the goals indicated?
3. Write five questions you might ask while conducting a discussion in a mathematics lesson of your choice.
4. The following discussion exercises refer to the investigation presented in exercise 5 of Exercise Set 2. **A** What statement did you make about prime numbers? **B** Can you find a prime number that does not appear in the first or the third column? Can you find more than one? **C** $4 \times n$ is an algebraic expression. What algebraic expression can you devise to describe the prime numbers in the third column? in the first column? **D** Of the prime numbers less than 100, which type of prime occurs more often? **E** 113 is a prime number. Which type of prime is it?
5. Investigation questions may be open ("In how many ways can you measure a ball?") or closed ("Can you find the circumference and diameter of this ball?"). Discuss the merits of open and closed questions.

Utilization and Extension

The *utilization* phase allows each child to work on his own and to use the ideas developed in the investigation and discussion phases.

Often children need to practice recalling facts that have been developed or introduced in the lesson. Appropriate exercises requiring written answers are often valuable in providing this practice.

In another lesson, a child may have learned an algorithm or a skill. In order to refine this skill, he may need considerable practice using it. Appropriately designed written exercises which children complete independently can be quite helpful in polishing these skills.

In another lesson, a new idea may have been presented. In order to become more familiar with this idea and to understand how it relates to other ideas, the child may need thought-provoking problems which involve the idea. The *utilization* phase presents an opportunity for the child to solve problems which involve ideas that have been presented previously or to look at an idea that is different but closely related to one he has already encountered.

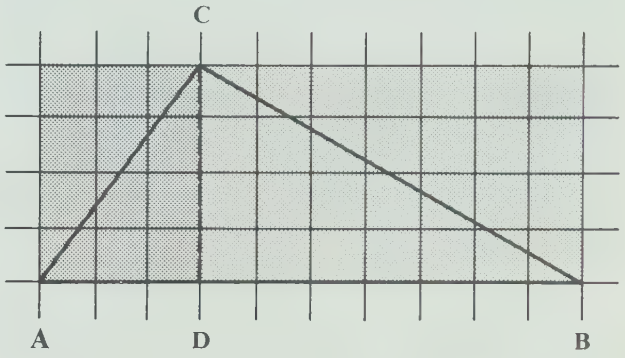
Creative activities for independent work can do much to extend the learnings developed in the investigation and discussion phases. The utilization exercises in examples A and B below are sequenced in such a way that the child has an opportunity to discover a new procedure or new ideas as a result of his work.

EXAMPLE A

Find the differences.

75	75	75	75	75	75	75
−32	−33	−34	−35	−36	−37	−38
43	42	41				

EXAMPLE B

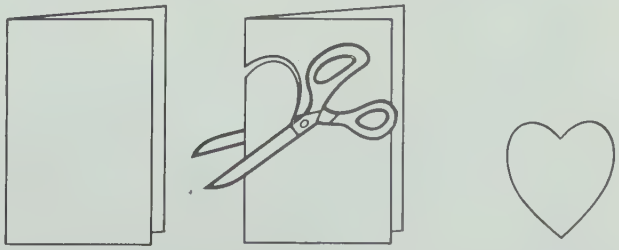


What is the area of the region shaded dark gray?
What is the area of the region shaded light gray?
What is the area of the two regions together?
What is the area of triangle ADC?
What is the area of triangle BDC?
What is the area of triangle ABC?
The area of triangle ABC is what part of the entire shaded region?

The teacher should appreciate the great potential value of discovery-sequenced exercises such as these, and should look for opportunities to make his own exercise sets using such sequences. Another set of utilization exercises might encourage the child to independently delve more deeply into the idea initiated in the investigation. Further activities with mathematical materials often provide opportunities for the child to use and extend the idea of the investigation. Example C provides an opportunity for the child to reinforce his concept of symmetry.

EXAMPLE C

Do this to make symmetrical figures.



Fold a piece of paper.

Make a cut that starts and ends on the fold.

Unfold the piece you cut out. It will be symmetrical.

Make cuts so that the unfolded shape will be:

A a rectangle	**D** a square	**G** a rocket
B a leaf	**E** a house	**H** a hexagon
C a triangle	**F** a pumpkin	**I** a butterfly

Regarding the utilization phase, it should be noted that on occasion it may be more valuable to have pairs or small groups of children work the exercises together.

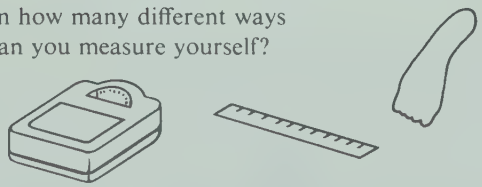
Finally, the *extension* phase provides for use of remedial, maintenance, or enrichment activities to further individualize the learning opportunities. This individualization offers numerous advantages. The slower children can avoid the frustration of having to proceed to new ideas before the previously presented ideas are understood, and the more capable children are spared the tedium of completing long lists of drill problems involving ideas they already understand.

The teacher might look for creative ways to meet individual differences in the ability to learn mathematics. For example, the slower child might profit from additional drill on certain facts and skills. Drill tapes or audio cassettes made by the teacher might provide a novel way to present the necessary practice. Duplicator masters and commercial workbooks are also available to provide extra work for those who need it. For other situations, an appropriate programmed instructional unit might serve the needs of the slower child. Single-concept film loops, which the child can play again and again, often are useful in helping him grasp an important concept. Appropriately conceived tutorial situations, in which classmates who understand the ideas work with children who do not, can be quite effective. Simple investigations utilizing physical objects which clarify more abstract ideas can also provide remedial work for certain children.

The teacher must also be concerned with those children who understand the basic ideas of the lesson and who can quickly work all the utilization exercises provided. These children can often become quite interested in activity cards which contain "open-ended" questions, such as the card shown below. (You are encouraged to try the suggested activity yourself.)

ACTIVITY CARD 10

In how many different ways can you measure yourself?



Make as many different measurements of **you** as you can and make a chart to show the information. Here are just a few suggestions:

Pulse	Length of step
Height	Number of calories used
Weight	Area of bottom of foot
Arm span	Distance you can jump

Activities such as these give the child an opportunity to make his own decisions about which ideas he uses from the lesson and how he uses them.

Puzzles or riddles can also provide a useful extension of ideas for your children. Consider, for example, those shown in Figure 6.

Think

Draw a figure like this one on your paper. Place the numbers 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 in the circles so that the sum along any line is 21.

Think

I can be found halfway between Twenty-seven and seventeen.

WHO AM I?

27 ? 17

Figure 6

Conceptually fertile games can also provide valuable experiences to supplement the basic lesson. For example, the game "Sleuth" (3M Company) is fun for children and gives them valuable experience in classification and drawing logical inferences.

The methods suggested for extending the ideas for slower children are often suitable for use in certain situations with more capable children. Similarly, the more exciting modes of extension suggested for faster children can often be quite stimulating and valuable if used appropriately for the slower children.

It is to be hoped that the teacher will share a sense of excitement in providing extra stimulation to broaden the mathematical perspective of the children. Perhaps, he will also see that much of the extension activity can truly be fun for children while at the same time inspiring new interest and involvement in mathematical ideas. In using this suggested lesson outline, if the teacher chooses to maximize the investigation phase while deemphasizing the others, it might justly be said he is using the laboratory approach. On the other hand, should he maximize the discussion phase, he may find increased options for a guided discovery approach to mathematics learning. Also, it is possible that maximization of the utilization phase accompanied by appropriate student materials would allow the teacher to embark on a course of individually prescribed instruction.

EXERCISE SET 4

1. Find an example of an exercise set in which a learning sequence occurs in an *Investigating School Mathematics* text.

2. Choose a mathematics topic and write a set of exercises which might lead the student to discovery of a central idea.
3. Can you find a lesson in an *Investigating School Mathematics* text in which the "Using the Ideas" section provides for varying degrees of student ability.
4. Choose a learning experience appropriate for your children and list some possible specific activities for use in the extension phase of this learning experience.
5. Describe your views concerning the role of drill for slow, average, and bright children.
6. Select and play a game that could be used to extend a lesson with children.
7. In Exercise Set 2, you investigated an idea of mathematics. In Exercise Set 3, you had an opportunity to discuss this idea. The exercises below enable you to use the idea you learned, and suggest an extension of the idea.

Complete each exercise.

A List five prime numbers of the "$4n + 1$" type that are greater than 50.

B List five prime numbers of the "$4n - 1$" type that are greater than 50.

C 997 is the largest prime number less than 1000. Is it a "$4n + 1$" or a "$4n - 1$" prime?

D Suppose you used a continuation of the array of numbers shown below and circled all the prime numbers. What does this suggest about another way to classify the primes?

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
...					

III. A Focus on Specific Types of Learning

In considering the more specific aspects of mathematics learning it is helpful to categorize the general types of things children learn. A simplified categorization is given below.

Concepts
Skills
Generalizations
Facts
Attitudes

It is important to recognize that each of these types of learnings has unique characteristics. Because of this, the approaches and children's activities chosen to promote these learnings may often be quite different. In the sections that follow, we will consider each of these types of learning and suggest possible approaches and activities.

Concepts

Suppose that a child is having difficulty and comes to the teacher for assistance. When the teacher asks what the difficulty is, the child points to the multiplication 9×6 and says, "I can't do this because we haven't had it yet." This reflects a common attitude among children who have been in school for a few years. Somehow they learn to feel that they are incapable of figuring out anything new in mathematics. Literally, they can do nothing that they "haven't had yet."

If this child had confidence in his ability to "figure something out" and had a clear understanding of the *concept* of multiplication, he could have found the product by perhaps adding sixes, using sets, or making jumps on the number line. Another child who knew no division "facts" but who had a clear concept of division (as illustrated below) could use his knowledge of multiplication to find any of the basic quotients desired.

$$\begin{array}{lll} \mathbf{P} & \mathbf{F} & \mathbf{F} \\ 72 \div 8 = n & \leftarrow & \text{You find this quotient,} \\ & & \mathbf{F} \quad \mathbf{F} \quad \mathbf{P} \\ \text{when you find this factor.} & \rightarrow & n \times 8 = 72 \end{array}$$

A *concept,* then, may be thought of as an idea which, when properly understood, will help the child to solve problems he "hasn't had yet," to figure something out for himself. As another example, consider the concept of prime number. Once a child understands that a prime number is a whole number with exactly two factors, he has the power, providing he understands how to find the factors of a number, to seek out and list those numbers that are prime. Of course, the task of deciding whether or not a given number is prime may be quite laborious, but understanding the concept does give the child the power to succeed.

To look more carefully at what concepts are and how they are taught, consider a model in which concept learning is relatively easy, namely, that of a set of attribute pieces. Suppose there are pieces of four different shapes (triangles, squares, circles, and rectangles), of three different colors (red, blue, and yellow), and of two different sizes (large and small), as pictured in Figure 7. (In the figures the colors red, blue, and yellow are denoted by the initials R, B, and Y.)

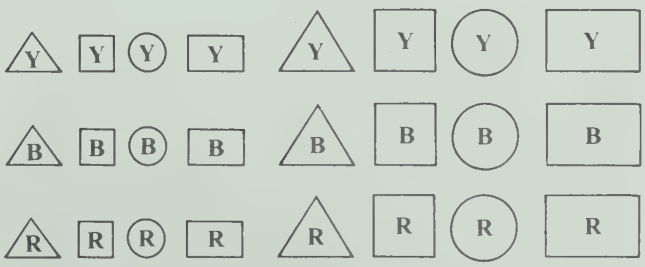


Figure 7

Now consider the Concept Card in Figure 8.

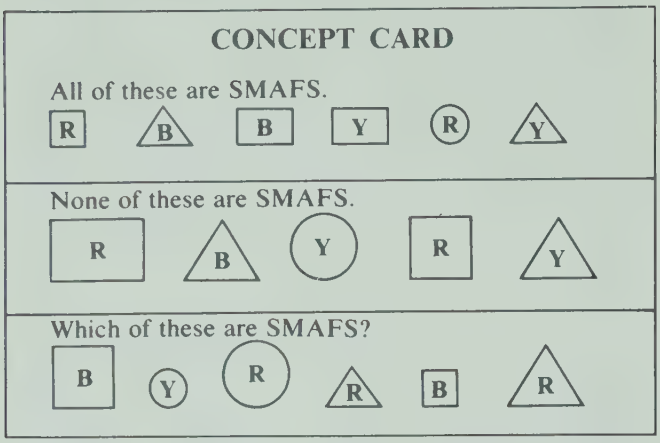


Figure 8

If you study the preceding Concept Card carefully, you will develop the simple concept of a SMAF. Notice that the key means used to teach this concept is by examples, along with *non-examples*. Both examples and non-examples play important roles in teaching many concepts in mathematics. The concept of a triangle may be taught to young children by using the Concept Card in Figure 9.

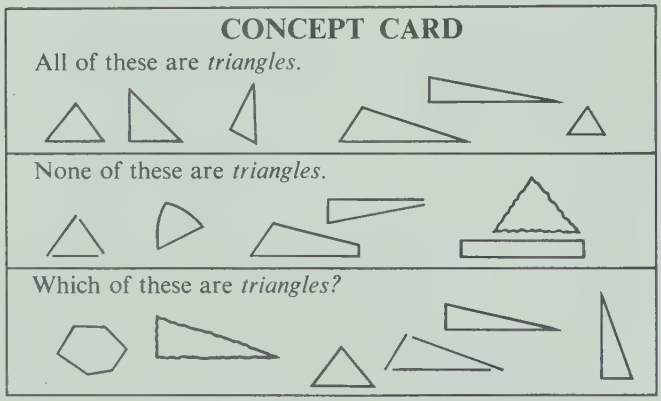


Figure 9

Clearly, the child would need further experiences in order to develop fully the concept of triangle, but the beginnings are embodied in the Concept Card shown in Figure 9.

One of the important ideas to remember when considering concepts is that concepts, unlike some other things that children learn, are developed over a period of time. Simple concepts may be developed very quickly, but other, more complicated concepts must be germinated when the child is very young and broadened through a spiralling return to the concept at various stages throughout the child's development. Many concepts are not fully developed until the child becomes an adult and encounters the idea in a variety of situations. For example, the concept of a fraction or fractional number may be introduced in grade 1 or grade 2, but a full understanding of this concept may not come until many years later. The child may acquire only an embryonic idea of a concept the first time it is presented, so it is important for the teacher to recognize the true nature of concepts and be willing to return often to the idea and carefully nurture its growth within the child. If he does not expect complete mastery after the initial presentation, he will spare himself considerable frustration when he recognizes later that the child needs further development of the basic idea.

Another key feature of concept learning suggested by the experiments of Piaget and supported and extended by the theories developed by Z. P. Dienes concerns the role of physical manipulative materials in young children's concept learning. In general, the implication of these authors' works is that it is through child involvement with physical environment that a firm basis for the development of more abstract concepts is laid. In fact, it is suggested that concept learning is facilitated by exposing children to as many different physical situations which embody the concept as possible.

It should be recognized that there are different levels of concept development and different types of concepts within these levels. For example, in the very earliest stages of mathematical learning, most concept learning involves the *concept of physical objects* such as balls, blocks, and circular or triangular objects. Very soon, the concept of certain *relations between objects* is developed: above, below, taller, shorter, larger, wider, longer, behind, and so on. A subsequent stage involves the concept of a *set of objects* such as a set of golf clubs, a set of dishes, a box of crayons, a set of blocks, a collection of stamps, or the children in a classroom. A slightly higher level of concept learning involves *relations between sets of objects:* equivalent, equal, has more than, has less than, and so on. It is at this stage that the important concept of *number* arises. For, in a sense, the concept of number involves a consideration of a set of equivalent sets. At a higher level of abstraction, the concept of certain *relations between numbers* (is less than, is greater than, is equal to, and so on) is developed. Ascending the ladder of abstraction, another level of development might involve the concept of *sets of numbers,* such as odds, evens, primes, composites, and perfects.

Clearly, the realm of concepts is vast, and the elementary teacher need not concern himself directly with many of the types of higher-level concepts. He must recognize, however, that the beginning stages in the development of many important concepts occur in the elementary school and that, through utilization of a variety of manipulative materials and appropriate strategies, he can do much to help the children learn concepts appropriate for their level.

EXERCISE SET 5

1. Use the attribute pieces shown in Figure 13. Invent a concept, name it, and make an appropriate concept card for it.

2. Choose at least two *Investigating School Mathematics* concepts from the list given below and develop concept cards which illustrate the use of examples and non-examples to teach the concepts you have chosen.
   - **A** quadrilateral
   - **B** simple closed curve
   - **C** odd number
   - **D** greater than (the relation)
   - **E** right triangle
   - **F** is congruent to (the relation)
   - **G** lowest-terms fraction
   - **H** parallelogram
   - **I** diagonal of a polygon
   - **J** parallel lines
   - **K** one half
   - **L** isosceles triangle
   - **M** equivalent sets
   - **N** symmetrical figure
3. Answer the questions on the sets of Creature cards from the set of attribute materials published by the Webster Division of McGraw-Hill Book Company (if available).
4. Choose an unusual concept of your own invention and make a concept card from which a person might discover your concept.
5. The investigation in Figure 1 was used to teach the concept of congruent segments. Make a card to teach this concept using examples and non-examples.
6. Complete "Learning a Concept" on pages I-18 and I-19; then answer the following questions.
   - **A** What are some examples of the concept you learned?
   - **B** Give some characteristics of the concept you learned.
   - **C** What were your feelings about the lesson? How could the lesson be improved to make the learning of the concept easier?

Skills

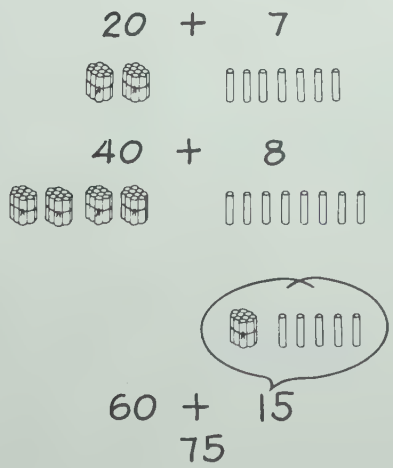
Broadly speaking, there are several types of skills that children develop in the elementary school. Hopefully, many children will develop a skill in estimating distance, weight, capacity, and time. Some teachers may wish to help children develop skill in drawing geometric figures. Some teachers set goals for upper-grade children which include developing skills in reasoning and even in "proof" of simple ideas. In elementary mathematics the most fundamental skill, by far, is that of computation with whole and rational numbers. It is these specific computational skills involving addition, subtraction, multiplication, and division and the processes related to these operations with which we are particularly concerned in the discussion that follows.

Two types of skills, power skills and speed skills, are available for completing each arithmetic process. A *power skill* is any effective way to find an answer. A *speed skill* is the most efficient way to find an answer. A power skill is a process through which a given problem is attacked by means of some technique which, though possibly quite inefficient, can produce a correct solution. This power skill may involve a long, tedious process, one which may be totally unrelated to the most efficient method for arriving at the solution. On the other hand, when a speed skill is employed, the problem is attacked with the most efficient technique available, and the problem is solved relatively quickly, usually in a mechanical fashion.

For example, suppose a child wants to find the sum of 27 and 48. If he simply starts at 48 and counts on 27 more, he is using a power skill. If, however, he finds the answer by using the usual algorithm for addition, then a speed skill is being employed.

Two additional points are worth noting about the previous example. First, in order to utilize the power

POWER SKILL B
– Bundles and Grouping



POWER SKILL C
– Expanded Notation

$$\begin{array}{r} 27 \\ +48 \\ \hline \end{array} \qquad \begin{array}{r} 20 + 7 \\ 40 + 8 \\ \hline 60 + 15 \\ 75 \end{array}$$

POWER SKILL D
– Addition Algorithm with Intermediate Step

$$\begin{array}{r} 27 \\ +48 \\ \hline 15 \\ 60 \\ \hline 75 \end{array}$$

Figure 10

skill, the child needed a clear concept of addition as it relates to counting. Thus, a power skill relies on a previously learned concept. As the child uses the concept in a power-skill situation, he gains new confidence in his ability to do something he "hasn't had yet." Secondly, the teacher should observe the evolution from power to speed. In finding the sum of 27 and 48, the initial power skill involved a basic concept of addition and the counting process. In practice, the child may continue the evolutionary trek from power to speed by next utilizing power skills B, C, and D as shown in figure 10.

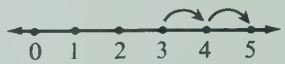
Note that each of these power skills represents a small step toward the ultimate, more efficient speed skill. When considering this process of evolution, it should also be noted that the earlier stages in a power-skill sequence often involve manipulative materials with subsequent power skills exhibiting a transition from the concrete to the more abstract. This physical beginning, which utilizes bundles and grouping, is illustrated as Power Skill B in Figure 10.

The use of power skill is available to all children. The slower child may well attempt the problem by the only means he knows, one which may often be quite laborious. For example, in finding the quotient $5863 \div 72$, the slower child might subtract 1 seventy-two at a time until he has reduced the dividend to some number less than seventy-two. The more able and creative child might tire of this method and attempt to subtract some multiple of seventy-two, such as 10 seventy-twos. Since each child is working on his own for a period of time, the development of power skill is extremely helpful in working with individual differences.

One decision that the teacher must make in relation to each child is the extent to which he should be encouraged to develop an efficient speed skill for a given algorithm. Obviously, skills are important and should be taught in elementary mathematics, yet it is the good judgment of the teacher that plays the crucial role in guiding a given child from power to speed. For certain processes, children should probably never be forced to attain a speed skill, but should be allowed to operate at the power-skill level. Other children should be directed toward the speed skill as quickly as possible in order that they may proceed to more interesting aspects of mathematics. In rare instances, a child might profit from an initial consideration of a speed skill with no previous power-skill development of a given process. The emphasis on the role of conceptual power in the performance of a skill is a key feature of the so-called "new" mathematics. It is quite probable that we cannot predict the future mathematical needs of children in our classes today, but we can help them develop the confidence, even in the area of learning skills, to utilize concepts previously learned to discover some of the basic processes for themselves.

EXERCISE SET 6

1. Write *power* or *speed* depending on the type of skill you think is being employed.

Specific Skill	Example
A Using sets to find sums	 $3 + 2 = \square$
B Using number line to find sums	 $3 + 2 = \square$
C Counting fingers to find sums	 $3 + 2 = \square$
D Memorizing that $3 + 2 = 5$	Think: 3, 2 → 5 $3 + 2 = 5$
E Thinking about "take away" to find differences	 $5 - 2 = \square$
F Using the inverse relation (missing addend) to find differences	Think: $? + 2 = 5$ $5 - 2 = \square$
G Memorizing that $5 - 2 = 3$	Think: 5, 2 → 3 $5 - 2 = 3$
H Using sets to find products	 $3 \times 6 = \square$
I Using the number line to find products	 $3 \times 5 = \square$
J Using logic (basic principles) to find products	Since $5 \times 5 = 25$, $6 \times 5 = \square$ or Since $3 \times 5 = 15$, $6 \times 5 = \square$

2. Four different power skills are shown for finding $91 \div 7$. These skills would lead up to finding this quotient by "ordinary short division."

$$7\overline{)91}$$

In what order should these be presented?

A
$$\begin{array}{r} 7\overline{)91} \\ \underline{70} \\ 21 \\ \underline{21} \\ 0 \\ \hline 13 \end{array}$$
⑩ ③

B Subtract 1 seven at a time.

$$\begin{array}{r} 91 \\ -\ 7 \\ \hline 84 \\ -\ 7 \\ \hline \vdots \end{array}$$

C
$$\begin{array}{r} 7\overline{)91} \\ \underline{70} \\ 21 \\ \underline{21} \end{array}$$

D Group 91 objects into sets of 7.

3. Complete the "Learning a Skill" lesson on pages I-19 and I-20; then do these exercises.
   - **A** Discuss the skill you learned and the way you learned in terms of power skills and speed skills.
   - **B** What part of the lesson helped you evolve a speed skill?
   - **C** What were your feelings about the lesson? How could it be improved?

Generalizations

Imagine that one of your students is engaged in an investigation in which he was asked to cut out a large quadrilateral and draw colored lines connecting the midpoints of each side of the quadrilateral. The question stimulating the investigation was, "Can you make an odd-shaped quadrilateral so that when you connect the midpoints you do not form a parallelogram?" As a result of this investigation and the subsequent discussion of his findings, the child was led to form a generalization: "The segments connecting the midpoints of any quadrilateral form a parallelogram."

In another lesson, a child might be responding to an investigation question which asked: "If you cut off the corners of a triangle and place the tips at the centre of a circle, what part of the circle can you cover? Can you find a triangle for which this is not true?"

As the child completes the investigation and engages in the discussion which follows, he forms this tentative, unproved *generalization:* "If a compass is used to draw arcs on the corners of any triangle and these corners are cut off along the arcs, then these corners will cover exactly one half of a circle drawn with the same compass opening." This tentative generalization, of course, is the forerunner of the familiar generalization that the sum of the degree measures of the three angles of any triangle is 180.

A generalization provides the economy of moving from consideration of isolated, specific cases to a general statement which holds true for a complete set of numbers or geometric figures. For example, the generalizations stated above deal with the set of all quadrilaterals and the set of all triangles. The regular occurrence of the word "any" in the generalization statements implies that the observation is true for *every* such geometric figure.

The key to teaching a generalization effectively is to provide children with appropriately chosen examples (or instances) which lead them to the generalization. An approach often used by teachers to help children learn generalizations is that of *guided discovery.* In this approach the teacher uses carefully sequenced questions and carefully chosen examples to focus the child's thought on the generalization to be discovered.

It is instructive for children in the upper elementary grades to have experiences in forming generalizations which seem obvious from a set of examples, but which, in fact, do not hold true. For example, consider the equations below.

$$\boxed{1} \times \boxed{1} - \boxed{1} + 11 = 11$$
$$\boxed{2} \times \boxed{2} - \boxed{2} + 11 = 13$$
$$\square \times \square - \square + 11 = ?$$

Figure 11

If 1 is written in the box and the operations are performed, the result is 11, which is a prime number. If 2 is written in the box, the result is 13, also a prime number. Upper-grade children are likely to conjecture that the sum is always a prime number. When they try 3, the sum is 17, also a prime. Similarly, the child finds that the numbers 4, 5, 6, 7, 8, 9, and 10, when written in the box produce a prime number. A child accustomed to forming generalizations from even fewer examples than this will likely conclude that this formula will always produce a prime number. It is instructive to note that when the next number, 11, is written in the box, the result is 121, which, being divisible by 11, is not a prime. This example illustrates the important idea that, even though the generalizations the child might make seem quite plausible and are most often true, it is only by means of a mathematical proof of a generalization that one can be completely sure that it is correct. These proofs, of course, are often not accessible to elementary school children. Thus, a healthy attitude might be characterized by references to generalizations which include phrases such as, "appears to be true," "is probably true," or "could most likely be proven."

Often a search for a generalization is initiated by a question such as, "Do you see any patterns?" For example, several simple generalizations might be formulated about the multiplication table in Figure 12. One child might observe that every number on the main diagonal of the table is a square number. Another student might observe that for every number on one side of this main diagonal, such as 10, there is a matching number symmetrically placed on the other side of the main diagonal.

×	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

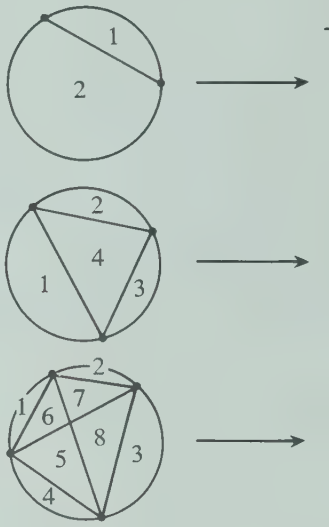
Figure 12

This last generalization is the table counterpart of the commutative principle for multiplication; that is, in the case of 12, $6 \times 2 = 2 \times 6$ or $4 \times 3 = 3 \times 4$. Another generalization that might be reached by careful consideration of the table is that the only primes in the table occur in the one-row or one-column of that table. Still another interesting generalization suggested by the table is that the sum of any number in the two-row and a number below it in the five-row will equal the number below these numbers in the seven-row. Of course, there are many other generalizations ranging from the very simple to the more complex that could be made about this multiplication table.

Perhaps the illustrations above will suggest that the mathematics available to the elementary school child is replete with possibilities for discovery of generalizations. The teacher's task is to create a learning environment in the classroom, not only in terms of physical materials and situations, but in terms of attitude toward learning and toward children, which provides opportunities for discoveries of generalizations and an atmosphere in which it is rewarding to make these discoveries. The teacher should be ever aware of the possibility that the habit of seeking generalizations may well be one of the most valuable things the child learns from his experiences in mathematics.

EXERCISE SET 7

1. Choose a text from the *Investigating School Mathematics* series and list some generalizations which the students who study this text might discover.
2. Investigate the Madison Project shoe boxes and complete the activities for at least two boxes.
3. The illustrations and the table which follow show that if you connect two points on a circle, you divide the interior of the circle into two regions; if you connect three points on a circle, you divide its interior into four regions; if you connect four points on a circle, you divide its interior into eight regions. Note that the points chosen should not be evenly spaced on the circle.



Number of points on a circle	Number of regions formed inside circle
2	2
3	4
4	8
5	
6	

   - **A** Fill in the table to show how many regions are formed if five points on a circle are connected.
   - **B** Form a generalization about the right-hand column of the table.
   - **C** Test your generalization by finding out how many regions are formed inside when six points on a circle are connected.
4. Devise an investigation which might enable a student to discover this generalization: "The sum of the degree measures of the angles of a quadrilateral is 360."
5. Write some questions you would ask and show some examples you would use in guiding a child to discover one of the following generalizations.
   - **A** The commutative principle for multiplication
   - **B** The volume of a "box" is found by multiplying length times width times height.
   - **C** In measuring length, the shorter the unit, the greater the measure.
   - **D** Any angle inscribed in a semicircle is a right angle.
   - **E** Every even number ends in 0, 2, 4, 6, or 8.
6. Complete the "Learning a Generalization" lesson on page I-20; then answer the following questions.
   - **A** What generalization did you learn from the lesson?
   - **B** How many specific examples did you consider before you understood the generalization?
   - **C** In what way did you use the generalization after you discovered it?

Facts

In elementary mathematics, there are certain bits of information that are used so frequently that it is

beneficial for the child to be able to recall them quickly when they are needed. These items are ordinarily called *facts*. There are three main types of facts that are of major concern. The first type of fact is one which evolves from a concept. It might be an example of a specific concept ("Two is a prime number," "25 is a square number," "A parallelogram is a quadrilateral"), or it might be a characteristic of a specific concept, possibly even a part of the definition for the concept ("An isosceles triangle has two congruent sides," "An even number is a number divisible by two," "A pentagon has five sides"). Examples of, or characteristics of, concepts are not always considered as facts; only if such an example or characteristic is deemed important enough to be remembered for immediate recall, is it considered to be a fact and committed to memory.

A second type of fact is a fact derived from a generalization; that is, if a generalization is simple, or deemed important enough to remember for immediate recall, it might often be considered a fact. For example: "The sum of the squares of the lengths of the legs of a right triangle is equal to the square of the length of the hypotenuse of the right triangle"; or "The length of the segment joining the midpoints of two sides of a triangle is one half the length of the third side." Each of these statements might be considered facts since they are sometimes useful for immediate recall. A third type of fact—one that is given a great deal of attention in the elementary school mathematics program—is the type of fact derived from a power skill. For example, the child may have utilized a sequence of power skills for finding sums such as $4 + 3$. He may have used sets of counters, centimetre strips, jumps on the number line, or reasoning from facts such as $3 + 3 = 6$. These power skills, based on certain important concepts, provided the evolutionary progression toward the final speed skill used in finding sums. In this particular case, however, the speed skill used is simply that of memorizing the sum. Whenever the speed-skill stage involves memorization, the particular learning which was classified as a skill or a process during the power-skill stage is reclassified as a fact. The basic addition and multiplication facts fall into this category, and they are given major attention in the elementary school. It is these facts to which primary attention will be given in this section.

A first important point to be made in discussing the teaching of facts is that extensive power-skill work preceding the memorization stage can pay valuable dividends. The broad base of understanding provided by the power-skill work removes the aura of magic from this aspect of mathematics and not only makes the task of memorization of the facts easier, but helps the child view it as a "reasonable thing to do." Figure 13, for example, shows some of the power skills that might be utilized in the initial development of procedures for finding products. Careful development using some or all of these power skills can give the child a basic feeling for a procedure by which products may be found.

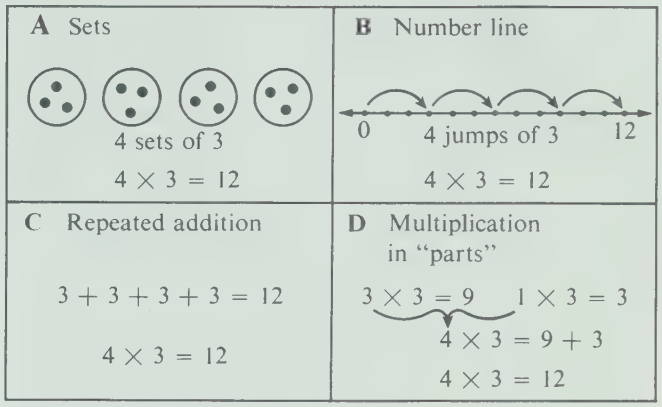


Figure 13

The teacher must use good judgment in deciding when a given child should move from this power-skill stage to memorization of the facts. The appropriate time could vary extensively depending upon the ability and experiences of the child. If the power-skill work is started early in the elementary grades, the child will have ample time to reap the benefits of this basic experience with materials and concepts before the transition to speed skill is made.

When the time has come to memorize the facts, it is important for the child to have a clear idea of the nature of this goal and the reasons it is appropriate. The teacher should even take time to help the child see the very clear difference between "figuring out the fact" and "memorizing the fact." Hopefully, he could help the child develop a feeling for situations in which the facts will be used and in which immediate recall would be quite valuable and time-saving for the child.

After the addition or multiplication facts to be memorized have been placed in perspective, the teacher should seek interesting situations and creative ways in which to practice recalling the facts. For example, the children might make their own flash cards and use a timer to see how long it takes them to give these facts. If desired, two children could work together and see which of them could go through the flash cards most quickly. Another game utilizes a pair of homemade colored dice and an empty multiplication table (Figure 14) for each child. As the game

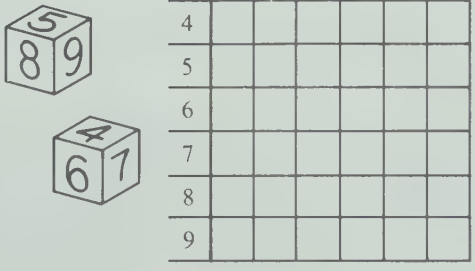


Figure 14

proceeds, a child rolls his dice and writes the product of the numbers on the dice in the appropriate space on his multiplication table. His partner then does the same thing when it is his turn. If a child arrives at an incorrect product or writes the product in the wrong space in the table, he is penalized by missing a turn. The object of the game is to see who can complete the table first. Various modifications of this game are possible, including one in which each child works independently and keeps a tally of the number of times he rolls the dice and also keeps track of the time it takes. The basic objective, of course, is to provide an interesting situation in which the child is motivated to recall multiplication facts rapidly.

Some children may need to spend considerable time in the power-skill stage before they begin to memorize. If there are children who have attempted to memorize the facts and find the job more difficult than anticipated, the teacher may want to consider allowing them to prepare a fact card on which they write the facts that they still do not know. Perhaps it would be realistic and beneficial to let some children use this fact card during the year whenever they desire, thus relieving the tension that could result from difficulties they encounter in memorizing the facts at one specific time. As the school year progresses, the teacher may want to suggest from time to time that a particular child concentrate on one of the troublesome facts and attempt to memorize it so that he can remove it from his fact card. The accomplishment of this goal, of course, would merit recognition and reward. After one fact is removed, the child might start working on removing another fact. The ultimate goal would be to remove all of the facts by the end of the year. Teachers who are interested in helping children learn mathematics in a comfortable way may find that a more realistic, less pressured approach to learning facts may enable the child to find greater enjoyment and success in his mathematical experience.

EXERCISE SET 8

1. Invent a game that could be used to help children practice recalling addition or multiplication facts.
2. Find a commercially produced game that is designed to help children practice recalling facts.
3. Complete the "Learning Some Facts" lesson on pages I-20 and I-21 of this text; then answer the following questions.
   - **A** How many of the facts did you know?
   - **B** What techniques did you use to help you memorize the remaining facts? Did you find this lesson difficult?
   - **C** Can you imagine some of the difficulties your children might have in learning facts?
   - **D** Did you find any mnemonic devices which were helpful in remembering the facts?

Attitudes

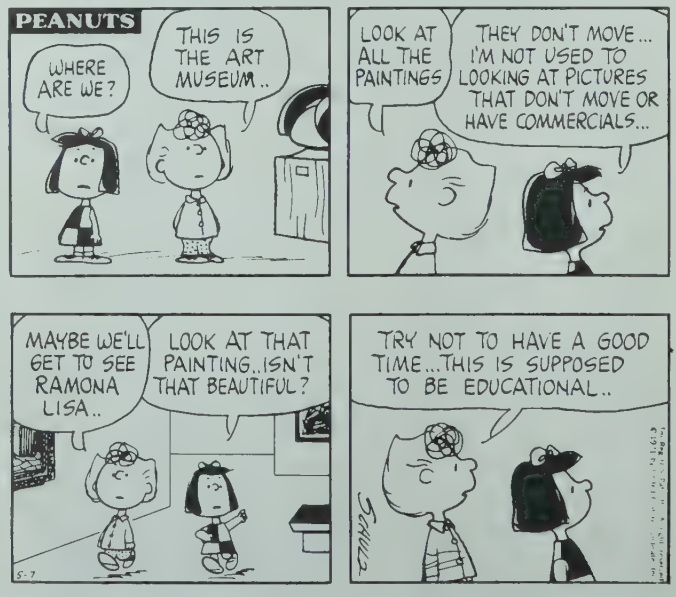
In his poem "Arithmetic," Carl Sandburg wrote:

> Arithmetic is numbers you squeeze from your head
> to your hand to your pencil to your paper
> till you get the answer.
> Arithmetic is where the answer is right
> and everything is nice
> and you can look out of the window
> and see the blue sky—
> or the answer is wrong
> and you have to start all over and try again
> and see how it comes out this time.
>
> . . . . .
>
> Arithmetic is where you have to multiply—
> and you carry the multiplication table in your head
> and hope you won't lose it.*

The attitude toward mathematics, school, one's ability, and learning in general that one senses on reading this part of Sandburg's poem is surely typical of many children in classrooms today. Perhaps, it was a feeling similar to this that caused Huckleberry Finn to say:

> I had been to school 'most all the time and could spell and read and write just a little and could say the multiplication table up to six times seven is thirty-five, and I don't reckon I could get any further if I was to live forever. I don't take no stock in math, anyway.

There are many different kinds of attitudes exhibited by children who have been exposed to classroom mathematical experiences in different parts of the world. There are, of course, the more general attitudes that a child has toward his teacher, toward his school, toward his fellow students, and toward the process of education. All too often the child's attitude toward education in general is that suggested by Charles Schulz in this *Peanuts* cartoon.



*From *Complete Poems*, copyright, 1950, by Carl Sandburg. Reprinted by permission of Harcourt Brace Jovanovich, Inc.

Two of the attitudes to be considered here, however, are the child's attitude toward mathematics and the child's attitude toward himself as he relates to mathematics. It has been said that the mathematical experiences of a child before the age of 11, and the responses he has been encouraged to make to those experiences, largely determine his potential mathematical development. If this is so, then a child's attitude toward mathematics and his feelings about how he relates to mathematics are extremely important considerations for the classroom teacher.

A moment's reflection on the number of people who are willing to say that they hate mathematics and on the multitude of others who seem to harbor a fear regarding their inability to cope with ideas of mathematics leads the teacher to realize that he does indeed teach attitudes, whether he tries to or not. Clearly, the teacher who conducts a classroom in which children's achievements are evaluated almost exclusively on the basis of how many right answers they can come up with must surely engender attitudes in children which differ greatly from those engendered by the sensitive teacher who recognizes the child's need to think his own thoughts and to become involved in an exciting exploration of ideas that interest him. Or, consider the difference between the teacher who teaches only speed skills and facts and the teacher who recognizes the central importance of concepts and generalizations, as well as the facts and skills. The child exposed to the first teacher must surely have a feeling toward mathematics, and his ability to interact with it, that is far different from that of the child who learned with the second teacher.

If what happens in the classroom is of such importance in developing attitudes within the child, then the teacher may want to reevaluate his approaches to instruction by reconsidering certain fundamental questions. What subject matter and methods most effectively instill within the child the feeling that mathematics is interesting, fun, and a source of adventure? Will these means provide an opportunity for the child to exercise his freedom of choice and to make decisions about what he does with mathematics? Aldous Huxley said: "A child is a genius until the age of ten." Could it be that our classroom approaches squelch this genius? Can we select mathematical experiences and materials that enable the children to experience success and thus maintain that sense of worthiness and prestige with peers that is of such importance? Can we structure these experiences in such a way that the child maintains within this atmosphere of freedom a sense of security and safety, thus avoiding the fear that can erode his ability to approach mathematical situations with confidence? Can we help children see the usefulness and importance of mathematics without boring them?

Clearly, the questions just raised are difficult to answer and specific techniques for developing healthy attitudes are hard to come by. But even though prescriptions for developing attitudes are scarce, many of the ideas about teaching suggested in earlier sections of this text can provide assistance for the teacher. The investigation, for example, provides the child with an opportunity to make independent decisions and to interact with mathematics and materials and encourages him to take responsibility for his own learning. As difficult as it may seem at times, a child's acceptance of responsibility for his own learning inculcates an attitude that is ultimately invaluable. Also, the manipulative materials or activities that are made available to the child in the investigation situation provide an interaction with the physical world that is often extremely valuable in making mathematics real to a student. Unless a child is ready for more abstract thinking, he cannot be induced to sense the adventure in mathematics without a physical environment to explore. Opportunities for attitude development are implicit not only in the investigation phase of a lesson but in the discussion as well. If a teacher can convince the child that his ideas are important, then the child finds himself in a situation, albeit a mathematical one, in which *he* feels important. His prestige with his peers increases and he feels successful. Exercises in the utilization phase of a lesson that begin simply and gradually increase in difficulty can also help the child feel that he can do mathematics on his own; and, of course, carefully selected extension activities can provide the child with a variety of opportunities to experience the fun of mathematics.

Not only do the phases of the learning experience provide unique opportunities of attitude development, but the particular types of learnings involved within these phases also have their effect. The teaching of concepts and generalizations provides the child with a feeling of power regarding mathematics, for when he experiences the thrill of discovering a concept or a generalization, or when he uses these to solve a problem, he is also developing a useful and wholesome attitude toward mathematics learning. He is developing a habit of reacting to a mathematical situation which will be invaluable when he later encounters mathematical situations possibly undreamed of today. Also, careful teaching of skills and facts can provide the child with that basic sense of security that comes simply from being able to do something or to remember something.

Figure 16

Regarding the child's level of confidence in his ability to cope with mathematical problems, one of the child's paramount needs is to experience success, and as mentioned previously, having entertaining experiences with mathematics might decrease the fear that can erode his confidence. To provide these experiences, the teacher might create in the classroom a "Fun with Mathematics" centre (see Figure 16) that contains mazes, puzzles, design materials, and so on. This centre represents an extra effort to encourage the child to successfully play with mathematics. Some of the materials that might be in such a centre are as follows: the soma cube, the tangram pieces, 2-cm cubes, materials for curve stitching, a kaleidoscope, pattern blocks, Cuisenaire rods, multi-base arithmetic blocks, geoboards, a wide variety of counters, attribute blocks, scales and balances, timers, calendars, measuring tapes and rulers, yarn and string, an assortment of boxes and cans, magazines and catalogues, mirrors, dice, play money, graph paper, assorted plane and solid shapes, abacus, pegboard, compass, mathematical balance, etc.

Perhaps, as you consider the attitudes more carefully and reevaluate the effects of your approaches to instruction, you will find other ways to help children develop a healthy attitude toward mathematics and an enthusiasm for the enjoyment it can offer. Each day as the teacher enters the classroom with plans for a learning experience, he might well ask himself: "What effect will *this* have on the attitudes of the students in my classes?"

EXERCISE SET 9

1. Select a text from the *Investigating School Mathematics* series and find at least five activities which could contribute to the child's development of a positive attitude toward mathematics.
2. Explain how you think some of the other types of learning might also contribute to better child attitude toward learning in general and mathematics specifically.
3. Complete the "Learning an Attitude" lesson on page I-21 of this text; then answer the following questions.
   - **A** Was the lesson fun?
   - **B** How did you feel when you had finished the lesson?
   - **C** Did the lesson change any of your ideas about mathematics?

IV. Some Learning Experiences for the Teacher

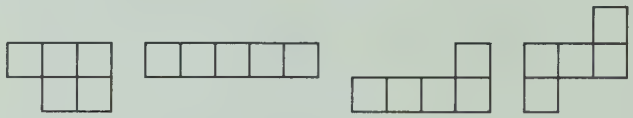
In Section II you were introduced to an outline for a learning experience which involved preparation, investigation, discussion, utilization, and extension. In Section III the types of things children learn—concepts, skills, generalizations, facts, and attitudes—were categorized. In this section, we combine these ideas and use them in presenting five learning experiences designed especially for the teacher. That is, in order to gain a first-hand view of lessons which develop these types

Lesson 1. Learning a Concept

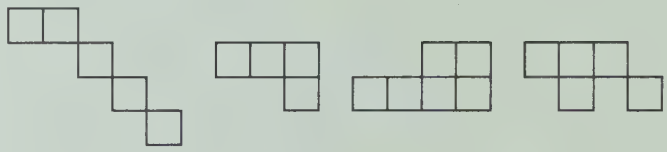
What is a pentominoe?

Investigating the Ideas

Each of these is a **pentominoe.**



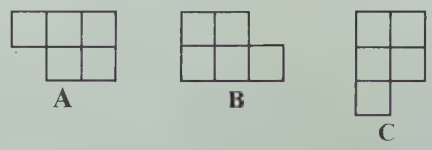
None of these is a **pentominoe.**



How many more pentominoes can you find and show on graph paper?

Discussing the Ideas

1. How many pentominoes did you find?
2. Can you give some characteristics of a pentominoe?
3. How would you "broadly classify" a pentominoe?
4. Can you define a pentominoe?
5. Are the pentominoes in Figures A, B, and C the same?



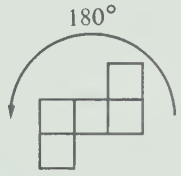
6. How could you convince someone that you have found all possible pentominoes?

of learning, the teacher will have experiences with each of these in the five lessons; and, in order to become more familiar with the suggested structure for a learning experience, each of these five lessons will involve an investigation, a discussion, a utilization, and an extension of the ideas.

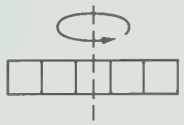
It might be valuable for the teacher, after he has become involved in each of these lessons and has completed the activities, to rethink and discuss his reactions to the various phases of the lesson structure and to the various types of learnings involved. In this way, he might gain a new insight into the way the children in his classes might react to these kinds of situations.

Using the Ideas

1. Which of the pentominoes can be folded to form a box with the "lid missing"?
2. Some pentominoes can be rotated about a point 180° and returned to their starting position. These pentominoes are said to have 180° rotational symmetry. Which pentominoes have 180° rotational symmetry?



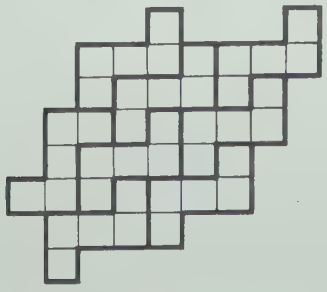
3. Some pentominoes can be flipped about a line and returned to their starting position. Such pentominoes are said to have reflectional symmetry. Which pentominoes have reflectional symmetry?



4. What do you think a hexominoe would be? How many hexominoes can you find?

Extension

Some pentominoes can be used to tessellate (fill without overlapping) the plane, as shown below. Can you find at least two more pentominoes that can be used to tessellate the plane? Show the tessellations on graph paper.

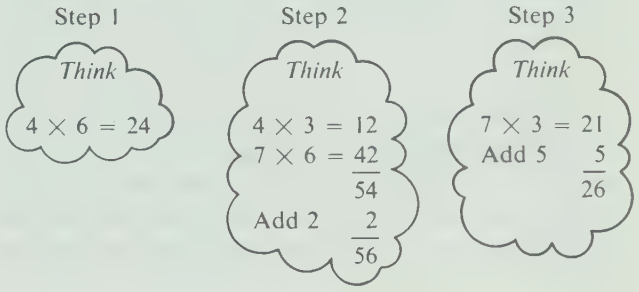


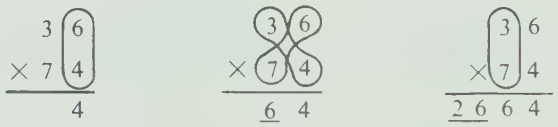
Lesson 2. Learning a Skill

Can you find the product of two 2-digit numbers "in your head"?

Investigating the Ideas

Follow these steps for writing the *answer only* for 74×36.





Can you use this method to write answers only for the products below? Check your answer using the "long" method.

53	37	45	67
×48	×62	×23	×32

Discussing the Ideas

1. Explain this statement: In Step 1 you are finding the number of ones.
2. In Step 2 you are finding the number of ?.
3. The 2 you remembered is really 2 ?.
4. Explain what you are finding in Step 3.

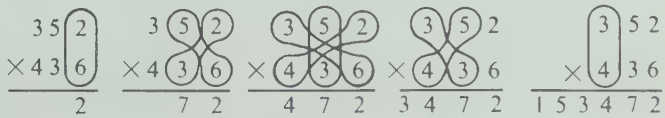
Using the Ideas

Write answers only for each product.

1. 28 ×42	**2.** 46 ×33	**3.** 37 ×42	**4.** 82 ×56	**5.** 53 ×34
6. 64 ×27	**7.** 29 ×63	**8.** 48 ×35	**9.** 53 ×53	**10.** 27 ×64

Extension

1. Study the figures below for finding the product of two 3-digit numbers.



2. Use the method shown in exercise 1 to find each product.

125	757	841	525
×365	×426	×215	×525

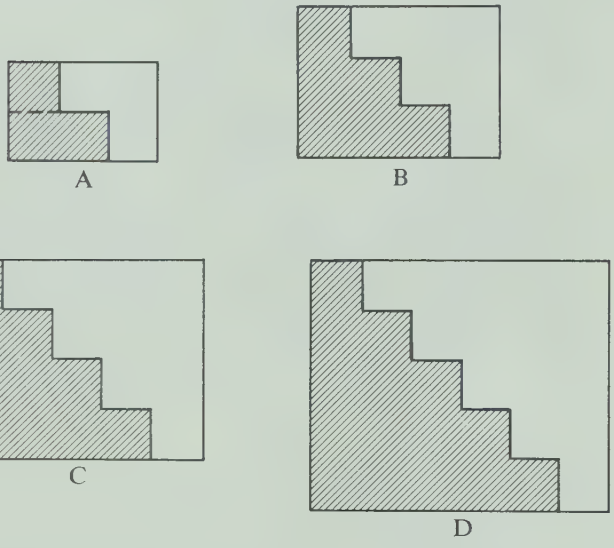
*3. Devise a rule for multiplying two 4-digit numbers.

Lesson 3. Learning a Generalization

Can you find a pattern?

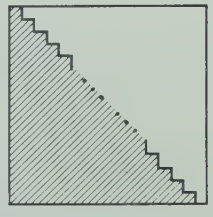
Investigating the Ideas

Use the small square in Figure A as the unit. Can you find the area of each shaded part in two different ways? For each part, write an equation to show that the two ways of calculating the area give the same result.



Discussing the Ideas

1. **A** Describe one way you found for finding area in the figures above.
   B Describe another way you found.
   C Did you find any other way?
2. Can you write an equation to show that these two methods give the same area?
3. **A** Suppose there are 50 vertical segments in the "stairsteps" of Figure E. What is the area of the shaded part?
   B Which of the two methods for finding the area would you use?
   C Can you write an equation about this?



4. Can you find the area of the shaded portion of Figure E if there are 100 vertical segments?
5. Can you use what you have learned so far to explain this generalization?

$$1 + 2 + 3 + 4 + 5 + \ldots + n = \frac{n \cdot (n+1)}{2}$$

Using the Ideas

1. Without adding each number, find the sum of the whole numbers through 25.
2. Find the sum of the first 75 whole numbers.
3. Find the sum of the first 200 whole numbers.
4. What is the sum of the first 1000 whole numbers?

Extension

1. What is this sum? $50 + 51 + 52 + 53 + \ldots + 99 + 100$
2. Can you find a short way to find the sum of
   A these even numbers? $0 + 2 + 4 + 6 + 8 + \ldots + 100$
   B these odd numbers? $1 + 3 + 5 + 7 + 9 + \ldots + 99$

*3. Can you state a rule for what you found in exercise 2 by using a variable?

Lesson 4. Learning Some Facts

Can you learn some "new" facts?

Investigating the Ideas

Many rapid "human Calculators" consider these products to be facts.

×	10	11	12	13	14	15
10						
11						
12						
13						
14						
15						

How many of these "facts" can you give without calculating?

(Record the facts you know and shade that portion of the table with a red pencil. Then fill in the remainder of the table by figuring out the remaining facts.)

Discussing the Ideas

1. Which facts in the table need not be memorized, provided you know the others and also know the commutative principle? Shade these facts blue.
2. **A** How many facts altogether are in the table?
   B How many facts remain to be memorized?
3. **A** What is the "largest" fact?
   B Which facts are over 200?
   C Which facts are in the 190's?
   D Do you notice other patterns in the table that might help you remember certain facts?

Using the Ideas

1. Give these products as quickly as possible.

A 15×15	**E** 13×13	**I** 11×13
B 15×14	**F** 14×12	**J** 11×14
C 14×14	**G** 11×11	**K** 11×15
D 13×15	**H** 11×12	**L** 12×13

2. Make flash cards for the "facts" in exercise 1 that you do not know. Practice with a friend.
3. In exercise 1, start with part L and, following reverse order, give each of the products as quickly as possible.

*4. Make a large multiplication table with all numbers up to 20. Mark out the "facts" you know. How many of these "facts" are left to memorize?

*5. A person who knew the distributive principle and the facts in the table referred to in exercise 4 looked at the multiplication 143×15 and wrote 2145. How did he do it so quickly?

Extension

Study the facts for these powers of 2.

$$2^2 = 2 \times 2 = 4$$
$$2^3 = 2 \times 2 \times 2 = 8$$
$$2^4 = 2 \times 2 \times 2 \times 2 = 16$$
$$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$$

1. Give the next six powers of 2.

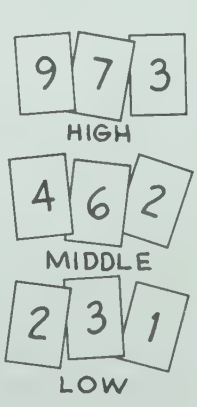
*2. Can you find some mnemonic aids to help you memorize the first ten powers of 2?

Lesson 5. Learning an Attitude

Let's try a place-value game.

Investigating the Ideas

Use 3 sets of 9 cards, each with the digits 1 through 9. Shuffle the 27 cards and deal 3 to each player. Each player then forms a 3-digit numeral, places his cards face down in order, and declares (starting to dealer's left and rotating clockwise) whether his number is high, middle, or low. Play the game in groups of three players.



Discussing the Ideas

1. One player arranged his cards like this and declared that he would try for the low hand. What was wrong with his strategy?

2. What is wrong with this arrangement for a middle hand?

3. If you were dealt these cards, would you try for a high or low? Why?

4. Suppose you are last to declare. Everyone else has declared either low or middle. What would you do with these cards?

Using the Ideas

1. Try playing this game with 2 or more other people.
2. Try the game with the rule that you can declare only high or low.
3. Make up rules for a game in which you turn up the cards one at a time starting with the ones' digit card.

Extension

1. Invent a place-value game in which 4 or 5 cards are dealt to each player.

*2. Find or invent another game or activity that strengthens understanding of the concept of place value.

V. Some Thoughts About Evaluation

The strategy of preparation, investigation, discussion, utilization, and extension is a flexible organizational plan that allows each teacher an opportunity to make a modest beginning toward an activity-oriented mathematics program. The lesson categorization of concept, skill, generalization, fact, and attitude provides a framework that allows each teacher an opportunity to apply the teaching strategy to various types of learning situations. Since there are different types of learning, it is reasonable to assume that there should be different types of evaluation used to measure these learnings.

When considering the facts and skills, for example, emphasis should be placed on child accountability. The teacher should determine the learning outcomes, consider performance objectives for these outcomes, and help the child attain these objectives. The evaluation of this attainment is most easily completed by use of fact and skill tests which determine the child's level of achievement. Since the child needs considerable practice in remembering facts and performing skills, the procedure for helping them is reasonably straightforward.

When evaluating concepts, generalizations, and attitudes, however, the desired performance objectives are often quite difficult to verbalize. We have mentioned earlier that concept learning often takes place

over a relatively long time span, that concepts are extended and broadened, and that concepts mature with each subsequent set of related experiences. Clearly, it is difficult to write a performance objective which specifies the exact level of concept maturity appropriate for a given child at a given time. Whenever possible, objectives for simple concepts should be written, and an attempt should be made to write test items which will show whether children understand these concepts. These items should involve requests for children to give examples of concepts, characteristics of a concept, and even, in some cases, a definition of the concept. For more difficult concepts, the evaluation of children's progress might be made through observation and recorded by means of a check-list which specifies certain levels of development for the given concept. The teacher should be alert for situations in which the child actually uses the concept correctly and should recognize also that understandings which are only partially developed indicate positive achievement. The teacher should also search for instances where the child has shown an ability to form concepts, for this is one of the desired learnings.

When evaluating the child's understanding of generalizations, the teacher should specify the simple generalizations which should be learned by all children. Specific performance objectives and the subsequent test items should be written to evaluate these generalizations. Beyond this, the teacher should again evaluate in greater depth through personal observations or interviews with the children. In the area of generalizations, the teacher should be ever aware that a child who is in the habit of looking for patterns or generalizations has learned a great deal. The teacher should also recognize that a child who can form a generalization from a sequence of specific examples has developed an understanding of a process that is extremely important. We would be remiss if we evaluated only the factual part of the learning of generalizations. As noted earlier, however, although these are important goals of mathematics learning, it is very difficult to write performance objectives for these goals. Whenever possible, objectives should be written which go deeper than facts and skills, but in the absence of objectives, the teacher should feel free to use other means of evaluation, including interviews to evaluate student learning.

While attitudes are not easy to measure in a conventional way, it is suggested that teachers frequently observe children and talk to them about their feelings about mathematics. It is important to realize that one's philosophy toward testing can also have a marked influence on the child's attitude toward mathematics. Testing should be reasonable and realistic, and the child should understand its purpose. The spirit of evaluation should be one of helpful assessment, rather than of critical evaluation. If children participate with teachers in understanding (if not in developing) the goals of instruction, the testing procedure can be a positive influence on the child's attitude and ability to improve.

It is hoped that the teacher will constantly take a broad view toward evaluating mathematics learning among his children. In the long run, evaluation of a child's learning should depend upon the interaction of that child and his teacher. For this interaction to be successful, it may be necessary for the teacher to reexamine his own beliefs about how children learn mathematics. As each teacher makes modest beginnings toward an activity-oriented approach to mathematics learning, he might ask himself the following questions:

1. Do I respect each child as an individual with unique interests, abilities, sensitivities, and significant thoughts?
2. Does the learning environment of my classroom provide a natural, free atmosphere in which children can explore, make decisions, be independent, and encounter exciting new experiences?
3. Does the learning experience also include a supportive, non-judgmental atmosphere in which children have enough routine activities to provide a comfortable threshold of security?
4. Is the child's need for earned success recognized in my classroom?
5. Do I recognize and treat mathematics as a dynamic, ever-growing discipline which offers limitless new vistas to be explored and an inexhaustible variety of new problems to be investigated and solved?
6. Do I view mathematics as a subject of beauty and a source of pleasurable fulfillment of intellectual curiosity?
7. Do I appreciate the significance of my role as a fellow-learner rather than merely a source of information?
8. Is my overall attitude toward mathematics one that encourages a basic freedom to learn through use of manipulative materials in an investigative environment, and through free discussion and exchange of ideas?

As a teacher evaluates the children in his class, he should also reevaluate his approach to mathematical learning. The goal of this short text has been to help in that reevaluation by encouraging the teacher to read, study, observe, experience, experiment, and reconsider. If that goal has been achieved, perhaps his resulting basic beliefs about children, mathematics, and evaluation methods will help him create a new climate of interaction that will spark more effective learning experiences in his classroom.

EXERCISE SET 10

1. Give a set of performance objectives for each lesson completed in Section IV.
2. Create an evaluation tool for each set of behavioral objectives given in exercise 1.

General Bibliography

Biggs, Edith, and James MacLean, *Freedom to Learn: An Active Learning Approach to Mathematics* (Don Mills, Ontario: Addison-Wesley, 1969).

Boyer, Carl B., *A History of Mathematics* (New York: John Wiley & Sons, Inc., 1968).

Copeland, Richard, *How Children Learn Mathematics: Teaching Implications of Piaget's Research* (New York: The Macmillan Co., 1970).

Dienes, Z. P., *Building Up Mathematics* (London: Hutchinson Educational Ltd., 1960).

Elliott, H. A., James MacLean, and Janet Jorden, *Geometry in the Classroom: New Concepts and Methods* (Toronto, Ontario: Holt, Rinehart and Winston of Canada, Ltd., 1968).

Forbes, Jack, and Robert Eicholz, *Mathematics for Elementary Teachers* (Menlo Park, Cal.: Addison-Wesley, 1971).

Members of the Association of Teachers of Mathematics, *Notes on Mathematics in Primary Schools* (New York: Cambridge University Press, 1967).

National Council of Teachers of Mathematics, *Insights into Modern Mathematics* (23rd Yearbook, 1957); *The Growth of Mathematical Ideas,* Grades K–12 (24th Yearbook, 1959); *Enrichment Mathematics for the Grades* (27th Yearbook, 1963); *Topics in Mathematics for Elementary School Teachers* (29th Yearbook, 1964); *More Topics in Mathematics for Elementary School Teachers* (30th Yearbook, 1969), Washington, D.C.: National Council of Teachers of Mathematics).

Newman, J. R., *The World of Mathematics* (New York: Simon and Schuster, 1956).

Nuffield Mathematics Project, *I Do, and I understand* (New York: John Wiley & Sons, Inc., 1967).

School Mathematics Study Group, *Studies in Mathematics, Vol. IX, A Brief Course in Mathematics for Elementary School Teachers,* Revised Edition (Stanford University, 1963).

The Schools Council, *Mathematics in Primary Schools* (Curriculum Bulletin No. 1. Available from Selective Educational Equipment, Newton, Mass., 1964).

Williams, E. M., and Hilary Shuard, *Elementary Mathematics Today, Grades 1–8* (Menlo Park, Cal.: Addison-Wesley, 1972).

Bibliography of Resources for Active Learning

Abbott, Janet, et al., Franklin Mathematics Series: *From Fingers to Computers; Learning About Measurement; Learn to Fold—Fold to Learn; Making and Using Graphs and Nomographs; Mirror Magic; Patterns and Puzzles in Mathematics; Pencil and Paper Geometry* (Chicago: Lyons and Carnahan, 1970).

Bates, John, Donald Irwin, and Garry Hamilton, *Developmental Math Cards* (Don Mills, Ontario: Addison-Wesley, 1970).

Clarkson, Dave, *Math Activity Cards* (New York: Macmillan Co., 1969).

Cohen, Donald, *Inquiry in Mathematics via the Geo-Board* (New York: Walker, 1967).

Cohen, Donald, *Maths Mini Lab* (Newton, Mass.: Selective Educational Equipment, 1971).

Davis, Robert (Madison Project), *Discovery in Mathematics: A Text for Teachers; Student Discussion Guide* (Menlo Park, Cal.: Addison-Wesley, 1964).

Dienes, Z. P., *Multibase Arithmetic Blocks,* Tasks and Manual (New York: Herder and Herder, 1961).

Elementary Science Study: *Attribute Games and Problems; Mirror Cards; Tangrams* (St. Louis: Webster Division, McGraw-Hill Book Co., 1968).

Fletcher, Harold, Arnold Howell, and Ruth Walker, *Mathematics in Modules* (Menlo Park, Cal.: Addison-Wesley, 1973).

Galton, Grace, Arlene Fair, and Patricia Davidson, *Chip Trading Activities,* Set I (Fort Collins, Colorado: Sigma, Scott Scientific, 1970).

Huff, M. Elizabeth, Donald Irwin, *Activities in Geometry* (Don Mills, Ontario: Addison-Wesley, 1973).

Knaupp, John, and Gary Bitter, *Mathematics Activities Kit* (Menlo Park, Cal.: Addison-Wesley, 1972).

Mathex: Matching and Graphing No. 1; Numeration No. 2; Operations No. 3; Geometry No. 4; Measurement and Estimation No. 5; Graphing and Probability No. 6; Numeration No. 7; Operations and Problem Solving No. 8; Geometry No. 9; Measurement No. 10 (Toronto, Ontario: Encyclopaedia Britannica Publications Ltd., 1970).

Nuffield Mathematics Project: *Environmental Geometry; Mathematics Begins* 1; *Computation and Structure* 2, 3, 4; *Beginnings* 1; *Shape and Size* 2, 3; *Pictorial Representation* 1; *Graphs Leading to Pictorial Representation* 1; *Graphs Leading to Algebra* 2 (New York: John Wiley, 1967–1969).

Turner, Ethel, *Teaching Aids for Elementary Mathematics* (New York; Holt, Rinehart and Winston, 1966).

Wirtz, Robert, Morton Botel, and Max Beberman, *Toward Improving Computation* (Washington, D.C.: Curriculum Development Associates, 1970).

Wirtz, Robert, et al., *Math Workshop:* Games and Enrichment Activities (Chicago: Encyclopaedia Britannica Educational Corp., 1964).

Wirtz, Robert, Morton Botel, and B. G. Nunley, *Discovery in Elementary School Mathematics* (Chicago: Encyclopaedia Britannica Educational Corp., 1963).

INTRODUCING THE METRIC SYSTEM

Canada is committed to the metric system of measurement. You may be aware of this but may not have a clear idea of exactly what the metric decision means to you as a *teacher*. It is hoped that this section will serve three purposes—

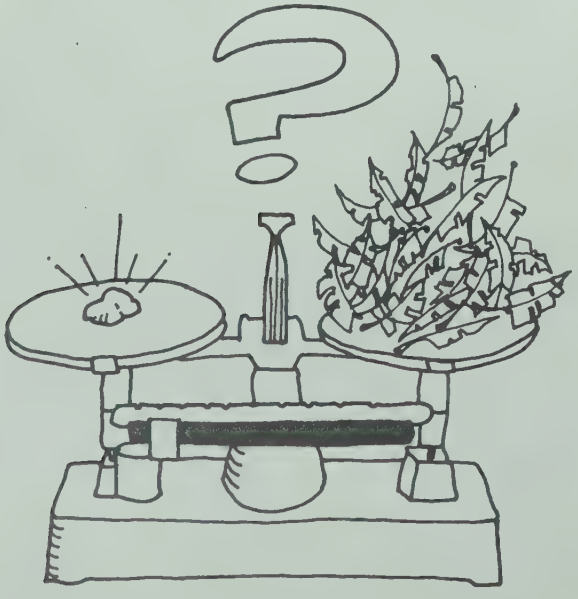
1. give you an idea of how the metric decision will affect you,
2. help you understand the metric system of measurement, and
3. give you some hints for teaching the metric system of measurement to your students.

History and Rationale

The English system of measurement developed from man's need to measure size and distances using units from the most readily available object—himself. He utilized his palm, span, finger, an ell, and a fathom for length; his foot, step, pace, an arrow's flight, and a day's journey for distance; and a handful, shellful, hornful, or gourdful for capacity.

There was little need for standardization until man began to travel and trade with other men. When "standard units" were developed, a new problem arose. Different countries used different definitions for the same unit. The foot was, at first, the length of any man's foot. In some countries, it was the length of the king's foot (since he was the "ruler") and this foot could change as the "rulers" changed. Later an effort was made to standardize some units; for example, England and Scotland decreed the foot to be 12 inches. Unfortunately, England and Scotland didn't use the same definition for the inch.

Today, in the age of technology, one still finds different units in those countries which are not yet metric. Canada and the United States are neighbouring countries, yet they use two different definitions for the gallon. A question at which people in metric countries must laugh is "Which is heavier, a pound of gold or a pound of feathers?" A pound of feathers is heavier since feathers are weighed by the avoirdupois pound (1 avoirdupois pound—7 000 grains) and gold is weighed by the troy pound (1 troy pound—5 760 grains). Which is heavier, an ounce of gold or an ounce of feathers? An ounce of gold is heavier. There are 12 ounces in the troy pound, so one ounce of gold weighs 480 grains; there are 16 ounces in the avoirdupois pound, so an ounce of feathers weighs 437.5 grains.



Out of such confusion there developed a need for a simple, standardized system of measurement. In 1670 Gabriel Mouton, a French abbé, developed a system of measurement organized according to the decimal system of numeration. It took over a hundred years for a system of measurement like the one Mouton put forth to get official sanction. In 1790 the French National Assembly appointed a committee to study the measurement situation and see if a rational system of measurement was possible. In 1795 France adopted a decimal system of measurement, defining the base unit of length to be the *metre* (from the Greek word *metron*, "a measure").

The metric system did not use parts of the human body as units. The metric system did not develop haphazardly adding more and more units as the need arose. The metre was defined as one ten-millionth of the distance from the North Pole to the equator, along the meridian passing near Dunkirk, Paris, and Barcelona. One can see that such a definition would be difficult to replicate in any one country. Also, the length of the metre changes as the position of the North Pole changes; at the time that the metre was defined, scientists were unaware that the position of the North Pole changed.

In 1870, because of the problem of replicating and comparing metric units from country to country, France called a meeting of the metric countries to develop a "unified metric system of measurement". In 1875, the *Treaty of the Metre* was signed to establish the General Conference on Weights and Measures which meets to determine the official definitions for the units used in the metric countries. In 1960 the Conference adopted the Système International des Unités (SI). It is this SI metric system that is most used throughout the world.

A Popular System

The popularity of the metric system stems from two characteristics—the high degree of standardization and its simplicity.

In the entire metric system there are only seven base units! They are **metre** (length), **kilogram** (mass), **second** (time), **ampere** (electric current), **degree kelvin** (thermodynamic temperature), **candela** (luminous intensity), and **mole** (amount of substance).

All units used in the metric system are related to these seven base units. The units you will be most concerned with (because they are the ones used in everyday living) appear in Table 1:

Table 1: Metric Units to be Studied

Quantity	Unit	Symbol
Length	metre	m
Mass	kilogram	kg
Capacity	litre	ℓ*
Temperature	degree Celsius	°C

*As a rule of thumb, the cursive letter (ℓ) is used as a symbol for the litre to avoid confusion with the numeral (1), however, in symbols such as ml (millilitre), kl (kilolitre) the cursive form is not used.

All other units to be discussed can be represented by the product of one of the units and a power of 10. For example, every possible unit of length can be developed by multiplying the number of metres by the appropriate power of 10.

Table 2: Metric Units of Length

Name (Symbol)	Metres
*kilometre (km)	10^3m or 1000 m
hectometre (hm)	10^2m or 100 m
decametre (dam)	10^1m or 10 m
*metre (m)	10^0m or 1 m
decimetre (dm)	10^{-1}m or $\frac{1}{10}$ m
*centimetre (cm)	10^{-2}m or $\frac{1}{100}$ m
*millimetre (mm)	10^{-3}m or $\frac{1}{1000}$ m

*preferred units

To make the system simpler the same prefixes are used with all units. For example, a millimetre (mm) is $\frac{1}{1000}$ of a metre, a millilitre (ml) is $\frac{1}{1000}$ of a litre, a milligram (mg) is $\frac{1}{1000}$ of a gram, etc.

According to the class, you may want to introduce the symbol "m" for metre, "cm" for centimetre, etc. The plurals, metres and centimetres, are also symbolized "m" and "cm", not "ms" or "cms." Remember, these are symbols and not abbreviations and no period is used after a symbol.

Countries which have been completely metric for several years find that some terms such as "decimetre" are not used in everyday living. People will talk of a book being 28 centimetres long rather than 2.8 decimetres long. You may wish to explain the term "decimetre," but it is not necessary.

Most people who feel that the metric system is complex are those who convert back and forth between the metric and English systems of measurement. When teaching the metric system, conversion to the English system is not necessary and should be avoided!

The metre is defined world-wide to be 1 650 763.73 wave lengths in a vacuum of the orange-red line of the spectrum of krypton 86. This is quite a definition! There are two reasons why such a complex definition was adopted –

1. the length never varies and

2. this measurement can be replicated in laboratories throughout the world.

From this brief history of the metric system it is hoped you will take three main thoughts –

1. The metric system resulted from concentrated effort to develop a rational system of measurement. It did not develop haphazardly.

2. The problem of standardization has been solved in the metric system.

3. The metric system is both popular and useful because of its simplicity.

Activities

Experience and activity are key words in the teaching of measurement. Measure things! The success of this material will depend upon the amount of experience each participant has with the activities. The limited number of activities that are presented should stimulate possibilities for many more. Although the content is approached through activities and measuring experiences, there is a need for exercises to further these experiences and to structure metric thinking. Two points should be emphasized –

1. It is *important* that *you* as well as your class do the activities in this section.

2. The activities will be more fun if done in a group situation.

Looking at Table 1 in the *History and Rationale* section, you will notice that you have to be concerned with only four base units. So, let's use the frontal attack, start right in on length, and begin inching our way down the metric road.

Length, Area, and Volume

In the groups where the metric system has been argued for years, there were two camps. One group wanted to use the centimetre, gram, and second for the core of the system and the other the metre, kilogram, and second. The latter group has prevailed.

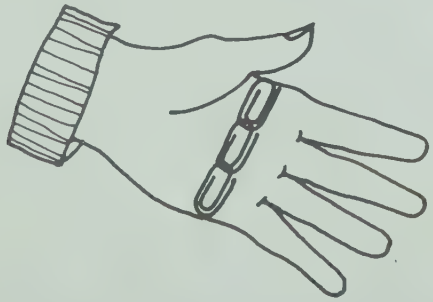
It is strongly urged that first grade teachers **not** start with the metre. It is very difficult for first graders to handle a metre ruler. The same argument may be advanced for the kilogram and litre. Length will be approached as it should be covered with students, i.e., first measure with arbitrary units, then use the centimetre, next use the 10-centimetre (decimetre), and finally the metre. All measurement should be approached as a three step process—

1. Select a unit.
2. Partition the object to be measured into units.
3. Count the number of units used. That number is the measure of the object.

ACTIVITY 1

Measuring objects with an arbitrary unit. Students should do several activities of this type using arbitrary units such as their thumb, a paper clip, pencil, crayon, cutout of their shoe, width of their hand (a unit in the English system used for measuring the height of horses), cubit (another "English" unit, the length of the forearm from the elbow to the tip of the middle finger), or other selected units. For your experience measure the chalk eraser, the width of your hand, the width of this book, and the length of a pencil using a paper clip as the unit.

In the illustration, a "paper clip train" is being used to measure the width of a hand. Follow the three steps mentioned previously in the measurement process.



Record all answers. Then measure the object again using pieces of paper the length of a thumbnail. Repeat the process measuring other objects.

In class emphasize four points—

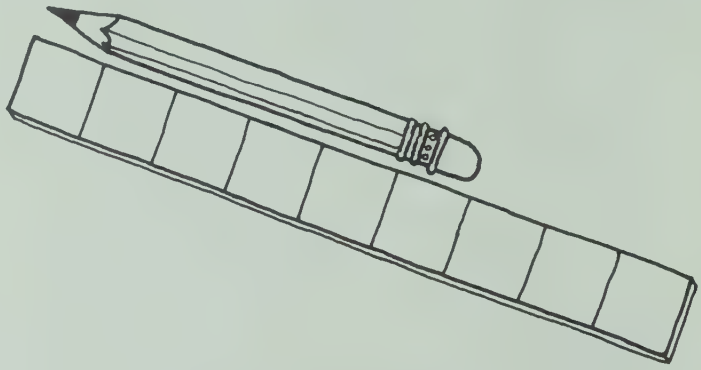
1. The first unit should be lined up with the "starting point" of the object.
2. The units should touch, but not overlap.
3. The "train" should be straight.
4. The units should be "rounded off" to the unit that has its right end nearest to the "finishing point" of the object.

In doing activities where arbitrary units are used, the need for standardized units becomes obvious. Ask several children to measure the same object, each with his own pencil. On the chalkboard, place their statements such as "The table (or whatever object you pick) is 5 pencils wide." "The table is 7 pencils wide." "The table is 8 pencils wide." Children will soon see that when pencils of differing lengths are used, different answers will result.

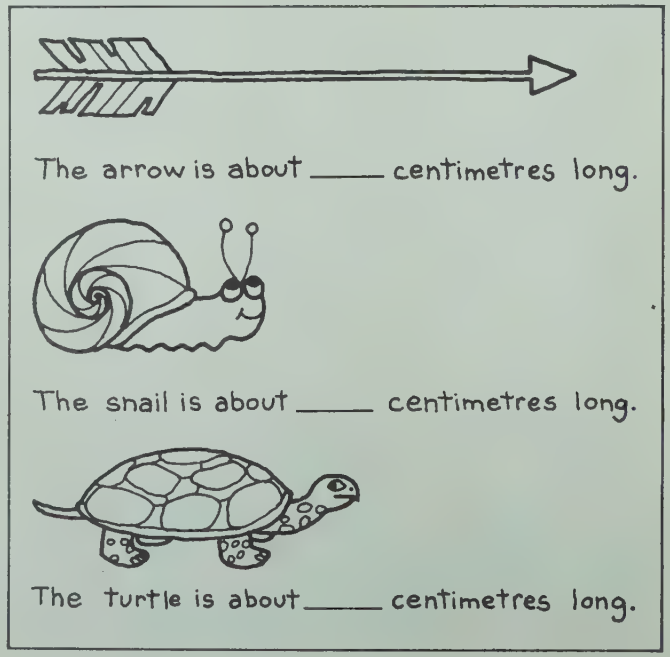
ACTIVITY 2

Developing the concept of a centimetre. Probably the first metric unit the children will make use of is the centimetre. You will need (and each student in the class will need) 9 centimetre strips—9 pieces of paper or cardboard 1 cm by 1 cm square.

The children, especially the younger ones, should have the experience of measuring many objects using centimetre strips. (If at the time you present this activity your students have studied two-digit numbers, have them measure objects longer than 9 cm.)



Using the centimetre strips, measure the length of a paper clip, a piece of chalk, the Cuisenaire 6-rod, the width of a hand, and the width of a thumb to the nearest centimetre. In this initial activity, actually use centimetre strips and not a ruler marked in centimetres. An exercise the children can do at their desks is to measure the pictures of objects drawn on a duplicator master. The pictures can be of predetermined length. Measure the pictures below.



In exercises like these, the length can be controlled. Some answers should require "rounding up," and some "rounding down." The word "about" is important in the sentence since a measurement is an approximation. As the children progress you can have them write not only the number but also the name of the unit.

ACTIVITY 3

Measuring with centimetre rulers. When the children have learned to use the centimetre strips in the measurement process, a ruler marked off in centimetres (not millimetres) should be introduced. It is strongly urged that the child construct his own 10-cm ruler during his first introduction to metric measure. He can do this by constructing a 10-cm train on a 10-cm long piece of paper, pasting the train on the paper, then numbering the cars from 1 to 10. Another approach is to construct a 10-cm ruler in front of the class. Then hand out 10-cm long pieces of paper already marked off in centimetres and have the children number the centimetres from 1 to 10.

The next few activities should involve the measuring of an object with a centimetre train, a 10-cm ruler, and finally with only a 10-cm ruler. When measuring an object with a 10-cm ruler work toward getting your students to "read the ruler" rather than counting the centimetres as they did with the trains.

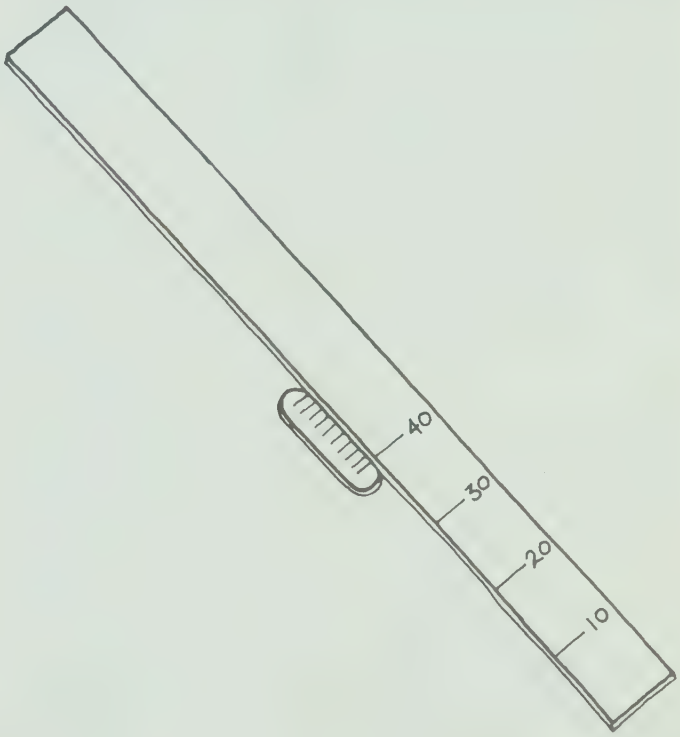
In the example illustrated the child should learn to round off to the nearest centimetre and then read the ruler, "8 centimetres," instead of counting "1, 2, 3, 4, 5, 6, 7, 8 centimetres."

After the children have become skilled in using a 10-cm ruler, they should be given activities requiring them to measure objects which are longer than 10 cm. When working with 5-and-6-year olds, be careful that the measure of the object is not a number the children haven't studied. In the activities concerning measurement it is the process that should be emphasized; the numbers themselves should never be a source of difficulty.

Now, using your 10-cm ruler, measure the length and width of this book and length of your forearm, the length of your foot, and length of your span (what is your span?).

ACTIVITY 4

The metre and notation. Initially, you may want to have your students measure objects with metre-long strips of unmarked cardboard. Then ask them to number the centimetres on the metre strip in groups of 10 using their 10-cm strips. Before proceeding



further, have the class subdivide these cardboard metre rulers into centimetres. It is important that you do the activities with the same type of ruler your students will use. If you have a classroom set of wooden metre rulers, use one of them. Ideally, the rulers used should be marked off in centimetres, but if the ruler is marked off in centimetres (cm) and millimetres (mm) no harm is done. Measure the length, width, and height of your desk rounding off to the nearest metre.

The measurements for a desk, accurate to the nearest metre, might be 2 m long, 1 m wide, and 1 m high. Such measurements would not be helpful. The metre is used for much longer measurements, such as the length and width of the classroom, the playground, the school, the block, etc. To measure the dimensions of objects such as desks, tables, bookshelves, and people, a metre ruler may be used and the results recorded in centimetres. For example, a desk may be 152 cm long, 76 cm wide, and 74 cm high.

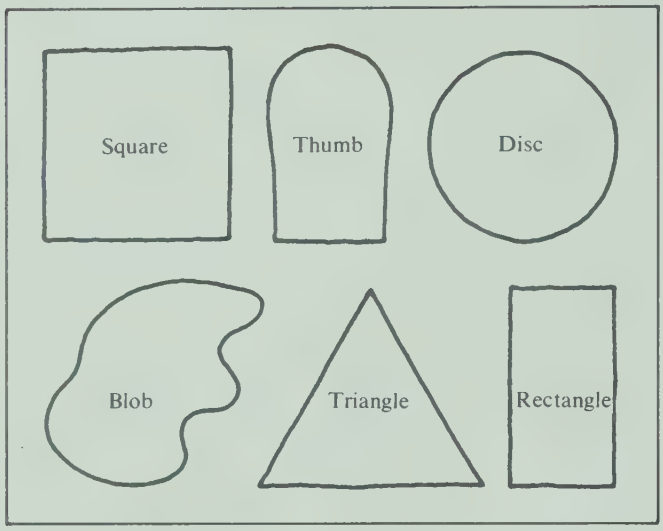
You might say: I am 178 cm tall; what is your height (in centimetres)?

Just as 153 cents is written as \$1.53, 153 centimetres is written as 1.53 metres. This can be interpreted as 1 metre and 53 centimetres which is read as "one point five three" metres. Do not dwell on the mathematical use of the notation—it is not necessary!

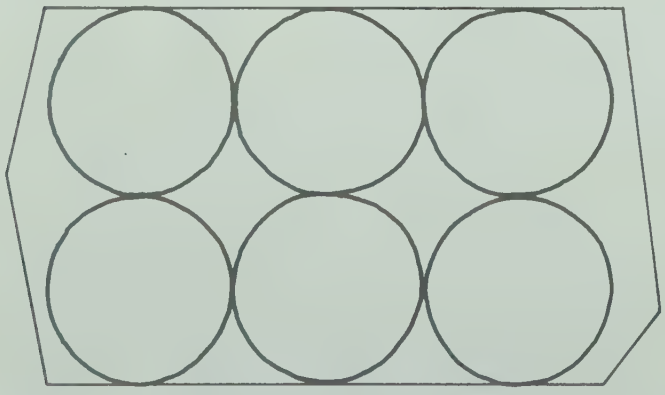
With your class, record the dimensions of your classroom, your desk, their desks, your height, and their heights in terms of centimetres, then in terms of metres using the decimal notation.

ACTIVITY 5

Area using arbitrary units. Here are some examples of area units:



Let the children give names to the units. Then follow the measurement process: select one of these units, match it against the area of some object, and count the number of units used. For example, the irregular figure below has an area of about 6 discs



(if disc is the name given to the unit used). Emphasize that you are trying to "cover" the object. The units should be "even with the edge" of the object, the units should touch, but not overlap, each other. Direct the children's attention to the parts of the object that are not "covered."

Make a cutout of some irregular area such as your thumb and make copies of it out of paper. Use your "thumb" to find the area of the top of a chalk eraser, of the irregular figure measured with the discs, of a cutout of your shoe, and of figure X.

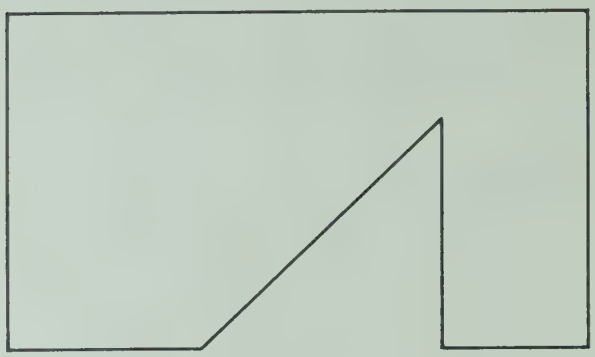


Figure X

Record the answers on the chalkboard in sentence form—

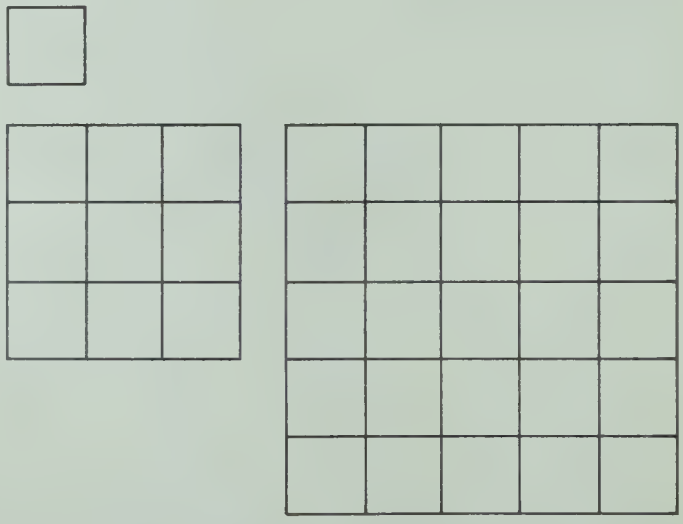
"The figure has an area of about _____ thumbs."

Have your class perform similar activities.

ACTIVITY 6

Area using the centimetre square (cm^2). Have the children make centimetre squares (or have them available for use). The children should have the experience of finding the area of many objects.

Make duplicator masters for some areas that the class can measure with their centimetre squares. The figures below are 1 cm^2, 9 cm^2, 25 cm^2, respectively.



You might point out that the square containing the 9 cm^2 has a side of 3 cm and the square containing the 25 cm^2 has a side of 5 cm.

Have the children use their centimetre squares to find the *area* of a stamp, a 10-cm ruler, the cutout of their thumb, the irregular figure which had an area of 6 discs, and figure X.

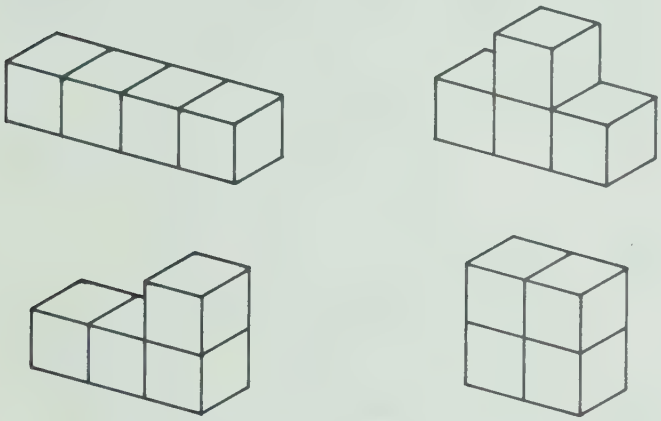
ACTIVITY 7

Volume, using the centimetre cube. In the initial development of the concept of volume, it is important

that children have the opportunity to construct several differently shaped objects each having the same number of volume units.

As with length and area, the study of volume should be introduced with activities making use of arbitrary units of volume, such as blocks, Cuisenaire rods, pencils, erasers, or even marbles.

Use 10 or 12 centimetre cubes in this activity. At first, let the children work on their own, constructing any objects they like. Encourage them to see that an object built of a specific number of cubes has a volume of the same number of cubes regardless of its shape. For example, the illustration shows 4 different constructions, each having a volume of 4 centimetre cubes (4 cm^3).



How many differently shaped objects can be constructed with a volume of 8 centimetre cubes? When those possibilities have been exhausted, try the activity with 10 cubes.

REVIEW: LENGTH, AREA, AND VOLUME

1. Have your class compare the length of their feet, spans, and cubits. Why are these units useless as standard units?
2. Complete these statements.

a.	128 cm = ____ m	**e.**	1.06 m = ____ cm
b.	108 cm = ____ m	**f.**	10.01 m = ____ cm
c.	15 cm = ____ m	**g.**	23.86 m = ____ cm
d.	1010 cm = ____ m	**h.**	0.09 m = ____ cm

3. What would be the length of the sides in a square containing:
   - **a.** 36 cm^2 → ____ cm?
   - **b.** 25 cm^2 → ____ cm?
   - **c.** 4 cm^2 → ____ cm?
   - **d.** 16 cm^2 → ____ cm?
4. How many different-shaped objects can you form with 6 centimetre cubes?

Capacity

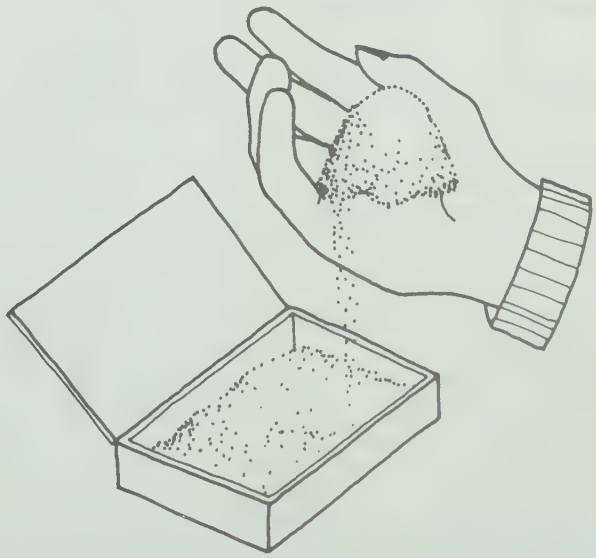
Capacity can be thought of as the amount of material a container will hold. Capacity is usually linked to liquid measure though you may have already had your classes measure capacity by using sand to avoid using liquids.

In the metric system of measurement, volume and capacity are directly related. A container with a volume of 1 cubic centimetre (1 cm^3) will hold 1 millilitre of water. One millilitre (1 ml) is one thousandth of a litre (0.001 ℓ).

The need for fractional names such as $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{4}$ etc. will diminish. The parts of the whole which need emphasis are—0.1, 0.2, 0.3, . . . , 0.9. Of course, in measurement, fractions could disappear completely, since $\frac{3}{8}$ of a meter is 0.375 m or 375 mm. However, when working with the litre (the unit of capacity in the metric system) don't worry now about using $\frac{3}{4}\ \ell$, $\frac{2}{3}\ \ell$, etc. if it is the amount you want the children to see or work with. Since the metric system is based on 10 and since 1, 2, 5 and 10 are the only divisors of 10, we will probably talk about halves, fifths, and tenths of metric units. The decimal notation ($\frac{1}{2}$ is 0.5) will prevail eventually, even at the primary level.

ACTIVITY 8

Capacity and arbitrary units. The most obvious capacity units are handfuls. Give each child a container to fill with water or sand or other material you prefer to use. Have the children fill the container



(milk carton, ice cream carton, cigar box, etc.) with "handfuls" of material. Have them record their results on a piece of paper: "My carton holds _____ handfuls of _____." Compare the wide range of results. Re-emphasize the need for a standard unit to measure capacity. If further experience is necessary, you may want to repeat the project with cups brought from home (since there are so many different sized and shaped cups). Try the activity yourself or get several containers such as an ice cream carton, a milk carton, a wastebasket, a big cooking pan, and a litre container.

On a piece of paper write a pair of sentences for each container:

"The (name of container) holds about (guess) litres.

The (name of the container) actually holds (result) litres.

In the first blank "guestimate" the number of litres the container will hold. In the second, write in the results of measuring the object.

Don't forget the three step measuring process—

1. Select the unit—the litre.
2. Match the unit against the object—fill the object using the litre.
3. Count the number of units (litres) used.

When the container is full (it is best to have a "fill line" just below the top of the container) round off to the nearest whole litre according to whether more or less than half of the last litre was used.

ACTIVITY 9

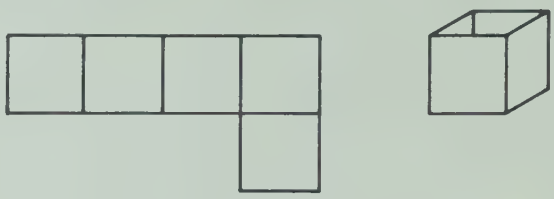
Working with the litre. Get a container that holds a litre of water(and, ideally, has submarkings for each 100 ml). When you are collecting containers for your classroom, try to get as many different shapes as you can. It is important, especially in early experiences, that the children see that litre containers can come in many different shapes. It is the quantity the container will hold, not its shape that determines a capacity of 1 litre.

Once you get a litre container you can make many more. Pour a litre of water into a container and mark a "fill line" for 1 litre on the outside with tape, or, if possible, cut the container so that it holds just 1 litre. Suggested existing containers which can be cut are quart, half-gallon, and gallon milk cartons, round quart, half-gallon, and gallon ice cream cartons. Containers that can be marked might be various shaped pans, cooking bowls, large tin cans, and bottles or jugs. Most activities for introducing the metric units should be accompanied by some estimation exercises. Have the students estimate and record how many litres a container will hold, then measure the container to see about how many litres it does hold. Compare records.

ACTIVITY 10

Introducing the millilitre. The litre is a unit for capacity that is used for milk, gasoline, paint, and other quantities of considerable size. The litre is not used to measure small quantities, such as toothpaste, soda pop, medicines, frozen orange juice, etc. The unit used for the smaller measures is the millilitre (ml). If your school is going to get a set of metric capacity containers, try to get them in these sizes—1 ℓ, 500 ml, 200 ml, 100 ml, 50 ml, 20 ml, and 10 ml. With such a set (whether bought, given, or constructed) one can do all the activities that are necessary.

Construct a container with a volume of 1 cubic centimetre (1 cm^3) to demonstrate the size of the millilitre (ml). Trace the figure below, then cut it out and tape it together along the edges. If you avoid spillage your cube will hold 1 ml of water.



The children need several activities measuring the capacity of objects and recording the results in millilitres. Have them first guess and then measure the capacity of a thimble, a match box, a tablespoon, and a teaspoon. Record the results in sentences like—

"I estimate that the thimble holds about _____ ml.
It actually holds about _____ ml."

Mass

As the metric system becomes the predominant system of measurement you may hear talk about the difference between mass and weight. A lunar example may be the best way to show the difference. Now that we are in the space age, practically everyone knows that a man weighs less on the moon than he does on the earth. For example, a 300-kg man on earth would weigh about 50 kg on the moon, but he would have the same mass on the moon as he does on earth. Weight is dependent upon gravity, mass is not. Begin to stress the use of the correct metric term, mass.

The base unit of mass in the metric system is the kilogram (kg). For example, we say "I have a mass of 78 kg.

ACTIVITY 11

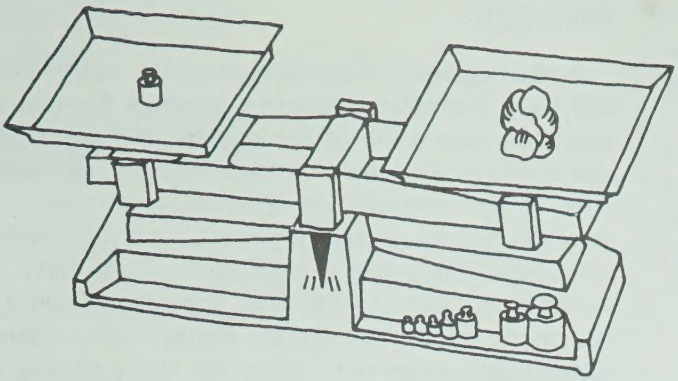
Arbitrary units of mass. To find the mass of an object you will need a balance and some arbitrary units such as paper clips, pencils, Cuisenaire rods, pennies, or other objects. Put a pencil on one side of the beam and then "balance the pencil" with pennies (or multiples of any other small unit). Record the results on paper in a sentence like:

"The pencil has a mass of about _____ pennies."

Repeat the activity with at least three other objects.

ACTIVITY 12

The unit used for small masses is the gram (g). This activity is very similar to the last. You will need gram masses. If you have a classroom set, that's great! If you don't, you can make one.



Put a gram mass on one side of the balance and balance it with a lump of clay or plasticine. Label your clay "1 g." In a similar manner make a set of clay or plasticine "masses" in multiples such as: 5 g, 10 g, 20 g, and 50 g. Use several small objects as test objects (a paper clip, a nickel, a penny, and a pencil). However, before you have the children put one of the test objects on the balance, ask them to estimate its mass in grams. Then find the mass of the object. Record both the guess and the result.

The quarter has a mass of about (guess) grams.

It actually has a mass of (result) grams.

Repeat the activity using other objects. Do you and the class get better at estimating mass?

ACTIVITY 13

Measuring mass using the kilogram. Hopefully, all schools will have metric scales available for finding the mass of children and other large objects using kilograms. For this activity, have each child find his own mass and then make and label a cutout of himself (perhaps using his projected shadow). Have him record his height and mass in metric units on the cutout.

Then you and your class might measure the mass of other objects, such as your own chairs, the textbooks used in the course of one day, litre of water (don't count the container—first find its mass when empty), a dictionary, and even the principal of the school (if he agrees). As mentioned earlier, there is a direct relationship between volume and capacity in the metric system of measurement. In fact, there is a direct relationship between volume, capacity, and mass. A container whose volume is 1 cubic cm (cm^3) holds 1 ml of water and the 1 ml of water has a mass of 1 g. A container whose volume is 1000 cubic cm (or 1 cubic decimetre) holds 1000 ml of water (or 1 litre), and the water has a mass of 1000 g (or 1 kilogram). What did you get for the mass of one litre of water?

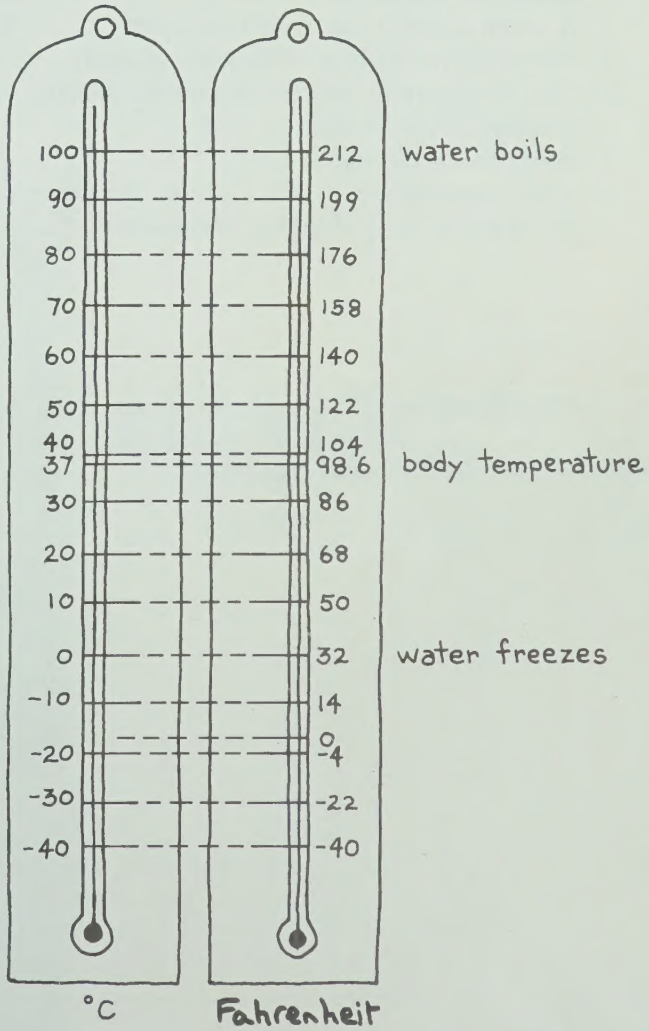
N.B. It is a good idea to label some of the objects in the room as you introduce each metric unit. For example, the aquarium may be 70 cm long, 40 cm wide, 35 cm high; have a water surface area of 2 800 cm^2, volume of 98 000 cm^3; a capacity of 98 ℓ of water and a mass of 12 kg. If the children label the objects as they study particular units, they will begin to think metric.

REVIEW: CAPACITY AND MASS

1. When finding the mass of something using a balance beam, how do you decide which unit to round off to?
2. Fill in the answers:
   - **a.** 28 ml of water has a mass of about _____ grams.
   - **b.** 170 ℓ is _____ ml.
   - **c.** 3.12 kg is _____ g and 438 g or _____ kg.
   - **d.** It would take _____ ml of water to balance 1 kg.
3. **a.** Will a car get a higher or a lower number of miles per litre than miles per gallon? (Is the litre larger or smaller than the gallon?)
   - **b.** Will a car get a higher or a lower number of kilometres per gallon than miles per gallon? (Is the kilometre longer or shorter than the mile?)
   - ★ **c.** Gasoline consumption rates will be given in kilometres per litre. Will a car get a higher or a lower number of kilometres per litre than miles per gallon?

Temperature

This last section covers the introduction of a metric unit, the degree Celsius (°C), for which there is no physical model. On the Celsius scale for temperature, water boils at 100° C and freezes at 0° C. The unit is named after the Swedish scientist, Anders Celsius, who created the centigrade temperature scale. The



Celsi[illegible] tigrade is no longer the proper term since the centigrade is a unit used to measure angles in the metric system.

The best way to get used to the Celsius temperature scale is to use it! It is almost a necessity that you have a Celsius thermometer. However, if you have a demonstration model of the Fahrenheit thermometer, you can rescale it using the nomograph shown here.

ACTIVITY 14

Graphing temperatures. Be sure to give the children lots of opportunities to read the temperature and record it in degrees Celsius (° C). Perhaps you could institute a morning weather report given by a different child each day to get the class to use Celsius thermometers and to give them a feeling for what the temperature is when expressed in degrees Celsius (° C). The previous day's high and low temperatures (taken from a newspaper account) could be recorded on a wall graph.

REVIEW: TEMPERATURE

1. My body temperature is about _____ °C.
2. Normal room temperature is about _____ °C.
3. Water boils at about _____ °C.
4. A warm summer day would be about _____ °C.
5. The temperature in a refrigerator is about ____ °C.
6. The temperature in the vegetable section of a supermarket is about _____ °C.
7. Water freezes at about _____ °C.
8. What temperature on the Celsius thermometer is the same on the Fahrenheit thermometer? ____ °C

Materials

At the beginning of metric conversion, many schools will have a problem gathering supplies. Certain materials are necessities in the teaching of measurement and metric measurement is no exception. Fortunately, most of the materials are inexpensive or easily constructed. In the section on length, the construction of some of the rulers is discussed. If you have one metric ruler, you can construct the rest. If you have one metric ruler, you can also construct the centimetre squares and cubes needed for the study of area and volume.

The construction of units of capacity and mass have also been discussed. When it comes to temperature you should have a thermometer available for classroom use. If it is a Fahrenheit thermometer, then you should rescale it to degree Celsius using the nomograph given earlier.

Following is a list of companies and government agencies that are currently producing materials or can give some assistance with this problem of teaching the metric system of measurement.

Addison-Wesley (Canada) Ltd.—Don Mills, Ontario
Buntin Gillies & Co. Ltd.—Ottawa, Ontario
Cameron Products—Bramalea, Ontario
Canadian Metric Association—(P.O. Box 35)—Fonthill, Ontario
Contrasts 20—Calgary, Edmonton, Vancouver, Winnipeg, Regina (Nearest Barber-Ellis Office)
Kruger Pulp and Paper Ltd.—Moncton, Toronto, Hull, Montreal (Nearest Office)
Information Canada (Under Government of Canada) (Nearest Office)
Jack Hood School Supplies Co. Ltd.—Stratford, Ontario
Lufkin Rule Co. of Canada Ltd.—Don Mills, Ontario
Lily Cups Ltd.—Scarborough, Ontario
MacLean-Hunter Learning Materials Co.—Toronto 101, Ontario
Metric-Aids Ltd.—Toronto, Ontario
Moyer-Vico Ltd.—Moncton Weston, Winnipeg, Saskatoon, Edmonton, Vancouver and the Longueuil Co. in Chambly (Nearest Office)
The National Council of Teachers of Mathematics—1906 Association Drive, Reston, Virginia 22091
Sargent-Welch Scientific Co. of Canada Ltd.—Weston, Ontario
Spectrum Education Ltd.—Toronto, Ontario
Spicars International Ltd.—Scarborough, Ontario
Toronto Dominion Bank (Nearest Office)

...ur	Book Five	Book Six	
	Sets: 4–7 Logic and sets: 8–11 Patterns: 12–17 Number line sequences: 58–61 Union and intersection of sets: 176–179 Sets of equivalent fractions: 190–195 Rational number sequences: 212–213, 232 Sets of ordered pairs: 302–319	Naming sets: 4–5 Classifying attribute pieces: 6–11 Logical reasoning: 10–11 Patterns: 12–15, 216–217 Union and intersection of sets: 130–135 Fractions and sets: 142–145 Sets of equivalent fractions: 144–145 Repeating decimal sequences: 248–249 Sets of ordered pairs: 282, 332	A
Union and intersection of sets: 184, 310 Odd and even numbers: 214–217 Informal logic: 16, 24, 61, 92, 134, 146–155, 156, 189, 296–297	Symbols for numbers: 20–21 Place value concepts: 22–27 Inequalities: 28–31, 214–215 Numeration systems other than base ten: 32–37 Fraction numerals: 186–189 Decimal numerals: 252–257 Integers: 316–319	Symbols for numbers: 18–19 Inequalities: 20–21, 150–153, 214–215, 228–229, 349 Billions: 22 Rounding numbers: 24–25, 234–235 Exponents and scientific notation: 26–29, 34, 96–97, 252–253 Number bases other than base ten: 30–33, 35 Decimals and scientific notation: 252–253 Integers: 340–352	B
Grouping by tens to 4-digit numerals: 30–45, 48 Inequalities: 42–43, 97, 120–121, 282 Place value in computation: 92, 98, 106, 226, 242, 248, 294, 300 Reasoning: 251 Prime numbers: 222–223 Negative numbers: 272–273	Solving equations: 40–43, 46–49, 54–57 Operations related: 44–45 Inverse relation (+ and −): 50–51 Basic facts: 52–53 Basic principles: 62–65 Estimating sums and differences: 104–109 Review of addition and subtraction: 114–121	Review of computation: 36–37, 40–45, 84–89 Basic principles: 38–39 Writing and solving equations: 52–53	C
Addition and subtraction related: 50–57, 66–71 Comparison subtraction: 58–59 Basic principles: 60–65 Differences and missing addends: 66–67 Adding and subtracting 2- and 3-digit numbers: 90–121	Solving equations: 40–43, 46–49, 54–57 Operations related: 44–45 Inverse relation (× and ÷): 50–51 Basic facts: 52–53 Basic principles: 62–65, 122 Estimating products and quotients: 98, 104–109, 155 Special products and quotients: 100–103 Multiplying 2- and 3-digit factors: 124–127 Dividing: 132–147, 156–163 Money problems: 164–165	Review of multiplication skills: 36–37, 40–45, 90–97 Basic principles: 38–39, 50–51 Special products and quotients: 46–49 Writing and solving equations: 52–53 Review of dividing skills: 100–115 Money problems: 116–117	D
Concepts of multiplication: 122, 162–163 Basic principles: 138–145, 236–239 Multiplication facts: 146–155 Pairing and multiplication: 162–163 Concepts of division: 170–181, 186–189, 276–281, 284–291 Multiples and factors: 218–221 Multiplying with multiples of 10: 226–231 Multiplication algorithm: 240–249 Division algorithm: 294–303 Remainders in division: 300–303	Meaning of fractions: 186–189, 198–199 Equivalent fractions: 190–197 Lowest terms fractions: 200–201 Concept of fractional numbers: 208–213 Inequalities: 214–217 Ratio: 218–219 Addition and subtraction using fractions: 226–245 Fractional numbers in measurement: 246–247 Decimal numerals: 252–257 Addition and subtraction using decimals: 258–261 Decimals and the metric system: 262–263 Multiplication using fractions: 288–291 Basic principles: 240–241, 292–293 Division using fractions: 294–296	Review of fractions: 142–143 Equivalent fractions: 144–145, 150–151 Lowest terms fractions: 146–147 Concept of fractional numbers: 148–149 Inequalities: 152–153, 214–215, 228–229 Addition and subtraction of fractional numbers: 156–175 Basic principles: 168–169, 186–187, 192–193 Mixed numerals and improper fractions: 166–177 Multiplication of fractional numbers: 182–200 Reciprocals: 194–195 Division of fractional numbers: 202–211 Decimal names for fractional numbers: 224–227, 248–251 Basic operations with decimals: 230–251 Decimals and scientific notation: 252–253 Ratio and percent: 304–327	E
Halves, fourths, eighths: 14–15, 18–23			
Science: 59, 110, 232–233, 243, 247, 249 Social Studies: 47, 102–103, 111, 159, 161, 165, 191, 291, 308 Measurement: 27, 46, 108, 115, 130–131, 234, 235, 245, 283, 293, 299 Picture Stories: 134, 156, 189, 190, 296–297 Short Stories: 23, 67, 117, 134, 157, 173, 193, 241, 253, 290, 298, 307 Miscellaneous: 119, 164, 177, 305	Science: 130, 143, 154, 167, 169, 249, 265, 267, 301 Social Studies: 113, 120–121, 126–127, 131, 151, 158–159, 220–221, 225, 251 Measurement: 104, 107, 108–109, 119, 217 Picture Stories: 207 Short Stories: 142, 20... 245, 297, 299 Miscellaneous: 43, ... 239, 261, 332	Science: 28–29, 94–95, 99, 108–109, 174, 198–199, 200, 210, 213, 233, 240, 247, 255, 257, 281, 309, 329 Social Studies: 24–25, 86–87, 123, 141, 175, 181, 211, 241, 320–321 Measurement: 114–115, 173, 219, 221, 277, 311 Short Stories: 89, 118, 153, 164, 172, 179, 191, 209, 218, 223, 232, 246, 307, 317	F
Measurement (not exact): 12–15 Estimating area: 18 Estimating weights: 108–109 Estimating products: 250–251	Estimating length: 8–9 Estimating perimeter: 14 Estimating 1000 things: 40–41 Meaning of estimation: 170–171 Estimating sums and differences: 172–173 Estimating products and quotients: 174–177 Estimation in story problems: 178–179 Using estimation to find quotients: 224–225, 228–229, 234–235	In measurement: 7... Story problems: 1... Rounding number... Estimating sums, ... quotients: 98–9... Estimating to help... 144–147, 155	
Odd and even numbers: 214–217 Multiples and factors: 218–221 Prime numbers: 222–223	Even and odd numbers: 250–251 Factors and products: 252–253 Greatest common factor: 254–255 Prime numbers: 256–259	Factors of a numb... Factor trees and ... Greatest common... Least common m... Least common de... Modular arithmet...	
Concept of measurement: 4–29 Centimetres: 8–15, 20–21 Area (square centimetres): 16–19, 20–21 Volume: 24–25 Litres: 26–27 Time: 245	Linear measure: 6–13, 30, 339 Perimeter: 14–17, 349 Area: 20–25, 194 Volume: 26–27, 195 Liquid measure: 28–29	Length: 72–73, 24... Angle measure: 7... Areas and perime... Decimals and the... Volume and surfa... figures: 276–28...	
Simple geometric figures: 74–79 Angles and triangles: 80–87 Parallel lines: 196–199 Angles: 80, 84, 198–199 Polygons: 200–207 Symmetry: 210–211, 264–265 Graphing: 256–263, 268–269, 272–273 Translations: 266–267 Circles: 208 Reading a chart: 108–111 Maps as scale drawings: 165, 283 Bar graphs: 270–271	Space figures: 98–103, 106–107, 206–207 Points, angles, lines, line segments: 102-105 Simple closed curves: 108-109 Polygons: 110–111 Symmetry: 112-113, 216–217 Circles: 208–213 Congruent figures: 214–217 Co-ordinate geometry: 296–313 Transformations: 304–307 Graphing negative numbers: 312–313 Maps: 18–19, 293, 317 Bar graphs: 91, 189, 308–309 Charts: 346	Number line: 58–... 226–229, 289–2... Points, lines, segr... Congruent figures... Angles: 76–81 Symmetry: 82–83... Perimeter and are... polygons: 88–9... Space figures: 26... Graphing points ... figures: 302–30... Relations, transla... enlargements:... Tessellations: 31... Graphing integer... Charts: 120, 121, ... Bar and circle gr... Ratio and scale d...	
Intuitive introduction to functions: 70–71, 150, 185 Graphing: 256–274 Negative numbers: 272–273 Mathematical Activities: 309–317	Intuitive work with functions: 64, 144–145, 258 Intuitive work with number pairs: 262–267 Distance on maps: 18–19, 156 Mathematical Activities: 347–355	Functions: 46, 54... Numeration syste... Average of a set ... Time, rate, and d... Maps: 220–221 Tessellations: 31... Negative numbe... Modular (clock) ... Mathematical Ac...	

Continued from inside front cover]

ISM

E D'DAFFER • FLEENOR

ADDISON-WESLEY (CANADA) LIMITED